Mem. S.A.It. Vol. 87, 249

Memorie della

Winds of AGB stars:

the two roles of atmospheric dust

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Abstract. The winds of AGB stars are commonly regarded as dust driven, i.e. due to radiation pressure incident on atmospheric dust grains, which are then accelerated and drag the gas along, creating an outflow. The formation of dust, however, requires some extra energy input to expand the atmosphere and push a sufficient fraction of it beyond the so-called condensation radius where the radiative heating of the grains is less than the critical value. This energy injection is provided by the pulsation of the star and the standard picture is therefore that the winds of AGB stars are pulsation-aided dust-driven outflows. The present paper compares the strong- and weak/slow-wind regime. It is argued that in the latter the winds are better described as 'dust supported' rather than dust driven. That is, the gas pressure and pulsation of the star seem to play more direct roles in sustaining the outflow. Taking this fact into account in stellar evolution models which include dust and wind formation will be important and may lower the resultant dust yields.

Key words. Stars: atmospheres – Stars: AGB and post-AGB – Stars: evolution – Stars: mass loss – Stars: variables: general

1. Introduction

Stars on the asymptotic giant branch (AGB) are characterised by their low effective temperatures, high and periodically varying luminosities and extended circumstellar envelopes. The majority of these stars have also significant winds and some, especially the carbon-rich ones, are completely enshrouded with dust, which makes radiation pressure on dust grains a natural driving mechanism. That is, a scenario with pulsation-enhanced dust-driven outflows (the "PEDDRO" scenario, see Höfner 2015, and references therein) as the explanation for the winds of AGB stars. The seminal works by Wood (1979) and Bowen (1988) clearly demonstrated the importance of the pulsation and non-linear dynamics in combination with radiation pressure for the formation of the dusty winds of AGB stars. The PEDDRO model has been very successful in explaining the wind properties of carbon-rich AGB stars (C stars) and seems to be a realistic model for the winds of oxygen-rich AGB stars as well (Höfner 2008; Mattsson et al. 2010; Bladh et al. 2013). The existence of so-called "detached shells" around some C stars can be explained in combination with stellar evolution on the thermal-pulse AGB phase, which lends further support to the PEDDRO scenario (Mattsson et al. 2007a). However, kinetic-enrgy input by pulsation has been identified as an important parameter (see, e.g., Mattsson et al. 2007b, 2008, for the case of C stars), and thermal gas pressure is not at all negligible.

Winters et al. (2000) presented wind models where they classified the resultant winds according to three categories: type A, the strong and certainly dust-driven winds; type B, the critical winds, where the radiative acceleration is more or less balanced by gravity; and type AB, which represents the rare cases of critical winds with high mass-loss rates. Type winds A are prototypical examples of what the PEDDRO scenario describes. But the other two, type B and AB, do not obviously fit the picture. For B-type winds, the radiative acceleration plays a minor role, which is why these critical winds should perhaps not be considered as "dust-driven" in the usual sense. The AB-type are peculiar winds as they also have low luminosities and yet high mass-loss rates.

In this paper, a mean-flow version of the equation of motion for the wind is considered, where the net effect of the pulsation is taken into account in terms of an additional "wave pressure". Using this mean-flow model it is argued that dust may have two different roles in sustaining AGB winds: (1) as the main driving force of the outflow, and (2) to support the atmospheric structure (counter-acting gravity) and thereby allowing for an outflow largely sustained by pulsation and thermal gas pressure.

2. Mean-flow theory

2.1. Averaged wind equation

In the theory of incompressible turbulence, so-called Reynolds decomposition of density, pressure, velocity etc. is used to construct a mean-field theory of a turbulent flow. That is, quantity Q_i is split into a mean $\bar{q}_i$ and fluctuating part q'_i, where $\overline{q'_i} = 0$, but $\overline{q'_i q'_j} \neq 0$. Struck et al. (2004) used this approach to study the pulsation and wave dynamics of AGB stars, although it required some further linearisation to be applicable to compressible flows.

A convenient way to handle short-term variations, which better suited for a compressible flow, is to use a mass-density weighted time average,

$$\tilde{q}_i \equiv \lim_{\tau\to\infty} \int_{t_0}^{t_0+\tau} \rho\, Q_i\, dt \bigg/ \int_{t_0}^{t_0+\tau} \rho\, dt = \frac{\overline{\rho Q_i}}{\bar{\rho}}, \quad (1)$$

which is known as the *Favré approximation*, where the decomposition is given by $Q_i = \tilde{q}_i + q''_i$ (Favré 1962). Then, $\overline{\rho q''_i} = 0$, while $\overline{q''_i} \neq 0$ and $\overline{\rho \tilde{q}_i} = \bar{\rho}\tilde{q}_i = \overline{\rho Q_i}$. With these relations, assuming a stationary mean flow[1], mass conservation implies $\bar{\rho}\tilde{u}_j$ = constant and the equation of motion (EOM) for a compressible inviscid fluid is

$$\frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_i\tilde{u}_j) = \overline{\rho f_i} + \frac{\partial}{\partial x_j}\left[-\bar{p}\delta_{ij} - \overline{\rho u''_i u''_j}\right], \quad (2)$$

where the last "wave pressure term" is analogous to Reynolds stress. An energy equation can be derived in a similar fashion.

Assuming spherical symmetry, mass conservation implies $r^2\bar{\rho}\tilde{u}_r$ = constant and the Favre-averaged EOM simplifies to

$$\bar{\rho}\tilde{u}_r\frac{d\tilde{u}_r}{dr} = -\frac{d\bar{p}}{dr} + \overline{\rho f_r} - \frac{1}{r^2}\frac{d}{dr}\left[r^2\overline{\rho\,(u''_r)^2}\right], \quad (3)$$

where u_r is the radial velocity component. Here, f_r is the effective gravitational acceleration $g_{\rm eff}$, which is the sum of the gravitational and radiative accelerations, i.e., $g_{\rm eff} = g_{\rm grav} + g_{\rm rad}$. The effective acceleration can conveniently be approximated with

$$g_{\rm eff} \approx (\Gamma - 1)\frac{GM_\star}{r^2}, \quad \Gamma \equiv \frac{\kappa_{\rm tot}\, L_\star}{4\pi c\, GM_\star}, \quad (4)$$

where $M_\star$ is the mass of the star, $L_\star$ is the bolometric luminosity of the star, $\kappa_{\rm tot}$ is the total opacity of the gaseous medium (the sum of gas opacity and dust opacity) and c is the speed of light. Since $\overline{\rho g_{\rm eff}} = \bar{\rho}\tilde{g}_{\rm eff}$, the EOM becomes

$$\tilde{u}_r\frac{d\tilde{u}_r}{dr} = -\frac{1}{\bar{\rho}}\frac{d\bar{p}}{dr} - \frac{GM_\star}{r^2}\left(1 - \widetilde{\Gamma}\right) - \frac{1}{r^2\bar{\rho}}\frac{d}{dr}\left[r^2\overline{\rho\,(u''_r)^2}\right], \quad (5)$$

where the last term is due to the pulsation, which requires an additional closure relation.

[1] Note that this is *assumed* – the mean flow may not exist as a unique solution if the system is chaotic.

2.2. Piston boundary and the net effect of pulsation

At the inner boundary of the model, the pulsation is modelled by a piston boundary condition, with radial displacement given by

$$r_{\rm in}(t) = r_{\rm in}(0) + 2\pi \mathcal{P}\,\Delta u_{\rm p}\,\sin\left(\frac{2\pi\,t}{\mathcal{P}}\right), \qquad (6)$$

where $\Delta u_{\rm p}$ is the piston (velocity) amplitude (typically a few km s^{-1}) and $\mathcal{P}$ is the pulsation period. Consequently, there is a net input of kinetic energy, because

$$\overline{(u''_{\rm in})^2} \approx \overline{(u'_{\rm in})^2} = \frac{1}{2}\,\Delta u_{\rm p}^2 > 0. \qquad (7)$$

However, using Favré averaging, the average must be taken over the kinetic-energy density of the fluctuations $\rho\,(u'')^2$. But since the density variation at the inner boundary follows the displacement due to the pulsation, a convenient approximation can be introduced. Series expansion of ρ around $r = r_0$, where r_0 is the location of the inner boundary at $t = 0$, yields

$$\rho(r_{\rm in}, t) \approx \rho(r_0, t) + (r - r_0)\left(\frac{\partial\rho}{\partial r}\right)_{r=r_0} + \ldots, \qquad (8)$$

which after averaging leads to $\overline{\rho_{\rm in}(u''_{\rm in})^2} \approx \frac{1}{2}\,\bar\rho_{\rm in}\,\Delta u_{\rm p}^2$, where $\bar\rho_{\rm in} = \overline{\rho(r_0, t)}$.

Assuming a locally isothermal EOS for the shocks one may introduce a simple energy equation (for the shocks only), which after averaging will be of the simple form

$$\frac{d}{dr}[r^2\overline{\rho(u'')^2}\tilde u_r] \approx 2\frac{d}{dr}[r^2 c_{\rm s}^2(r)\,\bar\rho\tilde u_r]. \qquad (9)$$

Upon integration, making use of the linearised relation above, this equation yields

$$r^2\overline{\rho(u'')^2}\tilde u_r \;\approx\; r_{\rm in}^2\,\bar\rho_{\rm in}\tilde u_{\rm in}\left[\Delta u_{\rm p}^2 - 2c_{\rm s}^2\right] + 2r^2 c_{\rm s}^2\bar\rho\tilde u_r, \qquad (10)$$

where $c_{\rm s}$ is the sound speed. Dividing by $\tilde u_r$ and taking the derivative w.r.t. r results in an expression for the last term in the EOM (5),

$$\frac{1}{r^2\bar\rho}\frac{\partial}{\partial r}[r^2\overline{\rho(u'')^2}] \approx -\frac{1}{2}\Delta u_{\rm p}^2\,\frac{d\ln\tilde u_r}{dr}. \qquad (11)$$

Thus, the wave-pressure gradient is expected to be small at large radii ($d\tilde u_r/dr \to 0$), although the wave pressure as such may not vanish.

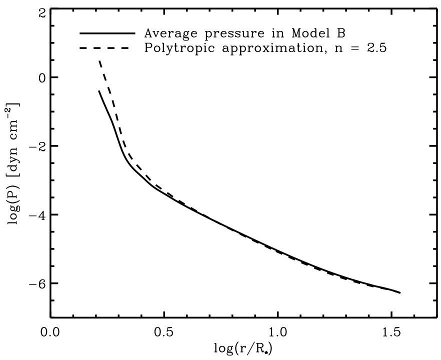

Fig. 1. Comparison of the average gas pressure profile for Model B (see Table 1) and that obtained by a polytropic EOS ($n = 2.5$) and the gas density profile of Model B.

2.3. Gas pressure

The average thermal gas pressure makes a non-negligible contribution to the outflow of an AGB star. For a polytropic equation of state (EOS), $P \propto \rho^{1+1/n}$, with n the "polytropic index", the sound speed is $c_{\rm s}^2 \propto \rho^{1/n}$. Hence, the average acceleration due to the gas pressure gradient can be estimated by

$$\frac{1}{\bar\rho}\frac{d\bar p}{dr} = -\bar c_{\rm s}^2(r)\left(\frac{2}{r} + \frac{d\ln\tilde u_r}{dr}\right), \qquad (12)$$

which holds also for an isothermal EOS. With a representative n, Eq. (12) is a good approximation of the acceleration due to gas pressure. For stars, $n = 2.5$ is often assumed, which seems appropriate in the present case (see Fig 1).

2.4. Radiative acceleration

The radiative acceleration of the gas in an AGB atmosphere is mainly a consequence of the momentum transfer from radiation to dust grains. Thus, $\kappa_{\rm tot}$ can be replaced by the opacity $\kappa_{\rm d}$ due to dust grains in the atmospheric gas. In the Rayleigh limit ($\lambda \gg 2\pi a$), the dust opacity of the gas at wavelength a λ becomes

$$\kappa_\lambda = \frac{\pi}{\rho}\int_0^\infty a^3\,Q'_{\rm abs}(\lambda)\,f(a)\,da, \qquad (13)$$

where f is the number density of grains of radius a, $Q'_{\rm abs}(\lambda) = Q_{\rm abs}(a,\lambda)/a$, where $Q_{\rm abs}$ is the ratio of the effective to the geometric cross-section. Note, however, that for critical winds this assumption puts a bias on the quantitative result (see Mattsson & Höfner 2011).

It is convenient to introduce the an average dust opacity defined as

$$\langle\kappa_{\rm d}\rangle = \frac{1}{L_\star}\int_0^\infty \kappa_\lambda\, L_{\star,\lambda}\, d\lambda, \quad (14)$$

where $L_\star$ is the bolometric luminosity. Similarly, the average opacity of the *grain material* can (in the Rayleigh limit) be defined as

$$\langle\kappa_{\rm gr}\rangle = \frac{3}{4\rho_{\rm gr}L_\star}\int_0^\infty Q'_{\rm abs}(\lambda)\, L_{\star,\lambda}\, d\lambda, \quad (15)$$

where $\rho_{\rm gr}$ is the bulk density of the grain material. Combination of Eqs. (13), (14) and (15) leads to a simple scaling relation between dust-to-gas ratio and the ratio of the average material density to the average dust opacity of the gas, $\rho_{\rm d}/\rho = \langle\kappa_{\rm d}\rangle/\langle\kappa_{\rm gr}\rangle$, which can be used to estimate the efficient of radiative acceleration, provided that $\rho_{\rm d}$ is known.

2.5. Strong-wind approximation

If the radiative acceleration is large (formally, if $\Gamma \gg 1$), the thermal gas pressure and pulsation of the star does not add significantly to the kinetic energy of the wind. Such winds are *dust-driven winds*, belonging the "type A" category according to Winters et al. (2000). The expected development of shocks that do no quickly dissipate, i.e., $\overline{\rho\,(u'')^2} \neq 0$ even quite far away from $r_{\rm in}$, which is confirmed by previous work (e.g., Mattsson et al. 2010; Mattsson & Höfner 2011, see also Fig. 2), implies that the root-mean-square of the fluctuations have a relatively weak radial gradient. An approximate EOM is therefore

$$4\pi\frac{d}{dr}(r^2\bar{\rho}\tilde{u}_r^2) \approx \frac{\overline{\rho\,\langle\kappa\rangle\, L_\star}}{c}. \quad (16)$$

Integrating and assuming that the time variations of $L_\star$ and the linear absorption $\rho\,\langle\kappa_{\rm d}\rangle$ are essentially (statistically) independent[2], one arrives at $4\pi\, r^2\bar{\rho}\tilde{u}_r^2 \approx \bar{\tau}_{\rm w}\,\bar{L}_\star/c$, where the average optical depth of the wind region is given by

$$\bar{\tau}_{\rm w} = \int_{r_{\rm in}}^{r} \overline{\rho\langle\kappa_{\rm d}\rangle}\, dx = \langle\kappa_{\rm gr}\rangle\int_{r_{\rm in}}^{r}\bar{\rho}_{\rm d}\, dx. \quad (17)$$

Thus, when $\Gamma \gg 1$, the velocity profile of the outflow is well approximated by

$$\tilde{u}_r^2(r) \approx \frac{\bar{\tau}_{\rm w}(r)\,\bar{L}_\star}{4\pi c\, r^2\,\bar{\rho}(r)}, \rightarrow \tilde{u}_r(r) \approx \frac{\bar{\tau}_{\rm w}(r)\,\bar{L}_\star}{c\dot{M}}, \quad (18)$$

which is essentially identical to a well-known result for stationary winds (see Lamers & Cassinelli 1999) and defines the character of a strongly dust-driven outflow. Note that this approximation works also for a moderately large Γ, but may in such case require some re-scaling to match the wind profile.

2.6. Critical and sub-critical winds

Critical winds ($\Gamma = 1$), where radiative acceleration plays a minor role, belong to "type B", according to the classification by Winters et al. (2000). $\Gamma > 1$ is often seen as a criterion for wind formation, but then the thermal gas pressure and the kinetic-energy input by the pulsation of the star is not taken into account. For $\Gamma \approx 1$ and assuming a polytropic (or isothermal) EOS (making use of eq. 12),

$$\left(\tilde{u}_r^2 - \bar{c}_{\rm s}^2\right)\frac{d\ln\tilde{u}_r}{dr} \approx \frac{2\bar{c}_{\rm s}^2}{r} + \frac{1}{2}\Delta u_{\rm p}^2\,\frac{d\ln\tilde{u}_r}{dr}, \quad (19)$$

Table 1. Stellar parameters and average outflow quantities at the outer boundary of two numerical simulations of a carbon-rich AGB star.

	Model A	Model B
$M_\star$ [$M_\odot$]	0.75	0.75
$\log(L_\star/L_\odot)$	3.85	3.85
$T_{\rm eff}$ [K]	2400	2400
$\log(\bar{\varepsilon}_{\rm c})$	-2.90	-3.80
$\Delta u_{\rm p}$ [km s^{-1}]	4.0	2.0
$\dot{M}$ [$10^{-6}M_\odot$ yr^{-1}]	9.24	1.05
$u_{\rm out}$ [km s^{-1}]	42.1	1.64
$\Delta u_{\rm out}$ [km s^{-1}]	2.31	0.03
$\rho_{\rm d}/\rho\cdot 10^3$	5.94	0.31
$f_{\rm c}$	0.56	0.23
Γ	~ 5	~ 0.2

[2] Near inner boundary, ρ and $L_\star$ are obviously correlated. Thus, $\rho\,\langle\kappa_{\rm d}\rangle$ and $L_\star$ cannot be completely independent throughout the whole atmosphere.

which in the isothermal limit yields

$$\frac{\tilde{u}_r^2}{\bar{c}_s^2} \approx 4\ln\left(\frac{r}{r_c}\right) + \left(\frac{\Delta u_p^2}{\bar{c}_s^2} + 2\right)\ln\left(\frac{\tilde{u}_r}{\tilde{u}_c}\right), \quad (20)$$

where r_c is the condensation radius, at which dust condensation becomes significantly more efficient than sublimation and $\tilde{u}_c$ is the mean-flow velocity at that radius. Here, one may note that $c_s \sim 1 - 5$ km s^{-1} for a typical AGB star. Thus, wind speeds of a few km s^{-1}, which is quite normal for AGB stars, can be obtained even for $\Gamma \approx 1$. Clearly, $\Gamma \approx 1$ does not a exclude a wind, neither does $\Gamma < 1$.

When the radiative acceleration is relatively weak, the gravitational, radiative and thermal (pressure) forces are almost in equilibrium. It can be shown that

$$\frac{1}{\bar{\rho}}\frac{d\bar{p}}{dr} \approx -\frac{GM_\star}{r^2}\left(1 - \delta\widetilde{\Gamma}\right) \quad (21)$$

where δ is a parameter describing the fraction of the radiative acceleration that is needed to support a sufficiently dense atmospheric structure reaching out to the wind region. Using this parameterisation one obtains an approximation for sub-critical winds,

$$\tilde{u}_r^2 \approx 2\delta\widetilde{\Gamma}\frac{GM_\star}{r_c}\left(1 - \frac{r_c}{r}\right) + \Delta u_p^2 \ln\left(\frac{\tilde{u}_r}{\tilde{u}_c}\right). \quad (22)$$

A closed-form expression exist in terms of the Lambert W function.

3. Results and discussion

Comparing a direct numerical simulation (see Mattsson et al. 2010), corresponding to a clearly dust-driven wind (Model A, Table 1), with the strong-wind approximation (Eq. 18; Fig. 2), one can clearly see that the approximation is well within the 1σ deviation (due to the pulsation dynamics) from the mean flow. This is an example of a wind where the radiation pressure incident on dust grains is dominating the acceleration of the outflow and where $\Gamma \sim 5$. If one instead considers a simulation of a sub-critical wind (Model B, Table 1), the acceleration is not dominated by the momentum transfer from radiation to dust grains. In fact, as can be seen in Fig. 3, such a wind is well

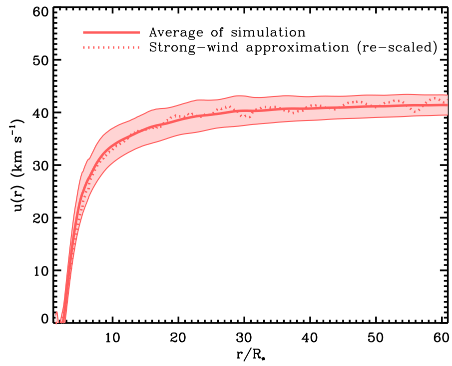

Fig. 2. Average wind profile of a strong-wind wind simulation (Model A) compared to the analytical prediction by eq.(18). The shaded area shows the 1σ-deviation from the mean of the simulated flow.

approximated by Eq. (22) with $\Gamma \approx 0.2$ and assuming negligible wave pressure. Due to the weak pulsation ($\Delta u_p = 2.0$ km s^{-1}) and negligible wave pressure ($\Delta\tilde{u}_r \leq 0.03$ km s^{-1}) outward acceleration is dominated by gas pressure, although aided by the presence of dust.

The kinetic-energy injection by pulsation and the thermal gas pressure play important roles in sustaining the slowest (sub-critical) winds, where $\Gamma < 1$. Even a more typical wind, with wind speeds $10 - 20$ km s^{-1}, does not necessarily require $\Gamma \gg 1$. If the radiation pressure is just barely enough to counteract the force of gravity, the gas-pressure gradient, aided by wave pressure due to the pulsation, is still sufficient to accelerate the gas and sustain a wind. However, the (outward) acceleration due gas pressure is always smaller than (inward) the gravitational acceleration in a typical AGB star. In principle one may argue that with enough wave pressure generated by the pulsation, a wind could be sustained, i.e., a marginally "pulsation driven" wind (such as the example shown by Struck et al. 2004). But the kinetic-energy injection by the pulsation required to create a sufficient wave pressure does not seem realistic according too previous attempts using frequency dependent radiative transfer (Mattsson et al., unpublished). Thus, "pulsation driven" winds of AGB stars are unlikely. Some other supportive force is

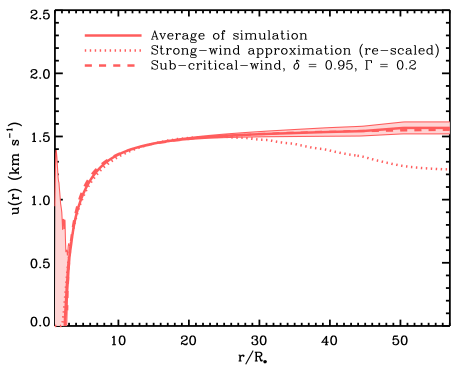

Fig. 3. Average wind profile of a sub-critical wind simulation (Model B) compared to the analytical prediction by eq. (20) omitting the wave-pressure term. Note that the re-scaled strong-wind model does not match the resultant wind profile.

needed, whether it be radiation pressure on dust or Alfvén waves (as suggested by Falceta-Gonçalves & Jatenco-Pereira 2002).

Simple AGB-wind models, such as those sometimes used in stellar evolution modelling (Ventura et al. 2012; Nanni et al. 2013) often neglect the effects of gas pressure as well as "wave pressure". In the case of stellar evolution modelling with atmospheric dust formation, this may put a bias on the efficiency of dust production. Since $\Gamma \propto \kappa_{\rm d} \propto \rho_{\rm d}/\rho$ (see Section 2.4), requiring that $\Gamma > 1$ forces the dust-to-gas ratio to be above a certain value, because there is a continuous mass loss on the AGB, which is prescribed rather than modelled. The dust yield, i.e., the total mass of dust lost from the star during the AGB, will obviously be lower if the dust-to-gas ratio is generally lower (Mattsson et al. 2015).

4. Summary and conclusions

What has been argued in this paper may seem quite obvious, but the implications are often overlooked. In fact, the notion of AGB winds as strongly dust driven has become the standard picture and $\Gamma > 1$ is often taken as a strict requirement for having a wind at all. However, as this paper has hopefully showed, this picture is incomplete. Also a low Γ value can sometimes be sufficient. Such winds cannot be regarded as dust *driven* (since most of the outward push on the circumstellar gas is not due to radiation pressure on dust grains which drag the gas along), but rather as being dust *supported* outflows.

Acknowledgements. J. D. Munkhammar and D. Mitra are thanked for valuable comments.

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