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Rotating and anisotropic models of globular clusters

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Abstract. We present a review of selected equilibrium models for the study of the structure and dynamics of globular clusters, with emphasis on the role of rotation and pressure anisotropy.

Key words. Galaxy: globular clusters – methods: analytical

1. Introduction

As a zeroth-order dynamical description, globular clusters may be considered as quasi-stationary systems, close to a thermodynamically relaxed state. These assumptions allows us to describe them in terms of a phase space distribution function that depends only on the single-star energy, thus characterized by isotropy in the velocity space. The additional assumption of spherical symmetry greatly simplifies the formal construction of the models.

Within the set of assumptions described above, a number of simple equilibrium self-consistent models can be defined. Among the historical results, polytropic spheres played an important role, but the most popular family in the class of quasi-relaxed models for globular clusters is defined as a Maxwellian distribution function, characterized by the presence of a truncation which is continuous in energy (King 1966). Such a phase space truncation prescription is non-unique, and it is essential in shaping the behavior of the potential, and, therefore, of all velocity moments, especially in the proximity of the truncation radius (Hunter 1977; Davoust 1977).

This class of models has been the backbone of our current understanding of the dynamics of globular clusters, but the soon available full phase space information (i.e., the synergy between Gaia and HST proper motion studies and ground-based spectroscopic surveys) *screams* for a proper treatment of some physical ingredients traditionally considered as "second order complications". More realistic equilibrium models are a possible tool to fulfill such a need.

Broadly speaking, two complementary paths can be followed for the construction of equilibrium dynamical models. In the first, "descriptive" approach, under suitable geometrical (on the intrinsic shape) and dynamical (e.g., on the absence or presence of dark matter) hypotheses, the available data for a given stellar system are imposed as constraints to derive the internal orbital structure (or phase space distribution function). This approach is often carried out in terms of schemes that generalize a method introduced by Schwarzschild (1979). In the second, "predictive" approach, one proposes a formation/evolution scenario in order to identify a physically justified distribution function for a class of systems, and then proceeds to investigate, ideally by comparison with observations of several individual objects, whether the data support the general physical picture that has been proposed. In this paper, we will focus mostly on the second approach, with the goal of reviewing some of the efforts that have been put forward by the communities studying the dynamics of globular clusters and galaxies, with emphasis of two physical ingredients: rotation and pressure anisotropy.

This paper should be considered as a summary of the narrative underlying the contribution presented at MODEST 16, and we encourage the reader to refer to the presentation slides to consult the appropriate equations, figures (and portraits!), which are omitted here.

2. Rotation

In consideration of the fact that globular clusters exhibit only modest amounts of flattening and given the success of the spherical, lowered isothermal models, relatively little work has been carried out in the direction of the construction of stationary self-consistent rotating models specifically designed for this class of stellar systems (with some exceptions, which we will mention below). Therefore, much of the currently available knowledge of the structure and dynamics of rotating stellar systems stems from studies originally intended to investigate the properties of elliptical galaxies.

Equilibrium models of stellar systems with non-vanishing total angular momentum may be classified into two fundamental categories, as characterized by rigid or differential rotation, respectively. In the first case, it is well known that in the presence of finite total angular momentum of the system, relaxation leads to solid-body rotation (e.g., see Landau & Lifchitz 1967), i.e., in the statistical mechanical argument that leads to the derivation of the Maxwellian distribution, one finds that, in the final distribution function, the single star energy is replaced by the Jacobi integral. Following this picture, we may consider the extension of many simple isothermal equilibria to the case of internal rigid rotation (for models with discontinuous and continuous truncation, see Kormendy & Anand 1971 and Varri & Bertin 2012, respectively). One of the beauties of this class of models lies in the fact that they may be constructed by means of an (almost completely analytical) perturbation approach. The construction of rigidly rotating configurations characterized by nonuniform density is indeed a classical problem in the theory of rotating stars (Milne 1923; Chandrasekhar 1933), but it is mostly limited to the study of fluid systems with a polytropic equation of state. In this context, a valuable contribution is represented by the work by Vandervoort (1980) on the collisionless analogues of polytropes, as well as the Maclaurin and Jacobi ellipsoids.

On the side of differentially rotating equilibria (which, almost inevitably, translates into models characterized by both rotation *and* pressure anisotropy), many of the currently available modeling tools go back to the pioneering work of Prendergast & Tomer (1970) and Wilson (1975), intended to describe ellipticals, and of Jarvis & Freeman (1985) and Rowley (1988), devoted to bulges. In all cases, the models are self-consistent, i.e., they are obtained by solving the relevant Poisson equation, often by means of an iterative approach; such a scheme typically requires, at each iteration step, the expansion in Legendre series of the density and the potential, and the associated radial Cauchy problems are often expressed in integral form.

Since the family of models initially proposed by Jarvis & Freeman (1985) reduces, in the limit of vanishing rotation, to King (1966) models, they have naturally become the preferred choice for the initial conditions of many numerical investigations of the role of angular momentum in the dynamical evolution of globular clusters (see Lagoute & Longaretti 1996; Einsel & Spurzem 1999, and many other Fokker-Planck and N-body studies). A remarkable effort for the construction of a family of models specifically designed for the description of globular clusters, and characterized by three integrals of the motion, has been put forward by Lupton & Gunn (1987), with an application to M13. More recent examples include the two-integral equilibrium models presented by Varri & Bertin (2012), with applications to selected Galactic globulars (Bianchini et al. 2013). An alternative approach for the construction of phase space equilibria is based on the use of actions instead of integrals of the motion; a refreshing example of this line of attack, with a possible application to rotating systems, has been proposed by Posti et al. (2015).

3. Anisotropy

The dynamical interpretation of globular clusters as quasi-relaxed stellar systems present some limitations also with respect to the assumptions on the velocity dispersion tensor. In this respect, possible deviations from isotropy in the velocity space may be explored along two complementary lines of argument. On the one hand, after many decades of progressively more realistic numerical simulations, it is well known that the dynamical evolution of collisional stellar systems, as driven by internal and external processes, has a strong effect on their phase space properties. On the other hand, we might also ask whether, especially for globular clusters characterized by long relaxation times, any signature of their formation process may actually be preserved in phase space.

In the first case, pioneering numerical investigations have shown that anisotropy is indeed a natural outcome of star cluster evolution, especially when the system is in isolation (Hénon 1971; Spitzer & Shapiro 1972). Spitzer (1987) showed that during their evolution, isolated globular clusters develop a structure composed by two distinct regions: an isotropic core, and a radially anisotropic halo of stars, resulting from the scattering of stars from the center, preferentially on radial orbits. Idealized models of star clusters based on gaseous and N-body methods have later confirmed that isolated systems tend to become progressively more anisotropic in their outer regions (e.g., see Bettwieser & Spurzem 1986; Giersz & Spurzem 1994).

This picture has been extended with the inclusion in the models of the presence of an external tidal field, the effect of which is typically to curb the degree of radial anisotropy developed, most likely as a result of the mass loss, which progressively exposes deeper and therefore more isotropic shells of the stellar system (e.g., see Giersz & Heggie 1997). As a result, the anisotropy profiles remains radially-biased at intermediate radii, while it becomes isotropic (or even mildly tangential, see Baumgardt & Makino 2003) in the outer regions. Given these two limiting cases, it should not come as a surprise that the degree of radial anisotropy developed in a system strongly depends on the strength of the tidal field in which it is evolving (Tiongco et al. 2016).

On the side of distribution function-based models, these ideas have inspired the construction of simple equilibria, defined as a direct generalization of lowered isothermal models, in which the second integral of the motion is considered to be the specific angular momentum, often introduced via an exponential dependence (see Michie 1963; Davoust 1977, and, more recently, Gieles & Zocchi 2015). These models have been successfully applied to the interpretation of both observational data (e.g., see Gunn & Griffin 1979) and numerical simulations (Zocchi et al. 2016).

As for the investigation of possible formation signatures, it might very well be that, especially for particularly rich and initially underfilled globulars, their outer structure is not too far from that of bright elliptical galaxies for which violent relaxation is thought to have acted primarily to make the inner system quasi-relaxed, while the outer parts are more and more dominated by radially-biased anisotropic pressure. This line of argument motivated the development of several families of dynamical models, to represent the final state of numerical simulations of the violent relaxation process (e.g., see van Albada 1982), as thought to be associated with the formation of bright elliptical galaxies via collisionless collapse (Stiavelli & Bertin 1985; Trenti et al. 2005).

These models show a characteristic anisotropy profile, with an inner isotropic core and an outer envelope that becomes dominated by radially-biased anisotropic pressure. After a first successful applicqation to the study of a sample of Galactic globular clusters under different relaxation conditions (Zocchi et al. 2012), these models have been recently modified to include an appropriate truncation in phase space, to heuristically mimic the limitation induced by the tidal field of the host galaxy (De Vita et al. 2016).

In this context, we wish to mention also some recent numerical experiments of violent relaxation, conducted in the presence of an external tidal field (Vesperini et al. 2014). Interestingly, the resulting configurations are characterized by differential rotation, and an anisotropy profile which is isotropic in the central regions, radial in the intermediate ones, and isotropic (or mildly tangential) in the outskirts, in qualitative agreement with Varri & Bertin (2012) models.

4. Conclusions

With these equilibria in hand, what should one do? First of all, in light of the growing importance of the kinematic and phase space exploration of globular clusters, these models should enable us to take up the new observational challenges of the forthcoming "era of precision astrometry". Second, their stability properties should be carefully studied. Third, they may be used as a controlled playground for developing our fundamental understanding of the phase space behavior of anisotropic, non-spherical, rotating stellar systems. Finally, we may consider to increase their complexity by introducing additional physical ingredients, such as the presence of (i) multiple mass components, (ii) intermediate-mass black holes, (iii) a more realistic prescription for the treatment of the tidal field, and (iv) the phase space contributions of a population of "potential escapers".

As parting thoughts, we wish to emphasize that study of the role of "classical" physical ingredients, such as rotation, anisotropy and tides, is a key step to understand any dynamical signature of more complex phenomena in star clusters, and that interesting (new) science may live at the (often unexplored) intersection of such classical ingredients.

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References

Baumgardt, H. & Makino, J. 2003, MNRAS, 340, 227
Bettwieser, E. & Spurzem, R. 1986, A&A, 161, 102
Bianchini, P., et al. 2013, ApJ, 772, 67
Chandrasekhar, S. 1933, MNRAS, 93, 390
Davoust, E. 1977, A&A, 61, 391
De Vita, R., et al. 2016, A&A, 590, A16
Einsel, C. & Spurzem, R. 1999, MNRAS, 302, 81
Gunn, J. & Griffin, R. 1979, AJ, 84, 752
Gieles, M. & Zocchi, A. 2015, MNRAS, 454, 576
Giersz, M. & Spurzem, R. 1994, MNRAS, 269,
Giersz, M. & Heggie, D. C. 1997, MNRAS, 286, 709
Hénon, M. 1971, Ap&SS, 13, 284
Hunter, C. 1977, AJ, 82, 271
Jarvis, B. & Freeman, K. 1985, ApJ, 295, 314
King, I. R. 1966, AJ, 71, 64
Kormendy, J. & Anand, S. 1971, Ap&SS,12,47
Lagoute, C. & Longaretti, P. 1996, A&A, 308, 441
Landau, L. & Lifchitz, E. 1967, Physique Statistique (MIR, Moscou)
Lupton, R. & Gunn, J. 1987, AJ, 93, 1106
McLaughlin, D. E. & van der Marel, R. P. 2005, ApJS, 161, 304
Michie, R. W. 1963, MNRAS, 125, 127
Milne, E. A. 1923, MNRAS, 83, 118
Posti, L., et al. 2015, MNRAS, 447, 3060
Prendergast, K. & Tomer, E. 1970, AJ, 75, 674
Rowley, G. 1988, ApJ, 331, 124
Schwarzschild, M. 1979, ApJ, 232, 236
Spitzer, L. & Shapiro, S. 1972, ApJ, 173, 529
Spitzer, L. 1987, Dynamical Evolution of Globular Clusters (Princeton Univ. Press, Princeton)
Stiavelli, M. & Bertin, G. 1985, MNRAS, 217, 735
Tiongco, M., et al. 2016, MNRAS, 455, 3693
Trenti, M., et al. 2005, A&A, 433, 57
van Albada, T. S. 1982, MNRAS, 201, 939
Vandervoort, P. O. 1980, ApJ, 240, 478
Varri, A. L. & Bertin, G. 2012, A&A, 540, A94
Vesperini, E. et al. 2014, MNRAS, 443, L79
Wilson, C. P. 1975, AJ, 80, 175
Zocchi, A. et al. 2012, A&A, 539, A65
Zocchi, A. et al. 2016, arXiv:1605.02032