Stellar evolution models:
current uncertainties and their impact on population synthesis tools

S. Cassisi

INAF – Osservatorio Astronomico di Teramo, Via M. Maggini, snc, I-64100 Teramo, Italy
e-mail: cassisi@oa-teramo.inaf.it

Abstract. The knowledge of the evolutionary and structural properties of stars has achieved an high level of accuracy and maturity. This occurrence has been possible thanks to an improved understanding of the physics at work in real stars. This notwithstanding, the current generation of stellar models is still affected by several - not always negligible - shortcomings related to our poor knowledge of some thermodynamical processes, nuclear reaction rates, as well as the efficiency of mixing/diffusive processes. These drawbacks have to be properly taken into account when comparing theory with observations, to derive evolutionary properties of both resolved and unresolved stellar populations. In present paper we review (some of) the major sources of uncertainty for the main evolutionary stages.

Key words. Stars: interiors – Stars: evolution – Stars: late-type

1. Introduction

The capability of the latest generation of stellar models to account for all the evolutionary phases observed in star clusters is undoubtedly an exciting achievement, that crowns with success the development of stellar evolutionary theory. Following this success, one is often tempted to use evolutionary results in an uncritical way, i.e., by taking these results at face value, without accounting for the associated uncertainties. However, theoretical uncertainties do exist, as it is clearly shown by the not negligible differences among the results obtained by independent research groups.

A careful discussion of the uncertainties affecting stellar models for low-mass stars was early addressed by Chaboyer (1995), who investigated the reliability of theoretical models for H-burning stars presently evolving in galactic globular clusters (GGCs). This type of investigation has been extended to advanced evolutionary stages by Cassisi et al. (1998, 1999), Castellani & Degl’Innocenti (1999), and Gallart et al. (2005). A discussion of the drawbacks of stellar models for low- and intermediate-mass stars and their impact on widely employed age, distance and chemical composition diagnostics has been also provided by Cassisi (2004, 2005). More recently, Valle et al. (2013) have addressed the problem of a quantitative and systematic evaluation of the cumulative propagation of physical uncertainties in current generation of stellar models of low mass stars by adopting a statistical approach.
Stellar evolution models are indispensable tools in many astronomical research areas. Much fundamental information on resolved stellar populations otherwise inaccessible, as for example the age and the metallicity, is obtained by comparing observational data and theoretical predictions. Furthermore, evolutionary models play a crucial role also in the studies of unresolved stellar populations, since they are a fundamental ingredient for the stellar population synthesis (SPS) tools. On the other hand, the accuracy of the adopted evolutionary framework affects our capability to derive robust insights about physical properties of galaxies when employing SPS techniques.

2. Stellar model uncertainties and SPS predictions

Although the uncertainties present in the stellar evolutionary framework can affect the SPS predictions, stellar models have been usually employed by the SPS community in an uncritical way. Therefore, when applying SPS tools to both resolved and unresolved stellar populations, the contribution coming from systematic uncertainties in the adopted stellar model library to the error budget on the derived ages and metallicities, is usually not properly taken into account. However, there is now an ongoing effort in this direction (Gallart et al. 2005, Coelho et al. 2007, Lee et al. 2009a, 2010, Cenarro et al. 2008, Conroy et al. 2009, Percival & Salaris 2009). The main conclusions of these analyses are that the various SPS diagnostics are affected differently and, sometimes, in the opposite sense, by systematic changes in the stellar model predictions, such as luminosity and/or effective temperature, and slight offsets between the metallicity scales of the adopted stellar model set and spectral library. This occurrence has a noteworthy implication for methods which fit simultaneously to several spectral indices for deriving ages and metallicities of unresolved stellar populations, since a failure to match several indices simultaneously could, spuriously, be interpreted for example as an indication of a non scaled-solar heavy elements distribution. It has been also proven that the inclusion of the Asymptotic Giant Branch (AGB) stage in SPS models is fundamental for understanding the physical properties of galaxies. However, a different treatment of this uncertain evolutionary stage alters the final results significantly as, for instance, the inferred galaxy masses (Bruzual 2007). Therefore, SPS models that do not account for the current uncertainties in AGB modelling are largely underestimating errors and may even be introducing systematic biases.

As for the possibility of testing the impact of independent stellar model databases in SPS tools, the situation has significantly improved in these last years thanks to the availability of updated sets of stellar models (Pietrinferni et al. 2004, 2006, Dotter et al. 2007, Bertelli et al. 2008) that can be easily incorporated in a SPS code. Ideally, the SPS community should now make the effort of considering these independent stellar model libraries in the SPS codes, in order to evaluate the effect of using independent model prescriptions on their SPS results.

3. Stellar models: the state-of-the-art

We address here only the major open problems affecting model computations, such as the uncertainties associated with the treatment of mass loss during the Red Giant Branch (RGB) and the AGB, and evolution of stars during the Thermal Pulses stage (TPAGB). However, we also provide some indications about the level of reliability of model predictions concerning ‘less problematic’ evolutionary stages:

3.1. The core H-burning stage

In the last decade, the accuracy of central H-burning (main sequence, MS) theoretical models has improved a lot. This occurrence is due to the availability of updated and accurate predictions concerning both the thermal and opacitive properties of matter in the relevant regime for both the interiors and atmospheres of low- and intermediate-mass stars. Some residual uncertainty is associated to (some) nuclear reaction rates. A large effort has been devoted to improve the measurements at energies as close
as possible to the Gamow peak, i.e. the energies at which nuclear reactions occur in the stars. Thanks to this effort the nuclear processes involved in the p-p chain have a small uncertainty. As a consequence, their associated uncertainty on the age - luminosity of the Turn Off (TO) calibration is also negligible (<2%). However, near the end of the MS stage, due to the paucity of H, the energy supplied by the H-burning becomes insufficient and the star reacts contracting its core in order to produce the requested energy via gravitation. As a consequence, both central temperature and density increase and, when the temperature attains a value of \( \sim 15 \times 10^6 \text{K} \), the H-burning process is controlled by the CNO cycle, whose efficiency is critically dependent on the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate, since this is the slowest reaction of the whole cycle.

In the past the rate for this reaction was uncertain, at least by a factor of 5, because all available laboratory measurements were performed at energies well above the range of interest for astrophysical purposes. The LUNA experiment (Formicola et al. 2003) has significantly improved the low energy measurements, obtaining an estimate which is about a factor of 2 lower than previous determinations. This new rate leads to a brighter and hotter TO for a fixed age (Pietrinferni et al. 2010), with the consequence that, for a fixed TO brightness, the new calibration predicts systematically older cluster ages of about 0.9 Gyr on average.

### 3.2. The Red Giant Branch

A correct theoretical prediction of the RGB spectral properties and colors is of paramount importance for interpreting observations of distant star clusters and galaxies using SPS methods, but also for determining the ages of resolved stellar systems by means of isochrone fitting techniques. In addition, being the location and the slope of the RGB in the CMD strongly sensitive to the metallicity, they are widely used as metallicity indicators.

The \( I \)-band brightness of the tip of the RGB (TRGB) provides a robust standard candle, largely independent of the stellar age and initial chemical composition, which allows to estimate distances out to about 10 Mpc using \textit{HST} observations (Tammann & Reindl 2013). Due to the lingering uncertainties on the empirical determination of the TRGB brightness, RGB models provide an independent calibration of this important standard candle (Salaris & Cassisi 1998). Moreover, theoretical predictions about the structural properties of stars at the TRGB play a fundamental role in determining the main evolutionary properties of their progeny: the core He-burning stars during the Horizontal Branch (HB) evolutionary phase. In particular, HB luminosities (like the TRGB ones) are mostly determined by the value of the electron degenerate He-core mass (\( M_{\text{He-core}}^{\text{He}} \)) at the end of the RGB evolution.

A detailed analysis of the existing uncertainties in theoretical RGB models, and of the level of confidence in their predictions has been performed by Salaris et al. (2002). As far as the location and slope of RGB evolutionary tracks is concerned, model predictions are affected by: the EOS, the low-T opacity, the efficiency of superadiabatic convection, the choice about the model outer boundary conditions and the initial chemical abundances. It has been already emphasized that in the thermal regime appropriate for RGB stars, big improvements have been achieved concerning both the EOS and low-T opacity. On the other hand, it is still worthwhile to discuss the issue related to the efficiency of the outer convection. The efficiency of superadiabatic convection parametrized by the mixing length parameter (\( \alpha_{\text{MLT}} \)) is usually calibrated by reproducing the solar \( T_e \), and this solar-calibrated value is then used for stellar models of different masses and along different evolutionary phases, including the RGB one. The adopted procedure guarantees that models always predict correctly the \( T_{\text{eff}} \) of at least solar type stars. However, the RGB location is much more sensitive to the value of \( \alpha_{\text{MLT}} \) than the MS. Therefore, it is important to verify that a solar \( \alpha_{\text{MLT}} \) is suitable also for RGB stars of various metallicities.

A source of concern about an a priori assumption of a solar \( \alpha_{\text{MLT}} \) for RGB computations comes from the fact that recent models
from various authors, all using a suitably calibrated solar value of $\alpha_{\text{MLT}}$, do not show the same RGB temperatures. This means that – for a fixed empirical RGB temperature scale – the calibration of $\alpha_{\text{MLT}}$ based on RGB $T_{\text{eff}}$ estimates values would not provide always the solar value. A comparison of independent sets of RGB stellar models (Salaris et al. 2002) - computed with the same initial chemical composition and solar calibrated values of $\alpha_{\text{MLT}}$ shows that these models can predict a different $T_{\text{eff}}$ scale for the RGB: a safe estimate of the current uncertainty on this $T_{\text{eff}}$ scale is of the order of $200 - 300$ K. The reason for this discrepancy must be due to some difference in the input physics which is not compensated by the solar calibration of $\alpha_{\text{MLT}}$ (Vandenberg et al. 2008).

This occurrence clearly points out the fact that one cannot expect the same RGB $T_{\text{eff}}$ scale from solar calibrated models that do not employ exactly the same input physics. Therefore it is always necessary to compare RGB models with observations to ensure the proper calibration of $\alpha_{\text{MLT}}$ for RGB stars (Salaris & Cassisi 2008, their fig. 5).

As for the uncertainties affecting theoretical predictions about the TRGB brightness, it is clear that, being this quantity fixed by the He core mass, any uncertainty affecting the predictions of $M_{\text{He core}}$ immediately translates into an error on $M_{\text{TRGB}}^{bol}$. The physical inputs that have the largest impact in the estimate of $M_{\text{He core}}$ are the efficiency of atomic diffusion and the conductive opacity. Unfortunately, no updates are available concerning a more realistic estimate of the real efficiency of diffusion in low-mass stars, whereas concerning the conductive opacity, large improvements have been obtained (Potekhin 1999 - P99, Cassisi et al. 2007 - C07). These conductive opacity predictions represent a significant improvement with respect to previous estimates.

The comparison of recent results (Bertelli et al. 2008 - Padua, Pietrinferni et al. 2004 - BaSTI, Vandenberg et al. 2000 - Victoria, Dotter et al. 2007 - Dartmouth, Yi et al. 2001 - Yonsei-Yale) concerning the TRGB bolometric magnitude and $M_{\text{He core}}$ at the He-flash is shown in fig. 1. When excluding the Padua models, there exists a fair agreement among the various predictions about $M_{\text{He core}}$: at fixed metallicity the spread among the various sets of models is at the level of 0.003$M_{\odot}$. Concerning the trend of $M_{\text{TRGB}}^{bol}$, all model predictions at a given metallicity are in agreement within ~0.15 mag, with the exception of the Padua models that appear to be brighter, at odds with the fact that they predict the lowest $M_{\text{He core}}$ values. When neglecting the Padua and Yonsei-Yale models, the ~0.1 mag spread among the different TRGB brightness estimates can be explained in terms of differences in the adopted physical inputs.

The comparison between theoretical predictions about the I-Cousins magnitude of the TRGB and empirical data is performed in fig. 2, where the data for the GGCs $\omega$ Cen. and 47 Tuc (Bellazzini et al. 2004), and theoretical calibrations of $M_{\text{TRGB}}^I$ as a function of the metallicity for various assumptions concerning the conductive opacity and the rate for the nuclear reaction $^{14}\text{N}(p,\gamma)^{15}\text{O}$ are displayed. The calibration based on the most updated physics appears in very good agreement with the empirical evidence.
4. Open problems

4.1. The mass-loss efficiency

One of the thorniest problems in current stellar evolution theory is that related to the efficiency of mass loss (ML) during both RGB and AGB stage. In fact, the efficiency of ML during the RGB strongly controls the $T_{\text{eff}}$ - and hence the color - of the star along the HB stage, while during the AGB by reducing the envelope mass, it truncates the AGB evolution - and hence the contribution of the star to the infrared flux of the global stellar population.

The astrophysical impact of ML in both Pop. I and II giants is huge and affects also the interpretation of the UV excess in ellipticals, or the interaction between the cool intracluster medium and hot halo gas. However, despite its importance, complete empirical determinations as well as a comprehensive physical description of the involved processes are still lacking. So far, there is a lack of any empirical law directly calibrated on Population II giants. Indeed, only a few, sparse estimates of ML for giants along the brightest portion of the RGB and AGB exist. From a theoretical point of view, our knowledge of the ML timescales, driving mechanisms, dependence on stellar parameters and metallicity is also very poor. The consequence is that there is little theoretical or observational guidance on how to incorporate ML into stellar model computations.

Without a better recipe, models of stellar evolution incorporate ML by using analytical formulae calibrated on Population I bright giants, the first and most used being the Reimers (1975) formula, extrapolated towards lower luminosity by also introducing a free parameter $\eta$, to account for a somewhat less efficient ML along the RGB. A few other formulae, which are variants of the Reimers one, have been proposed in the subsequent years (Catelan 2009) but there is no a priori reason for choosing among the different alternatives.

In these last few years, there is growing amount of empirical data concerning ML estimates for Pop. II red giants (Origlia et al. 2007). The preliminary empirical ML law appears significantly different (flatter) than the Reimers formula, that seems to be ruled out by current empirical estimates at the 3$\sigma$ level. It also seems that the ML phenomenon is not a continuous process along the RGB but an episodic phenomenon, and it does not appear to be strongly correlated with the metallicity.

The situation is still more complicated in the case of AGB stars, due to the link existing between the ML efficiency (and the physical processes that cause the ML) and the evolutionary, structural and pulsational properties of the evolving star (van Loon 2008).

4.2. The AGB stage modelling

The computation of AGB models is one of the most complicated task for the stellar evolution community. This is because the evolutionary properties of these stars are hugely dependent on the complex link existing among mixing processes (such as the third dredge up - 3DU), ML efficiency, nucleosynthesis and envelope opacity stratification. The results that can be obtained, strongly depend on the assumptions about the efficiency of these processes and their treatment in the numerical codes.

Concerning the 3DU, in spite of its fundamental relevance in determining the chemical enrichment of TPAGB star envelopes, its
treatment in stellar evolutionary code is quite uncertain. This is due to the fact that we are not yet able to properly describe convection inside the stars, and in particular, in case of mixing occurring in a region with a strong composition-opacity discontinuity. Many different - arbitrary - methods can be envisaged to treat the occurrence of the 3DU, but each one of these approaches has no robust physical ground. This has the important implication that in all fully evolutionary AGB models, the efficiency of the 3DU is managed by using one (or more) free parameter(s). An important physical implication of the occurrence of the 3DU is the huge change in the envelope C/O ratio (Ventura & Marigo 2010). The change in the C/O ratio when it approaches (and overcomes) unity has huge effects on the opacitive properties of the stellar envelope (Marigo 2002). As a consequence, the C/O ratio drives sharp discontinuities in many observational properties of AGB stars: $T_e$, colors, spectra, mass loss efficiency, etc. After many years during which only approximate evaluations for the C-rich mixture opacity were available, the situation is largely improving and new opacity tables suitable for AGB envelope computations are now commonly incorporated in evolutionary codes (Weiss & Ferguson 2009). These new opacities, although in good qualitative agreement with the previous estimates, show significant, quantitative differences. This has the effect that the $T_e$ scale for AGB models is expected to be significantly affected, with consequences on SPS modeling that are still to be fully exploited.

References
Cassisi, S., et al. 1999, A&ASS, 134, 103
Reimers, D. 1975, Mem. Soc. R. Sci. Liège 6 Ser., 8, 369