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Overshoot inwards from the bottom of the intershell convective zone in (S)AGB stars

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Abstract. We estimate the extent of overshooting inwards from the bottom of the intershell convective zone in thermal pulses in (S)AGB stars. We find that the buoyancy is so strong that any overshooting should be negligible. The temperature inversion at the bottom of the convective zone adds to the stability of the region. Any mixing that occurs in this region is highly unlikely to be due to convective overshooting, and so must be due to another process.

Key words. Stars: abundances – Stars: AGB – Stars: Evolution – Stars: mixing

1. Introduction

Thermal pulses on the AGB and SAGB drive a convective zone that reaches from the flashing helium shell almost to the convective envelope. This is known as the Inter-Shell Convective Zone (ISCZ). The composition of this region is mostly ashes from the hydrogen shell mixed with the products of the rapid He burning that is driving the convection. Typically we have about 70% He and 25% C. This material is later incorporated into the envelope by third dredge-up, and is responsible for much of the evolution of the surface composition of AGB stars.

2. Overshooting and its consequences

The Schwarzschild criterion for convection is commonly used to determine the borders of convective regions. However it simply finds a radial level where the buoyancy (the acceleration) is zero. Any fluid moving within the convective region toward the boundary reaches the boundary with a finite velocity. Conservation of momentum ensures that the fluid travels beyond the Schwarzschild border, and this is the original definition of overshooting. Whilst the existence of overshooting is clear, it's *extent* is another matter. It is common to use the diffusion equation to calculate the mixing in convective regions. This is not strictly correct because mixing is *advective* and not *diffusive*. Nevertheless, this provides a way to calculate the composition in a region where the mixing is not sufficient to produce homogeneity. The mixing is determined by the diffusion parameter D that is used in the diffusion equation

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = \left(\frac{\partial X_i}{\partial t}\right)_{\mathrm{nuc}} + \frac{\partial}{\partial M_r}\left[(4\pi r^2\rho)^2 D\frac{\partial X_i}{\partial M_r}\right]. \quad (1)$$

A simple way to include overshooting is to determine a (non-zero) D outside the convective region. The pioneering work by Herwig et al. (1997) was motivated by multi-dimensional hydrodynamical calculations of Freytag et al. (1996) which found a roughly exponential fall off in the velocity beyond the Schwarzschild boundary. This was implemented by using $D = D_{\rm os}$, where

$$D_{\rm os} = D_0 \exp\left(\frac{-2z}{H_{\rm V}}\right) \qquad (2)$$

and D_0 is the diffusion co-efficient near the Schwarzschild boundary. Here $D_0 = v_0 H_{\rm P}$ and the velocity scale-height $H_{\rm V}$ is assumed to be $H_{\rm V} = f H_{\rm P}$ where f is a constant. This overshooting algorithm was applied to the ISCZ in AGB stars by Herwig et al. (1997). Mixing into the CO core alters the composition of the ISCZ, increasing the C and O mass fractions. Indeed, Herwig et al. (1997) found that the typical intershell mass fractions changed from He:C:O $\simeq$ 70:25:1 to 25:50:25. This was found to be a better match to the observed abundances of central stars of planetary nebulae and the PG1159 stars (Herwig 2000). But there were other changes, produced by the feedback of the mixing on the stellar structure. First the extent of the dredge-up, as measured by the dredge-up parameter λ, changed from about $0.7 - 0.8$ without overshooting to values exceeding unity when overshooting was applied. This has a significant effect on both the structural and chemical evolution of the star. One very significant effect is seen in the production of s-process elements. In low- and intermediate mass stars we believe that the neutron source is ^{13}C, produced by partial mixing of protons inward from the bottom of the convective envelope during third dredge-up events (for a recent review see Karakas and Lattanzio 2014) . At higher masses the neutrons are thought to be provided by a different source: CNO cycling in the hydrogen shell produces ^{14}N. When engulfed by the ISCZ at the next pulse, two α-captures on ^{14}N produce ^{22}Ne. At sufficiently high temperatures, exceeding say 300 MK, a further α-capture can produce neutrons via ^{22}Ne$(\alpha,$n$)^{25}$Mg.

In the case of overshooting inwards from the ISCZ, the convective region is extended into the hot core with the result that the ISCZ experiences burning at higher temperatures. This was shown by Lugaro et al. (2003) to produce significantly different abundances of s-process elements. The higher neutron densities also affect some branching points in the s-process path with the result that some ratios no longer seem to match the observations (see Lugaro et al. 2003, for details).

3. Physics of the region

Thermal pulses drive the ISCZ, and the rapid injection of energy raises the temperature locally. This produces a discontinuity in the temperature (and so also the density) at the bottom of the ISCZ and also a temperature inversion. This is shown in Fig. 1, based on figure 5 of Herwig et al. (2006) who investigated the hydrodynamics of the ISCZ in 2D (and in one low resolution 3D calculation). They found that mixing outside the formal convective region was "orders of magnitude less efficient" and showed an "exponential decay" in moving away from the ISCZ. The calculations were not able to follow the evolution for very long, but did find that "mixing of material into the convection zone from below … shows a significant upward trend at late times (after several convective turnover times)."

We note that the temperature inversion means that the radiative temperature gradient is negative. When subjected to a standard Schwarzschild analysis we find that the region is unconditionally stable because the temperature decreases when the density increases. Of course this is already known, because "outside the unstable region the material is stable" (Denissenkov, private communication). The question is one of how efficient is the mixing at the bottom of the ISCZ. We have calculated the evolution of a $3M_\odot$ model of solar metallicity using the Monstar evolution code (e.g. Gil-Pons et al. 2013). We calculated two cases, one with no overshooting and one with the Herwig et al. (1997) prescription with $f = 0.02$. Fig. 2 shows the bottom of the ISCZ near the maximum of the 13$^{\rm th}$ pulse.

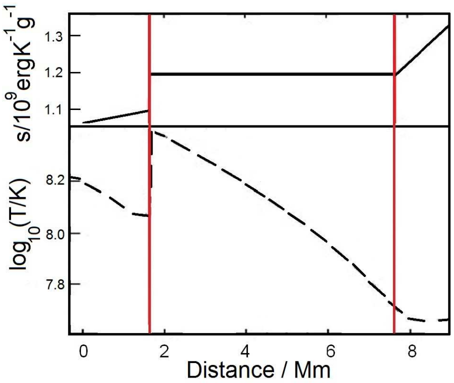

Fig. 1. Structure of the ISCZ during a thermal pulse. The top panel shows the entropy and the bottom panel shows the temperature distribution. The red lines show the Schwarzschild borders of convection. This plot is based on figure 5 of Herwig et al. (2006) for a $2M_\odot$ model with $Z = 0.01$ and $f = 0.016$.

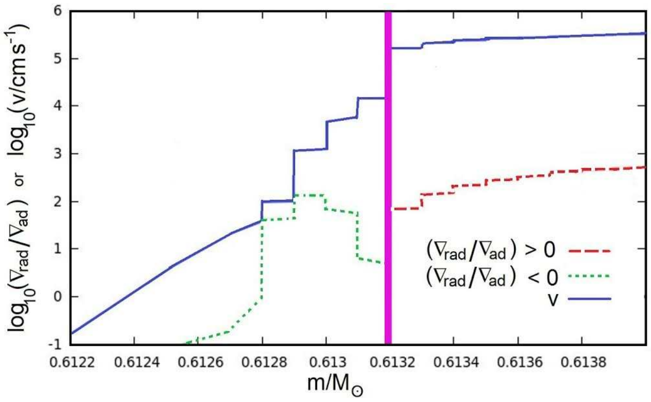

Fig. 2. Structure at the bottom of the ISCZ near the maximum strength of the 13th pulse for the model described in the text. The blue line shows log of the velocity while the red and green lines show log of the magnitude of $\nabla_{\rm rad}/\nabla_{\rm ad}$, coloured red and green where the ratio is positive and negative respectively. Thus the green line shows the region of the temperature inversion. The magenta line shows the Schwarzschild boundary.

Note that the overshoot algorithm produces a significant velocity in the region of the temperature inversion. The buoyancy here is opposed to the penetration of the convective motion, but the buoyancy is not included in the algorithm which simply assigns a velocity. Does a naive application of the overshooting formula produce mixing that is consistent with the structure? We now try to estimate the strength of the restoring force that opposes the mixing enforced by the algorithm.

4. Estimates of the overshoot

We make some basic estimates of the extent of the overshooting inward from the bottom of the ISCZ. Consider an eddy of volume $V_{\rm e}(r)$ and density $\rho_{\rm e}(r)$ at the bottom of the ISCZ. Let the

Table 1. Estimates of the extent of overshooting

Model	Phase	Dist. (m)
150	Start of pulse	1
170	Middle of pulse	6
1200	Maximum of pulse	200

background density of the star be $\rho(r)$. Hence the eddy will feel a restoring buoyancy force $F_\mathrm{b} = g\,\Delta m$ where $\Delta m = (\rho(r) - \rho_\mathrm{e}(r))\,V_\mathrm{e}$ and the equation of motion is

$$F = \rho_\mathrm{e} V_\mathrm{e} g = g\,\Delta m$$

so that

$$a_\mathrm{e} = \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{2}v^2\right) = g(r)\left(\frac{\rho(r) - \rho_\mathrm{e}(r)}{\rho_\mathrm{e}(r)}\right).$$

Integrating from the bottom of the ISCZ, where $r = r_0$ and $v = v_0$ to the point r_1 where $v_1 = 0$ we find

$$\int_{v_0}^{v_1} \mathrm{d}\left(\frac{1}{2}v^2\right) = \frac{1}{2}v_0^2 = \int_{r_0}^{r_1} g(r)\left[\frac{\rho(r) - \rho_\mathrm{e}(r)}{\rho_\mathrm{e}(r)}\right]\mathrm{d}r.$$

We make the usual assumption that the eddy moves adiabatically. Our procedure is to integrate the RHS downward until we reach a position r_1 where the integral is equal to $\frac{1}{2}v_0^2$. The depth r_1 is then the radial extent of the overshooting before buoyancy stops the eddy. We ignore the energy required to move material out of the path of the eddy, so our estimate for r_1 is a maximum. Further, for the velocity v_0 at the bottom of the ISCZ we take the largest velocity in the ISCZ, so that again our r_1 is a maximum.

We performed this procedure at three times during the 13^{th} pulse. The results are shown in Table 1. In each case the extent of the overshooting is less than one mesh point in the 1D model. Of course, if we were to resolve this region (and the tiny change of composition) it may cause some feedback such that these estimates are inaccurate. There are also many approximations and idealizations in the calculation. Nevertheless, it appears that the eddy faces a very strong restoring force below the ISCZ, largely due to the temperature inversion. It seems that the eddy finds it very difficult to mix far enough to change the composition in the way that seems favoured by comparisons with the composition of PG1159 stars for example.

5. Conclusion

Mixing within stars continues to be a problem. How to handle the expected overshooting, entrainment and other mixing processes at convective borders is a particularly pressing problem. It affects most phases of evolution. Together we call these processes convective border mixing. It is perhaps a more useful term than overshooting which may be just one of the processes taking place. We have investigated the effect of buoyant overshooting at the bottom of the ISCZ during thermal pulses in AGB stars. Our idealized 1D analytic theory argues for a small to negligible extent of mixing owing to *overshooting*.

We note that models which enforce mixing, despite the arguments presented in this paper, seem to match the observations better. How do we understand this? While we feel rather safe in concluding that momentum-based *overshooting* is unlikely to produce substantial mixing, we cannot rule out some other mixing process, such as gravity waves of shear instabilities, as being more effective. Perhaps the evidence is indicating that another process is responsible for the required mixing. In any case, this does not seem to be classical overshooting.

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