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The triple-alpha reaction and the A=8 gap in BBN and Population III stars

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Abstract. We investigated the effect of a variation of fundamental constants on primordial element production in Big Bang nucleosynthesis (BBN) and primordial stellar evolution. We focused on the effect of a possible change in the nucleon-nucleon interaction on nuclear reaction rates involving $A = 8$ and 5 unstable nuclei. While the triple–alpha process ^{4}He$(\alpha\alpha,\gamma)^{12}$C is normally not effective in BBN, its rate is very sensitive to the position of the "Hoyle state" and could in principle be drastically affected if ^{8}Be were stable during BBN.

1. Introduction

Constraints on the possible variation of fundamental constants are an efficient method of testing the equivalence principle (Uzan 2011), which underpins metric theories of gravity and in particular general relativity. These constraints are derived from a wide variety of physical systems and span a large time interval back to primordial (Population III) stars [z=10-14] and Big Bang Nucleosynthesis (BBN) [$z \approx 10^8$]. Here, we summarize our recent works on both subjects (Coc et al. 2007; Ekström et al. 2010; Coc et al. 2012b).

It is well known that, in principle, the mass gaps at A =5 and A = 8, prevent BBN to extend beyond ^{4}He. The presence of these gaps is caused by the instability of ^{5}He, ^{5}Li and ^{8}Be which are respectively unbound by 0.798, 1.69 and 0.092 MeV with respect to neutron, proton and α particle emission. Variations of constants will affect the energy levels of the ^{5}He, ^{5}Li, ^{8}Be and ^{12}C nuclei (Berengut et al. 2010; Ekström et al. 2010), and hence, the resonance energies whose contributions dominate the reaction rates. In addition, since ^{8}Be is only slightly unbound, one can expect that for even a small change in the nuclear potential, it could become bound and may thus severely impact the results of Standard BBN (SBBN). It has been suspected that stable ^{8}Be would trigger the production of heavy elements in BBN, in particular that there would be significant leakage of the nucleosynthetic chain into carbon (Bethe 1939). Indeed, as we have seen previously (Ekström et al. 2010), changes in the nuclear potential strongly affects the triple–alpha process and as a result, strongly affects the nuclear abundances in stars. The effects of the

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variation of fundamental constants on nuclear reaction rates is difficult to model. However, one can proceed in a two step approach: first by determining the dependencies of the light element abundances on the nuclear parameters and then by relating those parameters to the fundamental constants.

We have considered the very first generation of stars which are thought to have been formed a few 10^8 years after the big bang, at a redshift of $z \sim 10 - 15$, and with *zero initial metallicity*. For the time being, there are no direct observations of those Pop. III stars but one may expect that their chemical imprints could be observed indirectly in the most metal-poor halo stars (Pop. II). The observation of C and O in these Pop. II stars puts constraints on the variation of fundamental constants.

When using inputs from CMB observations for the baryon density (Komatsu et al. 2011; Ade et al. 2013), BBN yields are in excellent agreement between the theoretical predictions and astrophysical determinations for the abundances of D and ^{4}He (Cyburt et al. 2008; Coc & Vangioni 2010), despite the discrepancy between the theoretical prediction of ^{7}Li and its determined abundance in halo stars. Primordial ^{4}He and D abundances, deduced from observation, give constraints on the variation of fundamental constants.

2. Thermonuclear reaction rate variations

It would be desirable to know the dependence, w.r.t. fundamental quantities, of each of the main SBBN reaction rates, together with those involved in stellar evolution. In Coc et al. (2007) we considered the dependence the first two BBN reactions: the n↔p weak interaction and the p(n,γ)d bottleneck. Later, we extended this analysis to the ^{3}H(d,n)^{4}He and ^{3}He(d,p)^{4}He reactions that proceed through the $A = 5$ compound nuclei ^{5}He and ^{5}Li and to the ^{4}He($\alpha\alpha,\gamma$)^{12}C reaction that could bridge the $A = 8$ gap (Coc et al. 2012b). The latter was previously considered in the context of Pop. III stars (Ekström et al. 2010).

The weak rates that exchange protons with neutrons can be calculated theoretically and their dependence on G_F (the Fermi constant), Q_{np} (the neutron–proton mass difference) and m_e (the electron mass) is explicit (Dicus et al. 1982). The dependence of the n+p→d+γ rate (Ando et al. 2006) cannot be directly related to a few fundamental quantities as for the weak rates, but modeling of its dependence on the binding energy of deuteron B_D has been proposed (Dmitriev et al. 2004; Carrillo-Serrano et al. 2013).

For the ^{3}H(d,n)^{4}He, ^{3}He(d,p)^{4}He and ^{4}He($\alpha\alpha,\gamma$)^{12}C reactions, we used a different approach. In these three reactions, the rates are dominated by the contribution of resonances whose properties can be calculated within a microscopic cluster model. The nucleon-nucleon interaction $V(\mathbf{r})$ depends on the relative coordinate is written as:

$$V(\mathbf{r}) = V_C(\mathbf{r}) + (1 + \delta_{\rm NN})V_N(\mathbf{r}), \quad (1)$$

where $V_C(\mathbf{r})$ is the Coulomb force and $V_N(\mathbf{r})$ the nuclear interaction. The parameter $\delta_{\rm NN}$ characterizes the change in the nucleon-nucleon interaction. When using the Minnesota force (Thompson et al. 1977), it is related to the binding energy of deuterium by $\Delta B_D/B_D = 5.7701 \times \delta_{\rm NN}$ (Ekström et al. 2010). The next important step is to relate ΔB_D to the more fundamental parameters. To summarize, $B_{\rm D}$ has been related, within an ω and σ mesons exchange potential to quark masses and $\Lambda_{\rm QCD}$ by Flambaum & Shuryak (2003) and subsequently to more fundamental parameters [see Coc et al. (2007) and references therein], and in particular to the fine structure constant α.

2.1. The triple–alpha

The triple-alpha reaction is a two step process in which, first, two alpha–particles fuse into the ^{8}Be ground state, so that an equilibrium ($2\alpha \leftrightarrow {}^8$Be) is achieved. The second step is another alpha capture to the Hoyle state in ^{12}C. In our cluster approximation the wave functions of the ^{8}Be and ^{12}C nuclei are approximated by two and three-cluster wave functions involving the alpha particle, considered as a cluster of 4 nucleons. It allows the calculation of the variation of the ^{8}Be ground state and ^{12}C Hoyle state w.r.t.

the nucleon–nucleon interaction, i.e. δ_{NN}. In Ekström et al. (2010), we obtained $E_{g.s.}(^8\mathrm{Be})$ = (0.09208 – 12.208 × δ_{NN}) MeV, for the ^{8}Be g.s. and $E_R(^{12}\mathrm{C})$ = (0.2877 – 20.412 × δ_{NN}) MeV, for the Hoyle state. From these relations, it is possible to calculate the partial widths, and subsequently the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ rate as a function of δ_{NN} (Ekström et al. 2010). Indeed, variations of δ_{NN} of the order of 1%, induces orders of magnitude variations of the rate at temperatures of a few 100 MK. In addition, one sees that $E_{g.s.}(^8\mathrm{Be})$ (relative to the 2–α threshold) becomes negative (i.e. ^{8}Be becomes stable) for $\delta_{NN} \gtrsim 7.52 \times 10^{-3}$. In that case, we have to calculate the two reaction rates, $^4\mathrm{He}(\alpha,\gamma)^8\mathrm{Be}$ and $^8\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}$ for a stable ^{8}Be. The calculation of the rate of the second reaction can be achieved using the sharp resonance formula with the varying parameters of the Hoyle state from Ekström et al. (2010). For the first reaction, $^4\mathrm{He}(\alpha,\gamma)^8\mathrm{Be}$, we have performed a dedicated calculation following Baye & Descouvemont (1985) to obtain the astrophysical S-factor, and reaction rate, for values of the ^{8}Be binding energy of $B_8 \equiv -E_{g.s.}(^8\mathrm{Be})$ = 10, 50 and 100 keV.

2.2. The ^{3}He(d,p)^{4}He and ^{3}H(d,n)^{4}He reactions

The ^{3}He(d,p)^{4}He and ^{3}H(d,n)^{4}He reactions proceeds through the ^{5}Li and ^{5}He compound nuclei and their rates are dominated by contributions of $\frac{3}{2}^+$ broad resonances, well approximated by cluster structures (^{3}He⊗d or t⊗d). The level shifts obtained with the same cluster model as for the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ reaction are given by $\Delta E_R = -0.327 \times \delta_{NN}$ for ^{3}H(d,n)^{4}He and $\Delta E_R = -0.453 \times \delta_{NN}$ for ^{3}He(d,p)^{4}He (units are MeV) (Coc et al. 2012b). These energy dependences are much weaker, ∼ 20–30 keV for $|\delta_{NN}| \leq 0.03$, than for ^{8}Be and ^{12}C. This is expected for broad resonances which are weakly sensitive to the nuclear interaction. (Unlike in the case of ^{8}Be, the ^{5}He and ^{5}Li nuclei are unbound ∼1 MeV, therefore the issue of producing $A = 5$ bound states is irrelevant.) To calculate the S–Factor, we used a single pole R–matrix approximation that, for δ_{NN}=0, reproduces well our full R–matrix analysis (Descouvemont et al. 2004) of the experimental data. When the additional resonance shifts are introduced in the calculation, the rates are found to be little sensitive to N-N interaction ($\lesssim$5% for $|\delta_{NN}| \leq 0.03$). The corresponding effect on BBN is hence found to be negligible, contrary to Berengut et al. (2010) who use a different N-N interaction, not well adapted to broad resonances and a less elaborate parameterization of the cross–section.

3. Limits on variations of constants

3.1. Limits from Pop. III stellar evolution

Stellar production of ^{12}C through the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ reaction has been recognized as a way to set constraints the variation of constants. This is because of the high sensitivity of its rate to small shifts of the Hoyle state energy. This was investigated for red giant stars (1.3, 5 and 20 $M_\odot$ with solar metallicity) up to thermally pulsing asymptotic giant branch stars (TP-AGB) (Oberhummer et al. 2001) and in low, intermediate and high mass stars (1.3, 5, 15 and 25 $M_\odot$ with solar metallicity) up to TP-AGB (Schlattl et al. 2004). It was estimated that outside a window of 0.5% and 4% for the values of the strong and electromagnetic forces respectively, the stellar production of carbon or oxygen will be reduced by a factor 30 to 1000. We have considered (Ekström et al. 2010) instead, Pop. III stars, to extend the constraints to redshifts of $z \sim 10 - 15$. We used the Geneva code, assuming no rotation (Ekström et al. 2008) to model 15 and 60 $M_\odot$ stars with zero metallicity. The computations are stopped at the end of the core helium burning (CHeB).

Figure 1 shows the central abundances at the end of core He burning for those models showing that ^{12}C, ^{16}O or ^{24}Mg is dominant depending on the N-N interaction. These results show that the variation of the N–N interaction should be in the range $-0.0005 < \delta_{NN} < 0.0015$ (dotted lines) to insure that the C/O ratio be of the order of unity, as expected from observations in the most metal-poor halo stars.

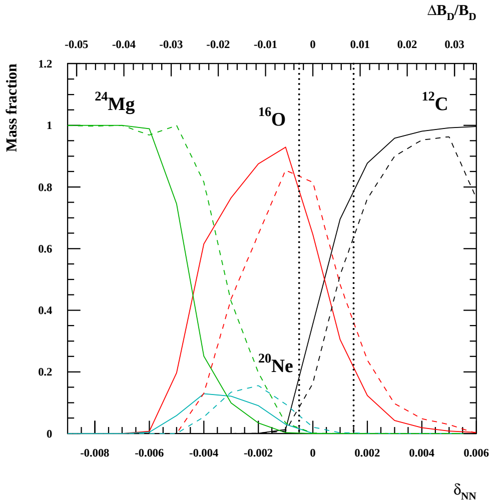

Fig. 1. Central abundances at the end of CHe burning as a function of δ_{NN} for 15 (solid) and 60 (dash) $M_\odot$ stars.

3.2. Limits from BBN

For ^{4}He, D, ^{3}He and ^{7}Li, we found that the effect of the ^{3}He(d,p)^{4}He and ^{3}H(d,n)^{4}He rate variations was negligible compared to the effect of the n$\leftrightarrow$p and n(p,γ)d reaction rate variations that we considered in our previous work (Coc et al. 2007). Then, with updated D and ^{4}He primordial abundances abundances deduced from observations, we obtained $-0.0025 < \delta_{\rm NN} < 0.0006$ (Coc et al. 2012b), for typical values of the parameters. We can easily extend our analysis by allowing both η_{10} and $\delta_{\rm NN}$ to vary. This allows one to set a joint constraint on the two parameters $\delta_{\rm NN}$ and baryonic density, as depicted on Figure 2. No combination of values allows for the simultaneous fulfillment of the ^{4}He, D and ^{7}Li observational constraints: those allowed variations in $\delta_{\rm NN}$ are too small to reconcile ^{7}Li abundances with observations. Note that the most influential reaction on ^{7}Li is surprisingly (Coc et al. 2007; Flambaum & Shuryak 2003) n(p,γ)d. The dependence of this rate to $B_{\rm D}$ that we have used comes from Dmitriev et al. (2004) more recent but options (Carrillo-Serrano et al. 2013; Berengut et al. 2013) need to be considered.

4. CNO production in BBN

4.1. Standard model

The main difficulty in BBN calculations up to CNO is the extensive network needed, including n-, p-, α-, but also d-, t- and ^{3}He-induced reactions in the A=1 to 20 range. Most of the corresponding cross sections cannot be extracted from experimental data only and we used extensively the theoretical rates provided by the TALYS code (Goriely et al. 2008). A detailed analysis of all reaction rates and associated uncertainties would be desirable but is impractical for a network of $\approx$400 reactions. So we first performed (Coc et al. 2012a) a sensitivity study to identify the most important reactions, followed by dedicated re-evaluations. The CNO production was found significantly sensitive to a few reaction rates only and the main nuclear path to CNO is displayed in Fig. 3. From the uncertainty on the (re-)evaluated reaction rates, we can estimate the range of CNO/H values to $(0.5-3.)\times10^{-15}$, consistent with those of Iocco et al. (2007).

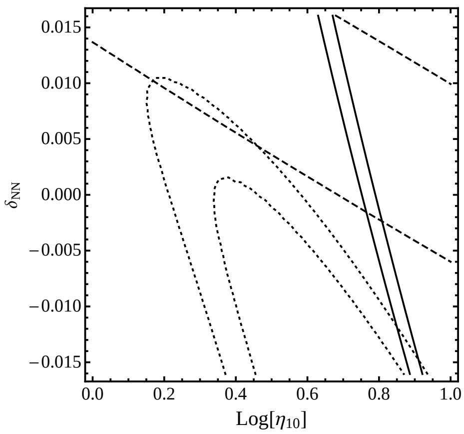

Fig. 2. Limits on of η_{10} (the number of baryons per 10^{10} photons) and $\delta_{\rm NN}$ provided by observational constraints on D (solid) ^{4}He (dash) and ^{7}Li (dot).

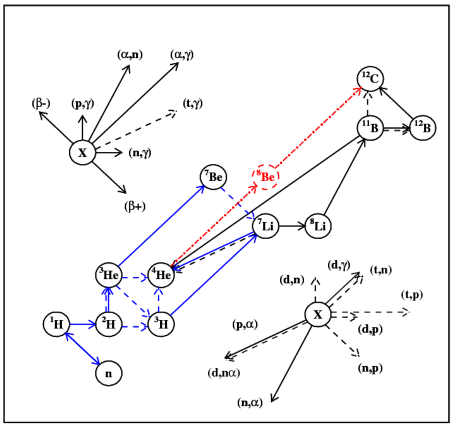

Fig. 3. Main reactions for ^{4}He, D, ^{3}He and ^{7}Li, and standard CNO (^{12}C) production (solid and dashed arrows) or through the 3–α reaction (dash-dot).

4.2. The 3–α reaction

Finally, we investigated the production of ^{12}C by the ^{4}He$(\alpha\alpha,\gamma)^{12}$C, or the ^{4}He$(\alpha,\gamma)^8$Be and ^{8}Be$(\alpha,\gamma)^{12}$C reactions as a function of $\delta_{\rm NN}$, to be compared with the CNO Standard BBN production presented above. To disentangle the ^{12}C production through the ^{4}He→^{8}Be→^{12}C link, from the standard ^{7}Li→^{8}Li→^{11}B→^{12}C paths (Fig. 3), we reduced the network to the reactions involved in $A < 8$ plus the ^{4}He$(\alpha\alpha,\gamma)^{12}$C, or the ^{4}He$(\alpha,\gamma)^8$Be and ^{8}Be$(\alpha,\gamma)^{12}$C reactions *only*, depending whether or no ^{8}Be would be stable for a peculiar value of $\delta_{\rm NN}$. The carbon abundance shows a maximum at $\delta_{\rm NN} \approx 0.006$, C/H$\approx 10^{-21}$ (Coc et al. 2012b), which is *six orders of magnitude* below the carbon abundance in SBBN (Coc et al. 2012a). Note that the maximum is achieved for $\delta_{\rm NN} \approx 0.006$ when ^{8}Be is still unbound so that contrary to a common belief, a stable ^{8}Be would not have allowed the buildup of heavy elements during BBN. This is illustrated in Fig. 4 which displays the evolution of the ^{12}C and ^{8}Be mass fractions as a function of time when ^{8}Be is supposed to be bound by 10, 50 and 100 keV (solid lines). They both increase with time until equilibrium between two α–particle fusion and ^{8}Be photodissociation prevails as shown by the dotted lines. For $B_8 \gtrsim 10$ keV, the ^{12}C production falls well below, out of the frame, because the ^{8}Be$(\alpha,\gamma)^{12}$C reaction rate decreases dramatically due to the downward shift of the Hoyle state. For comparison, the Standard BBN ≈400 reactions network is also plotted (dashed line).

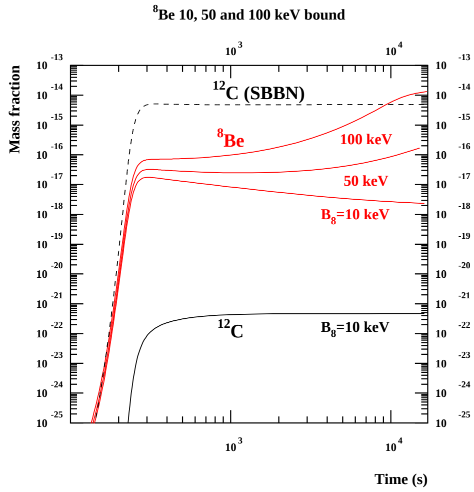

Fig. 4. ^{12}C and ^{8}Be mass fractions as a function of time, assuming ^{8}Be is bound by 100, 50 and 10 keV as shown by the upper to lower solid curves respectively (Coc et al. 2012b). (Only the ^{12}C mass fraction curve, for B_8 = 10 keV, is shown; others are far below the scale shown). The dashed line corresponds to the Standard BBN production (Coc et al. 2012a).

5. Discussion

We have investigated the influence of the variation of the fundamental constants on the predictions of BBN and Pop. III stellar evolution. Through our detailed modeling of the cross-sections we have shown that, although the variation of the nucleon-nucleon potential can greatly affect the triple–α process, its effect on BBN and the production of heavier elements such as CNO is typically 6 orders of magnitude smaller than standard model abundances. Even when including the possibility that ^{8}Be can be bound, at the temperatures, densities and timescales associated with BBN, the changes in the ^{4}He($\alpha\alpha,\gamma$)^{12}C and ^{8}Be(α,γ)^{12}C reaction rates are not sufficient. We have considered the ^{3}He(d,p)^{4}He and ^{3}H(d,n)^{4}He cross-sections involving ^{5}He and ^{5}Li but found that their effect remain small compared to the n(p,γ)d induced variation. Finally, relating $\delta_{\rm NN}$ to $\Delta\alpha/\alpha$ (Coc et al. 2007), we obtained limits of $-3. \times 10^{-6} < \Delta\alpha/\alpha < 10^{-5}$ from Pop. III (Ekström et al. 2010) and $-4 \times 10^{-6} < \Delta\alpha/\alpha < 1.6 \times 10^{-5}$ from BBN (Coc et al. 2012b) for the fine structure constant variations.

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