A novel method to estimate temperature gradients in stellar photospheres

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Abstract. Inversions utilizing one-dimensional atmospheric models provide information about the thermal stratification of stars, but these models are in general not unique nor sufficiently descriptive of the physical conditions of a star. Here we propose a novel model-independent method to better constrain the temperature stratification in a stellar atmosphere. In our method we employ intensities measured at opacity conjugate wavelength pairs to improve the estimate of temperature stratification that is obtained from radiation temperatures in combination with the Eddington–Barbier relation. This relation can lead to significant errors because of the non-linear dependence of the source function on optical depth, even in the case of continua. Such errors are substantially reduced by combining observations at pairs of conjugate continua, which have the same $H^+$ opacity between them, and therefore pairwise form at the same height.

Key words. Stars: photosphere – Stars: atmospheres

1. Introduction

The shape of the spectrum of a star is strongly dependent on the temperature stratification of its atmosphere. Usually, a mean atmospheric stratification is derived by interpreting the observed spectrum in the context of a one-dimensional static atmospheric model. However, such a description might not be unique nor sufficiently descriptive of the physical conditions of a star (Uitenbroek & Criscuoli 2011), as it is well known, for instance, in the case of the determination of chemical compositions of stellar atmospheres (Asplund et al. 2009).

Because of the strong dependence of the spectrum on the atmospheric stratification, estimates of temperature gradients are important for understanding the physical properties of a star, but are hard to make. For instance, in spite of the Sun’s vicinity to the Earth, the temperature stratifications of typical magnetic features observed on the solar surface are still uncertain, despite the fact that such temperature gradients of magnetic features play an important role in determining irradiance variations. In particular, temporal variations of the temperature gradient in the solar photosphere, induced by the variations of the magnetic filling factor, have recently been suggested to be the cause of temporal variations of solar irradiance measured by radiometers on-board the Solar Radiation and Climate Experiment (SORCE) in the NIR and other visible spectral ranges (Harder et al. 2009; Fontenla et al. 2011).
Verification of such hypotheses can rely, so far, only on reconstruction of irradiance variations through one-dimensional semi-empirical atmosphere models that schematically represent the physical properties of magnetic features, the appearance of which on the solar disk is known to modulate the irradiance signal (e.g. Domingo et al., 2009). Measurements of the temperature gradient within different magnetic structures would allow to validate such atmospheric representations, whereas measurements over the cycle of the variations of these gradients in turn would improve our understanding of the contribution that the different magnetic features provide to irradiance variations.

In their simplest form model-independent estimates of temperatures and temperature gradients are derived from photometrically accurate observations in continua formed at different heights, followed by inversion of the Planck function supplemented by the Eddington-Barbier approximation, which equates the emergent intensity to the source function at optical depth unity, and assumes a linear dependence of that source function on optical depth. Here we employ results from numerical Magneto-Hydro Dynamics (MHD) simulations to show that such method is prone to systematic errors, which can fortunately be reduced employing photometric measurements at two pairs of opacity-conjugate wavelengths. A similar method has previously been applied by Foukal & Duvall (1985).

2. The method

In solar like stars the main source of opacity is the $H^+$ ion, of which the absorption coefficient is shown as a function of wavelength in Fig.1. The absorption coefficient shows a maximum at $\approx 862$ nm, a minimum at $\approx 1600$ nm and a monotonic increase at longer wavelengths. Because of this peculiar dependence on wavelength, pairs of wavelengths exist, in the visible and NIR range, that are characterized by the same opacity and that therefore form at approximately the same height in the photosphere. Two of these pairs are illustrated in Fig.1 we notice that they are almost symmetrically located with respect to the maximum, at 862 nm. One then expects that temperature gradients derived from intensity measurements at the two wavelengths on the 'blue' side of 862 nm are the same as those derived from the two continua on the 'red' side of the maximum. In the following we show that these measurements show instead large differences because the shape of the source function around optical depth unity plays a role in the emergent intensity, not only its value at that location. We show that these estimation errors can be reduced by taking the average of the temperature gradients obtained from emergent intensity values at the 'blue' and 'red' sides of the $H^+$ opacity maximum.

2.1. Numerical simulations

To illustrate our method and investigate its feasibility we have analyzed synthetic solar radiation at the two opacity conjugate wavelengths pairs 668.7–1054.6 nm and 506.2–1240.8 nm, respectively, obtained through a snapshot from a MHD simulation (Stein & Nordlund, 1998) of the solar photosphere. The synthesis was performed in Local Thermodynamic Equilibrium with the RH code (Uitenbroek, 2002, 2003). Figure 2 shows the differences between the heights of optical depth unity for the two wavelengths in each of the two conjugates pairs. As expected, the differences are of the order of a few kilometers. Because of the slight dependence of the $H^+$ absorption coefficient on temperature, the opacity conjugate wavelength determination changes with height, resulting in slightly larger differences in formation height for the pair that forms deeper in the atmosphere (506–1241), where temperature variations are larger.

2.2. Validity of the Eddington-Barbier approximation

Under the Eddington-Barbier approximation the emergent intensity is equal to the source function (i.e., Planck function under LTE conditions) at optical depth unity. However, be-
cause the condition of linear dependency of the source function on optical depth in the Eddington–Barbier relation is in general not exactly fulfilled, other layers contribute in a non-negligible way to the emergent intensity. This is illustrated in Fig. 3 which shows the average contribution functions at 506 and 1240 nm within the analyzed MHD snapshot. We notice that the two contribution functions peak at approximately the same height but that the one for the 1240 nm continuum shows a larger contribution toward the higher layers of the photosphere. As a result, the radiative temperature \( T_{\text{rad}} \), i.e., the temperature estimated from the emergent intensity by inversion of the Plank function, and the temperature at optical depth unity \( T_{\text{form}} \) are not identical. This is illustrated in top panel of Fig. 4 which shows the scatter plot of \( T_{\text{rad}} \) and \( T_{\text{form}} \) for the 506 nm continuum. Because of the contribution of the higher layers of the photosphere, where the temperature is lower than in the deeper layers, to the emergent intensity, \( T_{\text{rad}} \) is systematically smaller than \( T_{\text{form}} \). Note that the largest difference we measured is below 3%. It is also worth to note that differences between the two temperatures are not constant; this is due to the fact that each spatial point in the simulation is characterized by different physical conditions and, therefore results in a different shape of the source function. For the same reason, although the \( T_{\text{form}} \) values at conjugate pairs show a very good agreement (as illustrated in the middle panel of Fig. 4 for the pair at 506-1240 nm, and as expected from Fig. 2) the corresponding \( T_{\text{rad}} \) values show larger differences (they do not exceed 4% for both pairs), as shown in the bottom plot. Note that the temperature derived from the 506 nm continuum is always larger than that derived from 1240 nm, because of the largest contribution of higher layers of the photosphere at this latter wavelength, as discussed above.
2.3. Temperature Gradients

Although the differences between $T_{\text{rad}}$ and $T_{\text{form}}$ for a given continuum, and between the $T_{\text{rad}}$ values derived from conjugate pairs are, overall, rather small (of order a few percent), such differences introduce large uncertainties in estimates of temperature gradients, which involve temperature differences between different heights. Figure 5 shows the temperature gradients computed from $T_{\text{rad}}$ values derived from the 506 and 668 nm intensity continua (black symbols) and the gradients derived from the 1054 and 1240 nm intensity continua (red symbols), respectively, versus the gradients computed from the corresponding $T_{\text{form}}$ values. In all cases the geometric heights employed for the computation of the gradients are those at unit optical depth. We notice a large spread of values, while the differences between temperature gradients derived from emergent intensity and those from temperatures at optical depth unity can be as large as 50%. Nevertheless, we notice that the temperature gradient estimated from the radiation temperature in the ‘blue’ 506-668 nm continua are mostly larger than the corresponding gradients derived from temperatures at optical depth unity, while they are mostly smaller in the case of temperatures derived from the ‘red’ 1054-1240 nm intensity continua. Both the systematic deviations, and the spread of results are reduced if we compute the geometric average of temperature gradients obtained from the $T_{\text{rad}}$ in the ‘blue’ and ‘red’ continua, as shown by gray symbols in Fig. 5.

3. Discussions and conclusions

Results obtained from continuum intensities synthesized through a 3-D MHD simulation snapshot show that the temperature gradients derived from emergent intensity under the Eddington-Barbier approximation are affected by systematic deviations compared to those obtained from temperatures at optical depth unity at the corresponding wavelengths. These devi-
Fig. 4. Top: scatter plots of radiative temperatures and temperatures at formation at optical depth unity for the 506 nm continuum. Middle: scatter plots of temperatures at optical depth unity for the conjugate pair 506-1240 nm. Bottom: scatter plot of radiative temperatures for the conjugate pair 506-1240 nm.

Fig. 5. Scatter plot of temperature gradients derived from $T_{\text{form}}$ values versus temperature gradients derived from $T_{\text{rad}}$ values. Black: gradients computed from 506-668 nm continua. Red: gradients computed from 1054-1240 nm continua. Grey: gradients computed as arithmetic average of gradients derived from 506-668 nm continua and 1054-1240 nm continua.

Measurements are introduced by the fact that several layers of the photosphere contribute to the observed emergent intensity, not only the layer strictly at optical depth unity. These deviations depend on the way in which the source function at the wavelength under consideration behaves, resulting in differences in emergent intensity even for continua that are expected to form at the same heights because they are characterized by the same $H^-$ opacity. Measuring errors are reduced if gradients are computed by combining radiative temperatures obtained from continuum observations at pairs of opacity conjugate wavelengths.

It is important to note that the determination of the temperature gradient in terms of K/km, in practice, must rely on some modeling in order to associate geometric heights to the differences in radiative temperature, thus introducing some model dependence. However, the difference in the average radiation temperature between two opacity conjugate pairs can be directly compared to those derived from emergent intensities in simulations. The simulations also tell us that the difference between the actual temperature at the formation height of the two pairs is more closely related to the difference between the average radiation temperatures (over the blue and red member of each
pair) than the difference in radiation temperature between the red or blue members of each pair individually.

In our example here we have diagnosed radiation temperatures at disk center, and compared them with actual temperatures in the simulation. When dealing with (spatially unresolved) stellar spectra we have to work with fluxes. More modeling work is needed to assess if a similar method can be worked out for such disk integrated spectra, and how radiation temperatures at opacity conjugate wavelengths relate to actual temperatures.

References