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Dynamical phenomena in the pulsation of type-II Cepheid models

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Abstract. We summarise the results of our recent calculations of non-linear convective type-II Cepheid models.

Key words. Stars: variables: Cepheids – Stars: Population II – Stars: oscillations – hydrodynamics – methods: numerical

1. Introduction

Type-II Cepheids are low mass, population II stars, pulsating in the radial fundamental mode. Depending on the pulsation period they are divided into three classes, which reflect different evolutionary stages (e.g. Gingold 1976): BL Her stars (1 − 4 d), W Vir stars (4 − 20 d) and RV Tau stars (> 20 d). For a recent review see e.g. Catelan & Smith (2015).

Pulsation dynamics of type-II Cepheids is very interesting. Characteristic feature of RV Tau stars is period doubling effect – light curves exhibit a periodic alternations of deep and shallow minima. Pulsation of these variables is not strictly periodic. The longer the pulsation period the more irregularities in the light curves appear. Detailed studies of stars for which sufficiently long observations are available indicate that deterministic chaos underlies their variability (Buchler et al. 1996; Kolláth et al. 1998). Irregularities are present in the shorter period W Vir stars as well. Period doubling effect was also detected in the prototype of the W Vir class (Templeton & Henden 2007) and in a single BL Her-type star (Soszyński et al. 2011; Smolec et al. 2012). In the latter case the effect was predicted to occur in BL Her stars based on radiative calculations by Buchler & Moskalik (1992). As analysed by Moskalik & Buchler (1990) period doubling effect may be caused by half-integer resonances between the pulsation modes.

The insight into pulsation properties of type-II Cepheids is possible thanks to non-linear hydrodynamic modelling. The published studies are scarce however; the majority were done using purely radiative hydrocodes (e.g. Kovács & Buchler 1988; Buchler & Moskalik 1992). Recently Di Criscienzo et al. (2007) published a set of convective BL Her-type models. Below we summarize our recent work on type-II Cepheid models.

2. The models

All models were computed with the Warsaw non-linear convective pulsation codes of Smolec & Moskalik (2008). These are Lagrangian, 1D codes implementing the one-equation turbulent convection recipe of Kuhfuß (1986). Several free parameters enter the convection model. The most common approach is to fix some of these parameters to the so-called standard values (resulting from the comparison with the standard MLT). Other parameters are used to match the observational constraints; in particular the eddy viscosity parameter, $\alpha_{\rm m}$, may be used to adjust the pulsation amplitudes. Its value strongly affects the dynamical behaviour of the models too. Radiation in our calculations is treated in the simple diffusion approximation.

3. BL Her-type models

3.1. BL Her star with period doubling

In their radiative, BL Her-type models, with periods in between 2 d and 2.6 d, Buchler & Moskalik (1992) detected a robust period doubling behaviour. The effect was discovered 20 years later in one star from the OGLE-III catalogue, OGLE-BLG-T2CEP-279 (Soszyński et al. 2011). Its period, 2.4 d, fits the predicted range. In Smolec et al. (2012) we have computed a grid of convective models to reproduce the light variation of this star. We have computed the models of fixed period; mass, metallicity and luminosity (equivalently temperature) were adjusted to match the light curve shape and constraints arising from stellar evolution theory. The best fitting model has M = 0.5M$_\odot$, L = 168L$_\odot$, Z = 0.01 and successfully reproduces the light variation of T2CEP-279 – see fig. 16 in Smolec et al. (2012). We also confirmed that the 3:2 resonance between the fundamental mode and the first overtone is responsible for the period doubling effect, as first analysed by Buchler & Moskalik (1992).

3.2. Modulation of pulsation in BL Her models with reduced eddy viscosity

While modelling T2CEP-279 we also computed a set of models with strongly reduced eddy viscosity ($\alpha_{\rm m}$ = 0.05 vs. $\alpha_{\rm m}$ = 0.25 used in the modelling of T2CEP-279). Such models do not fit the observations; pulsation amplitudes are excessive and light curves show spurious features, e.g. spikes or numerous bumps. Yet the dynamical behaviour, traced through radius variation which is smooth, turned very interesting. In a series of models (M = 0.55M$_\odot$, Z = 0.0001, $120 < L/{\rm L}_\odot < 160$) analysed in Smolec & Moskalik (2012) we detected long-term modulation of pulsation on top of period doubling effect. The modulation is quasi-periodic. The consecutive modulation cycles may differ, e.g. the length of the cycle may differ by as much as 10 % from the mean value.

The modulation of pulsation was not detected in any BL Her star so far. The effect is similar to the Blazhko effect in RR Lyr stars, which may be regarded as lower luminosity siblings of BL Her stars. Interestingly, period doubling was discovered in a significant fraction of Blazhko RR Lyr stars thanks to precise space observations (e.g. Szabó et al. 2010). Based on these observations Buchler & Kolláth (2011) proposed a new model behind the Blazhko effect, in which 9:2 resonance between the fundamental mode and the 9th overtone causes both period doubling and modulation. Their reasoning was based on the parametric study of the amplitude equations. Unfortunately, hydrodynamic models of RR Lyr-type stars do not show the modulation, only period doubling is computed (Kolláth et al. 2011; Smolec 2015). The BL Her models of Smolec & Moskalik (2012) show that the resonant mechanism may indeed work in stars and can cause the modulation akin to the Blazhko effect. In this case however, lower order resonance is in action, namely 3:2 resonance between the fundamental mode and the first overtone.

3.3. Deterministic chaos in BL Her models with reduced eddy viscosity

Another dynamical behaviour we have detected in BL Her-type models with the reduced eddy viscosity is deterministic chaos. In Smolec & Moskalik (2014) we discuss in detail the model sequence with constant luminosity (L = 136L$_\odot$, M = 0.55M$_\odot$, Z = 0.0001) and extending over only 170 K. A wealth of dynamical phenomena characteristic for the deterministic chaos is detected. At both the cool and the hot edge of the computation domain, simple, single-periodic pulsation is present. Then, period doubling cascades are detected on both sides of the domain leading to the chaotic band in between. Within the chaotic band several windows of order are present, in which stable, periodic, period-k pulsation is possible (k is even or odd). These windows are born through the tangent bifurcation, preceded by type-I intermittency. Within some of these windows period doubling cascade leads to chaos again. We also detect interior crises bifurcation (merging of separate chaotic bands), crises induced intermittency and type-III intermittency. Interestingly, the detected phenomena are very similar to that detected in much simpler dynamical systems, like in Lorenz equations or even in the iteration of the simple logistic map.

None of these phenomena was discovered in BL Her stars so far.

4. Survey of type-II Cepheid models

Recently we have started a model survey for type-II Cepheids, including BL Her, W Vir and RV Tau variables (Smolec, in prep.). Initial results are reported here. In the models we do not reduce the eddy viscosity; our intention is to compute as realistic models as possible. Hence, in our basic set of convective parameters eddy viscosity is set to $\alpha_{\rm m}$ = 0.25, just as in the successful modeling of T2CEP-279. In other set we increase the eddy viscosity to $\alpha_{\rm m}$ = 0.5. One of the goals is to check which phenomena we detected in the models with the decreased eddy viscosity (Sections 3.2 and 3.3) persist in more realistic models.

The models have [Fe/H] = −1.5 and adopt two values of mass $0.6\mathrm{M}_\odot$ and $0.8\mathrm{M}_\odot$. Results for the full model survey (other metallicities) will be published soon. The models cover the full temperature extent of the instability strip (IS), but are limited at high luminosities ($L \lesssim 600\mathrm{L}_\odot$ for $0.6\mathrm{M}_\odot$ and $L \lesssim 1000\mathrm{L}_\odot$ for $0.8\mathrm{M}_\odot$) due to dynamical instability detected in the models (Smolec, in prep.). This implies that we reach only the shortest period RV Tau domain. Our model grid is relatively dense; the default steps in effective temperature/luminosity are $25\,\mathrm{K}/25\mathrm{L}_\odot$. All models were integrated by at least 2000 pulsation cycles to reach stable, finite amplitude pulsation. The time series of radius variation for a few hundred last pulsation cycles was then analysed in detail.

The possible pulsation scenarios for the computed model grids are illustrated in Fig. 1. Period doubling is detected in the gray-shaded areas. The color codes the amplitude of alternations in solar radii (the amplitude of the subharmonic term in the frequency spectrum). The following pulsation scenarios are possible:

- single-periodic, fundamental mode pulsation; detected in the majority of the models (white areas in Fig. 1).
- period doubling in the gray-shaded areas in Fig. 1; two period doubling domains are detected. The lower luminosity one, which extends vertically over the BL Her and lower luminosity W Vir domains (pulsation periods ∼2–6.5 d) and higher luminosity period doubling domain (W Vir models with $P \gtrsim 9.5$ d). The lower luminosity domain is the same domain in which T2CEP-279 resides and includes the models of Buchler & Moskalik (1992) and Smolec et al. (2012). It disappears when eddy viscosity is increased ($\alpha_{\rm m} = 0.5$).
- period-4 pulsation, marked with diamonds in Fig. 1. These models are located within period doubling domains. They may be a part of period doubling cascade *en route* to chaos (as in Sect. 3.3), which does not fully develop however, as further period doubling bifurcations are not detected.
- periodic modulation of pulsation, in models marked with open circles. Detailed analysis shows that very narrow, ∼ 10 K-wide, modulation domains are present around these models.
- double-mode (beat) pulsation in models marked with squares. These models pulsate simultaneously in the fundamental and in the fourth radial overtone, which is a surface mode (trapped between the surface and the hydrogen ionization layers, e.g. Buchler et al. 1997).

No large amplitude chaotic behaviour was detected in the models, but we have not reached the longer period RV Tau domain in our calculations. The properties of the presented models and the origin of the detected

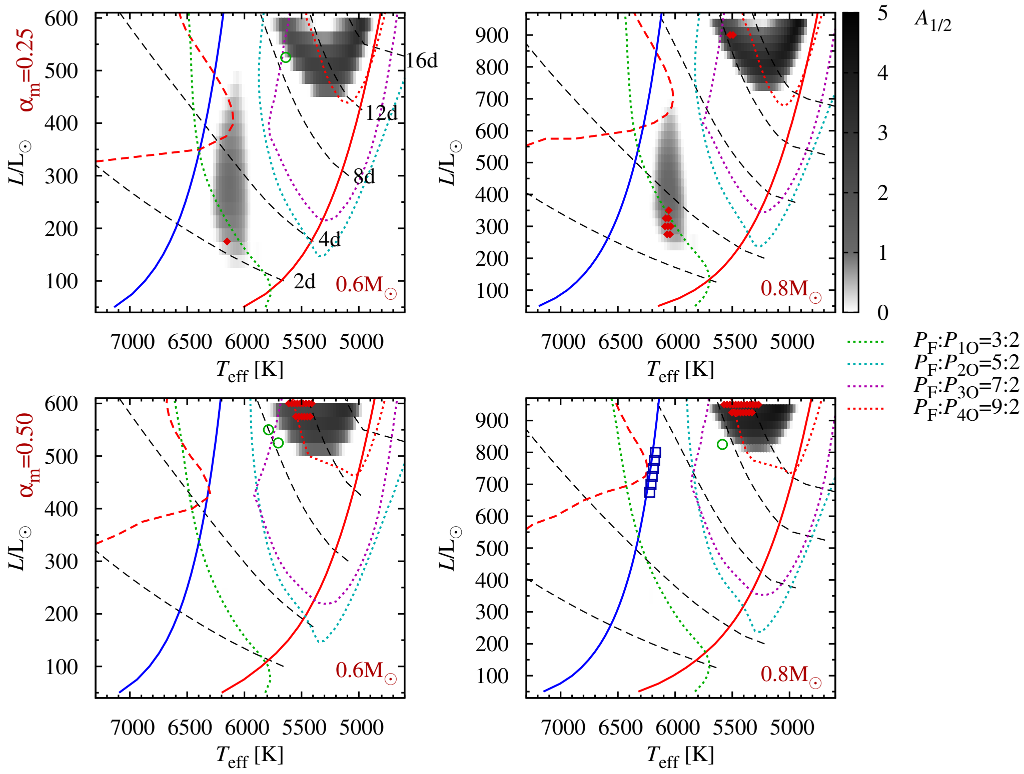

Fig. 1. Pulsation scenarios for the computed models. Thick blue and red lines mark the blue and red edge of the classical IS. Thick, red, dashed line delineate the 4th overtone IS. Black dashed lines are lines of constant period, labeled in the top left panel. Loci of four half-integer resonances are marked with dotted lines as indicated in the key. Pulsation scenarios: gray-shaded areas – period doubling; diamonds – cycle-4 pulsation; open circles – modulation; squares – beat pulsation.

phenomena will be studied in detail in the forthcoming publication.

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