



# On the radial variation in the stellar mass functions of star clusters

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**Abstract.** Observational studies of Galactic globular clusters have recently started measuring how the slope of a cluster's stellar mass function  $\alpha$  varies with clustercentric distance  $r$ . To better understand these observations, we present the evolution of  $\alpha(r)$  for star clusters with a range of initial conditions using a large suite of  $N$ -body simulations. We specifically focus on how  $\alpha(r)$  evolves as a function of both time and the fraction of mass lost by the cluster, and determine what effects initial size, mass, binary fraction, primordial mass segregation, black hole retention, an external tidal field, and the initial mass function have on the evolution of  $\alpha(r)$ . Our models suggest that the evolution of  $\alpha(r)$  depends primarily on a cluster's relaxation time and tidal filling factor (the ratio between its half-mass radius and tidal radius). Using the slope of a cluster's global mass function ( $\alpha_G$ ) as a tracer for the fraction of its initial mass that has been lost, we also find that clusters follow a well-defined track in the  $\alpha_G-d\alpha(r)/dr$  plane allowing factors like a cluster's initial structural properties, dynamical history, and stellar initial mass function to be constrained. Hence the  $\alpha_G-d\alpha(r)/dr$  plane represents an important tool for studying cluster dynamics using wide field studies of cluster stellar mass functions.

**Key words.** galaxies: star clusters (Galaxy):globular clusters: general stars: kinematics and dynamics stars:statistics

## 1. Introduction

The initial mass function (IMF) represents the initial distribution of stellar masses inside a star cluster. After formation, the mass function (MF) will evolve due to both stellar evolution and the escape of stars from the cluster. Stellar evolution mainly affects the high-mass end of the MF, as lower mass stars undergo very little mass loss over a Hubble time. The removal of stars from a cluster on the other hand can affect the entire mass spectrum, as stars can escape their host cluster via two-body relaxation, tidal stripping, tidal heating and tidal shocks.

Throughout the galaxy, clusters will experience a wide range of mass loss rates due to

each of the above mechanisms. Therefore it is no surprise that we find a wide range of mass functions in all types of star clusters. Knowing how the IMF can evolve allows for the dynamical clock of a star clusters to be rewound, revealing both its dynamical history and formation conditions. Modelling the evolution of star cluster MFs has been the focus of many studies (Vesperini & Heggie 1997; Baumgardt & Makino 2003; Kruijssen 2009; Trenti et al. 2010; Leigh et al. 2012; Lamers et al. 2013; Webb et al. 2014; Webb & Leigh 2015).

Observational studies can now measure the slope of a cluster's stellar mass function  $\alpha$  at different clustercentric radii  $r$ , with M10 (Beccari et al. 2010), Pal 4 (Frank et al.

2012), Pal 14 (Frank et al. 2014), NGC 6101 (Dalessandro et al. 2015), and NGC 5466 (Beccari et al. 2015) all following the prediction that  $\alpha$  will decrease with clustercentric distance from mass segregation due to two-body relaxation (e.g. Heggie & Hut 2003). Cluster's that have a higher degree of mass segregation are considered to be dynamically older than clusters with little to no mass segregation. However it should be noted that the degree that Pal 14 is segregated is surprising as its large size means it has a long present day relaxation time. Observations of NGC 6101 (Dalessandro et al. 2015) on the other hand show no signs of mass segregation despite having a shorter present day relaxation time than Pal 14. Hence theoretical studies on how the radial variation in  $\alpha$  evolves with time are necessary before such measurements can be used to study star cluster formation and evolution. In the following sections we present some of the key results from a recently submitted study which uses  $N$ -body simulations to show how radial variations in  $\alpha$  will evolve with time (Webb & Vesperini 2016).

## 2. Models

To understand how radial variations in  $\alpha$  evolve with time, we model the evolution of  $\alpha(r)$  for clusters with a wide range of initial conditions using NBODY6 (Aarseth 2003). The radial profile of each cluster initially follows a Plummer density profile (Plummer 1911), and the stellar population is assume to have an initial mass range between 0.1 and 50  $M_{\odot}$  and a metallicity of 0.0001. In addition to the position and velocity of Individual stars evolving with time, each star is able to evolve given the stellar evolution algorithms of Hurley (2008a) and Hurley (2008b). Clusters are evolved either in isolation or in a Milky Way-like tidal field. See Webb & Vesperini (2016) for further details regarding the full suite of simulations used in this study.

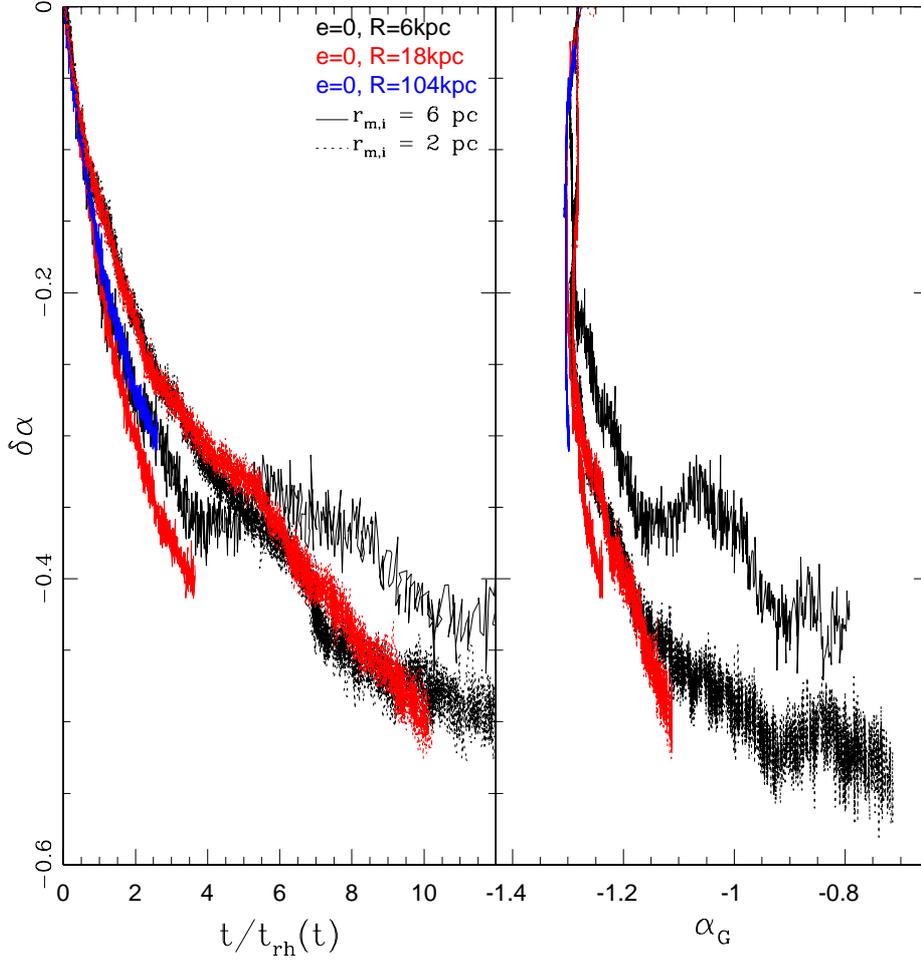
## 3. Results

We consider the evolution of  $\alpha(r)$  for  $6 \times 10^4 M_{\odot}$  clusters with Kroupa, Tout, & Gilmore (1993)

IMFs that range in  $r_{m,i}$  (2 pc and 6 pc) and orbital distance (6 kpc to 104 kpc). To quantify the evolution of  $\alpha(r)$  we measure the slope of the line of best fit to  $\alpha(r)$  versus  $LN(\frac{r}{r_m})$ , which we will refer to as  $\delta_{\alpha}$ , at different times for each model cluster. In Figure 1, we plot the evolution of  $\delta_{\alpha}$  with respect to both time (normalized by the cluster's instantaneous half-mass relaxation time  $t_{rh}$ ) and the slope of the global mass function  $\alpha_G$  (which serves as a tracer for how much mass the cluster has lost (e.g. Vesperini & Heggie 1997; Webb & Leigh 2015)).

The early evolution of  $\delta_{\alpha}$  is the same for all models, and even though mass loss may still be occurring  $\alpha_G$  will not evolve if no segregation has occurred and stars over the entire mass spectrum are escaping the cluster. Over time,  $\delta_{\alpha}$  will naturally slow as each cluster expands. Even for an isolated cluster,  $\delta_{\alpha}$  will continue to slow indefinitely unless mass segregation is halted by core collapse Giersz & Heggie (1996). However once  $\alpha_G$  does start increasing, the evolution of  $\delta_{\alpha}$  will slow further as the tidal stripping of low-mass stars balances the segregation of low-mass stars outwards and  $\alpha(r)$  stops decreasing in the outer regions.  $\delta_{\alpha}$  can even stop evolving if tidal stripping overcomes the mass segregation process and  $\alpha(r)$  starts increasing in the outer regions. Hence the more tidally filling a cluster, the sooner the evolution of  $\delta_{\alpha}$  slows and reaches its minimum value. Varying initial cluster mass simply serves as a method of changing the cluster's tidal filling factor  $\frac{r_m}{r_t}$ , and additional models with lower and higher initial masses evolve accordingly.

We also explore model clusters that range in orbital eccentricity (0 to 0.9), initial binary fraction (0%, 2% and 4%), black hole retention fraction (0%, 25% and 50%), IMFs, and degree of primordial mass segregation (S-0, 0.1, 0.25, and 0.5). Orbital eccentricity, initial binary fraction and black hole retention fraction were found to minimally affect the evolution of  $\delta_{\alpha}$  with respect to time or  $\alpha_G$ . Clusters with non-zero degrees of primordial mass segregation simply start with a non-zero initial  $\delta_{\alpha}$  and eventually reach the same value of  $\delta_{\alpha}$  as the non-segregated clusters within 5 relaxation times. Changing the IMF of each model cluster only has the affect of changing in the initial



**Fig. 1.** Slope of the radial variation in the stellar mass function for stars between  $0.1$  and  $0.5 M_\odot$  as a function of time normalized by current relaxation time (left panel) and  $\alpha_G$  (right panel) for globular clusters with circular orbits between 6 and 104 kpc, initial masses of  $6 \times 10^4 M_\odot$ , and initial half mass radii of 2 pc (dotted lines) and 6 pc (solid lines). Different model clusters are colour coded based on orbit, as indicated by the legend.

$\alpha_G$ , meaning the evolutionary tracks are offset by their initial  $\alpha_G$  values in the  $\delta_\alpha$ - $\alpha_G$  plane.

#### 4. Conclusions

Given the ability of recent observational studies to measure the slope of the stellar mass function at different clustercentric distances, we explore the evolution of  $\alpha(r)$  for a large

suite of  $N$ -body model star clusters that have a range of initial conditions in Webb & Vesperini (2016). We find that the evolution of the slope of the line of best fit to  $\alpha(r)$  versus  $LN(\frac{r}{r_m})$ ,  $\delta_\alpha$ , depends mainly on each cluster's initial relaxation time and tidal filling factor. For all clusters,  $\delta_\alpha$  initially decreases as the cluster expands at a rate that scales with the cluster's initial  $t_{rh}$  since clusters with short relaxation times

can segregate more rapidly. As cluster expansion continues, the evolution of  $\delta_\alpha$  will slow and may eventually stop if the cluster undergoes core collapse.

For a tidally filling cluster, once some segregation has occurred and the mean mass of escaping stars is less than the mean mass of all stars in the cluster, the decrease of  $\delta_\alpha$  will slow further as the tidal field removes low mass stars from the outer region as they segregate outwards.  $\delta_\alpha$  will reach its minimum value and stop decreasing completely once the cluster approaches dissolution and tidal stripping removes low-mass stars from the outer region faster than they segregate outwards.

Additional factors like orbital eccentricity, initial binary fraction, black hole retention fraction, IMF, and degree of primordial mass segregation were shown to have a minimal affect on the evolution of  $\delta_\alpha$ . However, since clusters with different IMFs have different initial  $\alpha_G$  values, the  $\delta_\alpha$ - $\alpha_G$  plane offers the possibility of identifying whether or not observed differences between Galactic globular clusters are due to their dynamical histories or a non-Universal IMF. And since our models already allow for the the age,  $\alpha(r)$ , and  $\alpha_G$  of a star cluster to be used to constrain the formation conditions and dynamical history of both star clusters and galaxies, the importance of future wide field studies of Galactic star clusters cannot be overstated.

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## References

Aarseth, S.J. 2003, Gravitational  $N$ -body Simulations: Tools and Algorithms

- (Cambridge Univ. Press, Cambridge)
- Baumgardt, H., Makino, J. 2003, MNRAS, 340, 227
- Beccari, G., Pasquato, M., De Marchi, G. et al. 2010, ApJ, 713, 194
- Beccari, G., Dalessandro, E., Lanzoni, B., et al. 2015, ApJ, 814, 144
- Dalessandro, E., Ferraro, F. R., Massari, D. et al. 2015, ApJ, 810, 40
- Frank, M.J., Hilker, M., Baumgardt, H., et al. 2012, MNRAS, 423, 291
- Frank, M.J., Grebel, E.K., Küpper, A.H. W. 2014, MNRAS, 443, 815
- Heggie, D. C., Hut, P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge Univ. Press, Cambridge)
- Giersz, M., Heggie, D. C. 1996, MNRAS, 279, 1037
- Hurley, J.R. 2008a, in The Cambridge  $N$ -body Lectures, ed. J. Sverre, et al. (Springer, Berlin), Lecture Notes in Physics, 760, 283
- Hurley, J.R. 2008b, in The Cambridge  $N$ -body Lectures, ed. J. Sverre, et al. (Springer, Berlin), Lecture Notes in Physics, 760, 321
- Kroupa, P., Tout, C.A., Gilmore, G. 1993, MNRAS, 262, 545
- Kruijssen, J.M.D. 2009, A&A, 507, 1409
- Lamers, H. J. G. L. M., Baumgardt, H., Gieles, M. 2013, MNRAS, 433, 1378
- Leigh, N., Umbreit, S., Sills, A., et al. 2012, MNRAS, 422, 1592
- Plummer, H.C. 1911, MNRAS, 71, 460
- Trenti, M., Vesperini, E., Pasquato, M. 2010, ApJ, 708, 1598
- Vesperini, E., Heggie, D. C. 1997, MNRAS, 289, 898
- Webb, J.J., Leigh, N., Sills, A., et al. 2014, MNRAS, 442, 1569
- Webb, J.J. & Leigh, N. 2015, MNRAS, 453, 3278
- Webb, J.J. & Vesperini, E. 2017, MNRAS, 464, 1977
- Zhang, C., Li, C., de Grijs, R., et al. 2015, ApJ, 815, 95