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White dwarf constraints on a varying G

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Abstract. A secular variation of G modifies the structure and evolutionary time scales of white dwarfs. Using an state-of-the-art stellar evolutionary code, an up-to-date pulsational code, and a detailed population synthesis code we demonstrate that the effects of a running G are obvious both in the properties of individual white dwarfs, and in those of the white dwarf populations in clusters. Specifically, we show that the white dwarf evolutionary sequences depend on both the value of $\dot{G}/G$, and on the value of G when the white dwarf was born. We show as well that the pulsational properties of variable white dwarfs can be used to constrain $\dot{G}/G$. Finally, we also show that the ensemble properties of of white dwarfs in clusters can also be used to set upper bounds to $\dot{G}/G$. Precisely, the tightest bound — $\dot{G}/G \sim -1.810^{-12}$ yr^{-1} — is obtained studying the population of the old, metal-rich, well populated, open cluster NGC 6791. Less stringent upper limits can be obtained comparing the theoretical results obtained taking into account the effects of a running G with the measured rates of change of the periods of two well studied pulsating white dwarfs, G117–B15A and R548. Using these white dwarfs we obtain $\dot{G}/G \sim -1.8 \times 10^{-10}$ yr^{-1}, and $\dot{G}/G \sim -1.3 \times 10^{-10}$ yr^{-1}, respectively, which although less restrictive than the previous bound, can be improved measuring the rate of change of the period of massive white dwarfs.

Key words. Stars – White dwarfs – Gravity

1. Introduction

General Relativity is the favorite theory of gravitation, and it is based on the equivalence principle, and in the end on the assumption that the gravitational constant, G, is indeed constant. However, this is just a hypothesis which needs to be verified. In fact, there are several modern grand-unification theories

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Send offprint requests to: E. García–Berro that predict that the value of G is a varying function of a low-mass dynamical scalar field (Lorén-Aguilar et al. 2003; García-Berro et al. 2007). Hence, we expect that if these theories are true the gravitational constant should experience slow changes over cosmological timescales. In recent years, several constraints have been placed on the variation of the fine structure constant, and other interesting constants of nature — see Uzan (2003), and García-Berro et al. (2007) for extensive reviews. However, very few works have been devoted to study a hypothetical variation of G. The most tight bounds on the variation of G are those obtained using Lunar Laser Ranging — $\dot{G}/G = (0.2 \pm 0.7) \times 10^{-12}$ yr^{-1} (Hofmann et al. 2010) — solar asteroseismology — $\dot{G}/G \simeq -1.6{\times}10^{-12}$ yr^{-1} (Guenther et al. 1998) — and Big Bang nucleosynthesis — -0.3×10^{-12} yr^{-1} $\lesssim \dot{G}/G \lesssim 0.4 \times 10^{-12}$ yr^{-1} (Copi et al. 2004; Bambi et al. 2005). Nevertheless, both Lunar Laser Ranging and asteroseismological bounds are eminently local, while Big Bang limits are model-dependent. At intermediate cosmological ages the Hubble diagram of Type Ia supernovae has also been used to put constraints on the rate of change of G, but the constraints are somewhat weaker $\dot{G}/G \lesssim 1 \times 10^{-11}$ yr^{-1} at $z \sim 0.5$ (Gaztañaga et al. 2002; García-Berro et al. 2006). In this work we summarize why on how white dwarfs can be used to place constraints on the rate of variation of a rolling G.

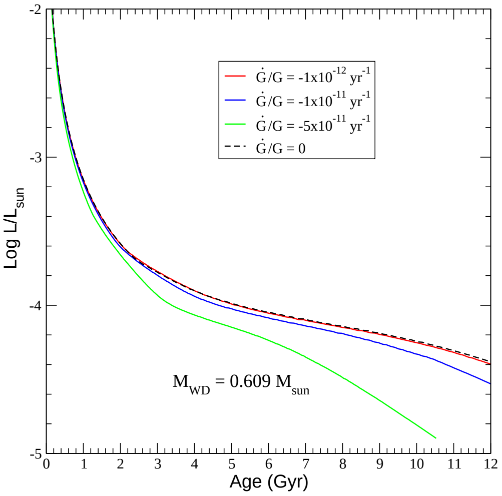

Fig. 1. Surface luminosity versus age for several 0.609 $M_\odot$ white dwarf sequences, adopting different values of $\dot{G}/G$.

2. White dwarf cooling times

A slowly rolling G affects both the cooling timescales of white dwarfs (García-Berro et al. 1995; Althaus et al. 2011) and those of their progenitors (degl'Innocenti et al. 1996). Consequently, the ages determined from the color-magnitude diagrams of globular or open clusters — namely, the main-sequence turn-off age and the age determined from the termination of the white dwarf cooling sequence — change accordingly, and depend on the precise value of $\dot{G}/G$ (García-Berro et al. 2011). This allows to put upper bounds on the rate of variation of G. To quantify the effects of a varying G on the derived ages, we computed the main sequence evolution of two model stars of 1.0 and 2.0 $M_\odot$ considering three values for the rate of change of G, namely $\dot{G}/G = -5 \times 10^{-11}$ yr^{-1}, $\dot{G}/G = -1 \times 10^{-11}$ yr^{-1}, and $\dot{G}/G = -1 \times 10^{-12}$ yr^{-1}. All the evolutionary calculations were done using the `LPCODE` stellar evolutionary code (Althaus et al. 2010; Renedo et al. 2010), appropriately modified to take into account the effect of a varying G. Despite the small rates of change of G adopted here, the evolution of white dwarf progenitor stars is severely modified. The evolutionary timescales can be modelled using rather simple arguments. In particular, it turns out that the main sequence lifetimes when a varying G is adopted is (García-Berro et al. 2011):

$$\tau_{\rm MS} = \frac{1}{\gamma\left|\frac{\dot{G}}{G}\right|}\ln\left[\gamma\left|\frac{\dot{G}}{G}\right|\left(\frac{G_0}{G_i}\right)^{\gamma}\tau^0_{\rm MS}+1\right]. \tag{1}$$

with $\gamma = 3.6$. The effect of a varying G on the white warf cooling times is displayed in Fig. 1. As can be seen, the cooling timescales are considerably modified, being the cooling accelerated in the case of $\dot{G} < 0$ (Althaus et al. 2011). This can be explained easily. A smaller value of G implies a smaller gravitational force, and thus a smaller degeneracy (and density) is needed to balance gravity.

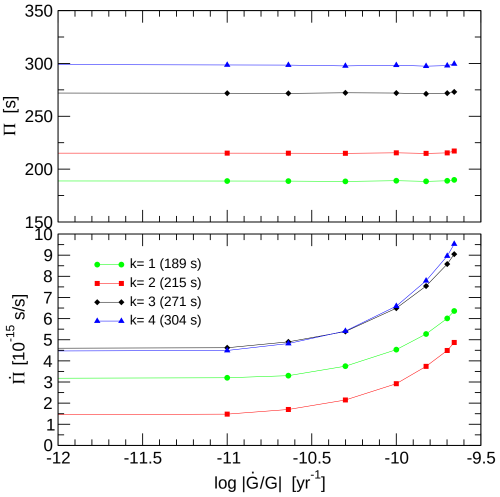

Fig. 2. Upper panel: periods of the several modes of G117−B15A as a function of the value of $|\dot{G}/G|$ with $\dot{G} < 0$. Lower panel: period derivatives of the same modes as a function of the secular rate of change of G.

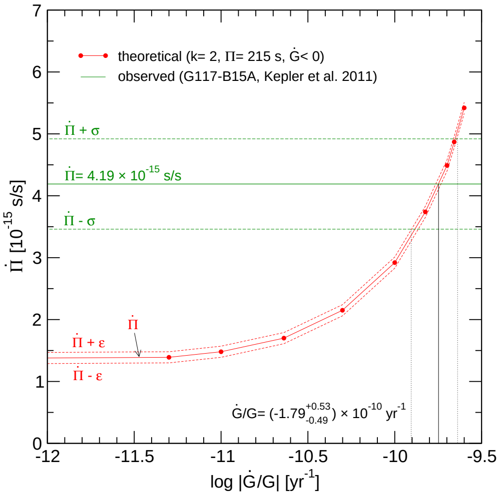

Fig. 3. Rate of temporal variation of the 215 s period of G117−B15A as a function of $\dot{G}/G$, red solid line. The observational value of the rate of change of the period — horizontal solid line — along with its observed error bars — horizontal dashed lines — is also displayed. The formal theoretical errors are also shown as dashed lines.

Hence, for $\dot{G} < 0$ the white dwarf expands as it evolves, and the cooling is accelerated. The cooling track shown in Fig. 1 is a representative example of a set of white dwarf cooling sequences which incorporate the most up-to-date physical inputs. Specifically, these cooling sequences consider ^{22}Ne diffusion and its associated energy release (García-Berro et al. 2010; Althaus et al. 2010; García-Berro et al. 2008), and together with the main sequence lifetimes given by Eq. (1) allow to derive an age for any cluster, and for each value of $\dot{G}/G$. Moreover, the grid of models has been computed for several initial values of G, to take into account that the evolutionary value of G must match its present value. With these sequences the effect of a running G on the color-magnitude diagram of any cluster can be studied.

We chose to employ the old, metal-rich, well populated open cluster NGC 6791, for which the ages derived from main sequence stars and from the termination of the degenerate sequence agree very well in the case of a constant G. When a varying G is adopted, the resulting age of NGC 6791 is modified, but then the position of the main sequence turn-off in the color-magnitude diagram is also significantly different if the same distance modulus is adopted. However, the distance modulus derived using an independent and reliable method (eclipsing binaries) which does not make use of theoretical models turns out to be 13.46±0.1 (Grundahl et al. 2008). Thus, large errors in the distance modulus seem to be quite implausible. Given the uncertainty in the distance modulus ($\simeq 0.1^{\rm mag}$), and the measured value, an upper limit to $\dot{G}/G$ can be placed (García-Berro et al. 2011). Since $\Delta t_{\rm MSTO}/\Delta(m-M)_{\rm F606W} \approx 4$ Gyr/mag, the maximum age difference with respect to the case in which a constant G is adopted is ~ 0.4 Gyr, which translates into an upper bound $\dot{G}/G \sim -1.8\times10^{-12}$ yr^{-1}. This upper limit considerably improves the other existing upper bounds to the rate of variation of G and is equivalent to the upper limit set by helioseismology.

3. Pulsating white dwarfs

Pulsations in white dwarfs are associated to nonradial g(gravity)-modes which are a subclass of spheroidal modes whose main restoring force is gravity. These modes are characterized by low oscillation frequencies (long periods) and by a displacement of the stellar fluid essentially in the horizontal direction. Hence, some characteristics of the pulsations are sensitive to the precise value of G, and to its rate of change. In particular, it can be easily understood that measuring the rate of change of the period is equivalent to measure the evolutionary time scale of the white dwarf. Thus, a slowly varying G should have an impact in the rate of period change of the observed periods. There are two white dwarfs for which we have reliable determinations of the rate of period change of its main periods, namely G117−B15A and R548. We performed an asteroseismological analysis of these white dwarfs using a grid of fully evolutionary DA models (Romero et al. 2012) characterized by consistent chemical profiles for both the core and the envelope, and covering a wide range of stellar masses, thicknesses of the hydrogen envelope and effective temperatures.

The pulsation periods for the modes with $\ell = 1$ and $k = 1, 2, 3$ and 4 of the asteroseismological model of G117−B15A for increasing values of $|\dot{G}/G|$ are shown in the upper panel of Fig. 2. The variation of the periods is negligible, implying that a varying G has negligible effects on the structure of the asteroseismological model, and that, for a fixed value of the effective temperature, the pulsation periods are largely independent of the adopted value of $|\dot{G}/G|$. In the lower panel of Fig. 2 we display the rates of period change for the same modes. At odds with what happens with the pulsation periods, the rates of period change are markedly affected by a varying G, substantially increasing for increasing values of $|\dot{G}/G|$. This is because that, for a decreasing value of G with time, the white dwarf cooling process accelerates (García-Berro et al. 1995; Althaus et al. 2011), and this is translated into a larger secular change of the pulsation periods as compared with the situation in which G is constant.

In Fig. 3 we plot the theoretical value of $\dot{\Pi}$ the mode with period $\Pi = 215$ s of G117−B15A for increasing values of $|\dot{G}/G|$ (solid curve). The dashed curves embracing the solid curve show the uncertainty in the theoretical value of $\dot{\Pi}$, $\epsilon_{\dot{\Pi}} = 0.09 \times 10^{-15}$ s s^{-1}. This value has been derived taking into account the uncertainty due to our lack of knowledge of the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction rate — $\varepsilon_1 \sim 0.03 \times 10^{-15}$ s/s — and that due to the errors in the asteroseismological model — $\varepsilon_2 \sim 0.06 \times 10^{-15}$ s/s Córsico et al. (2012). We assumed that the uncertainty for the case in which $\dot{G} \neq 0$ is the same as that computed for the case in which $G = 0$, which is a reasonable assumption. Considering that the theoretical solution should not deviate more than one standard deviation from the observational value, we conclude that the secular rate of variation of the gravitational constant obtained using the variable DA white dwarf G117−B15A is $\dot{G}/G = (-1.79^{+0.53}_{-0.49}) \times 10^{-10}$ yr^{-1} (Córsico et al. 2013). The same analysis applied to the other star, R548, results in $\dot{G}/G = (-1.29^{+0.71}_{-0.63}) \times 10^{-10}$ yr^{-1}, a very similar upper bound — see Córsico et al. (2013) for a detailed discussion. Clearly, these values are completely compatible each other, although currently less restrictive than those obtained using other techniques, and compatible with a null result for $\dot{G}/G$.

4. Discussion, conclusions and outlook

In this work we have reviewed the several constraints that white dwarfs can provide on the rate of change of a secularly varying gravitational constant. Specifically, we have shown that when a secularly evolving value of G is adopted, the white dwarf cooling tracks (and the main sequence evolutionary times of their progenitors as well) are noticeably affected, and depend sensitevely not only on the value of $\dot{G}/G$ but also on the actual value of G when the white dwarf was born. We find that for negative values of $\dot{G}$ the cooling is accelerated, due a less intense gravitational interaction. According to these results the main sequence turn-off age and the age derived from the termination of the white dwarf cooling sequence differ from those computed when a constant value of Newton's constant is adopted. This can be used to constrain the rate of variation of a rolling G. In particular, we have applied this technique to the metal rich, well populated, olg open cluster NGC 6791. It turns out that the resulting age of NGC 6791 is considerably modified modified, as it occurs with the position of the main sequence turn-off in the color-magnitude diagram, if the same distance modulus is adopted. Accordingly, the distance modulus necessary to fit the position in the color-magnitude diagram of the main sequence turn-off of the cluster needs to be changed as well. However, the distance modulus derived using an independent and reliable method (eclipsing binaries) which does not make use of theoretical models turns out to be 13.46 ± 0.1 (Grundahl et al. 2008). Thus, large errors in the distance modulus seem to be quite implausible. Given the uncertainty in the distance modulus ($\simeq 0.1^{\rm mag}$), and the measured value, an upper limit to $\dot{G}/G$ can be placed. Since $\Delta t_{\rm MSTO}/\Delta(m-M)_{\rm F606W} \approx 4$ Gyr/mag, the maximum age difference with respect to the case in which a constant G is adopted should be ~ 0.4 Gyr, which translates into an upper bound $\dot{G}/G \sim -1.8 \times 10^{-12}$ yr^{-1}. This upper limit considerably improves the other existing upper bounds to the rate of variation of G and is equivalent to the upper limit set by helioseismology.

We have also shown that individual pulsating hydrogen-rich white dwarfs can also be useful in setting upper limits to the rate of variation of G, although these upper bounds are currently less restrictive than those obtained using the color-magnitude diagram of clusters. In essence, we have found that the periods of the dominant modes of the two white dwarfs (G117–B15A and R548) for which we have reliable observational determinations of their rate of period change of their main modes, are not affected, thus allowing to derive an excellent asteroseismological fit of their pulsational spectra. However, the rates of period change of their dominant modes are severely affected, as a consequence of the fact that the rates of change of these periods reflect their evolutionary changes, and thus allow to measure their evolutionary time scales. Accordingly, in the case of an hypothetical smooth variation of the gravitational constant, and due to the sensitivity of the cooling time scales to the precise value of $\dot{G}$, the rates of period change are modified. Our calculations in the case of a running value of G allow to compare the predictions of the theoretical models with the observational rates of period change, and hence to derive constraints on the value of $\dot{G}/G$. Using this technique we found that the upper bounds are $\dot{G}/G = (-1.79^{+0.53}_{-0.49}) \times 10^{-10}$ yr^{-1}, and $\dot{G}/G = (-1.29^{+0.71}_{-0.63}) \times 10^{-10}$ yr^{-1}, for G117–B15A and R548 respectively. We emphasize that although these upper limits are less restrictive than those obtained using the previously described technique, they could be much improved should we have reliable observational determinations of the rate of change of the dominant periods of pulsating massive white dwarfs, as the effect of a running G is more evident for these white dwarfs, due to their larger gravitational field.

Last, but not least, we would like to emphasize here that there is still room for new (and possibly exciting) studies that have not been addressed here, and that the results of such studies could translate in interesting improved constraints. To be precise, we now have excellent observational luminosity functions of the white dwarf population of the Galactic disk, which are the result of both magnitude-limited large scale surveys — like the Sloan Digital Sky Survey (De Gennaro et al. 2008; Harris et al. 2006) or the SuperCOSMOS sky survey (Rowell & Hambly 2011) — or of volume-limited surveys (Giammichele et al. 2012). The completenesses of the large surveys is expected to be high ($\sim 80\%$), while the volume-limited sample is thought to be nearly complete. The white dwarf luminosity function reflects the characteristic cooling time of the population of white dwarfs as a function of the absolute bolometric magnitude, and has two distinctive features. The first of these properties is a monotonic increase until luminosities of the order of $\log(L/L_{\odot}) \sim -3.5$, which is simply a consequence of the the fact that due to the absence of energy sources other than the gravothermal energy of white dwarfs, the cooler a white dwarf the longer it takes to cool further. The second — and for our purposes most important feature — of the disk white dwarf luminosity function is the presence a sharp drop-off at $\log(L/_{\odot}) \sim -4.5$. This pronnounced cut-off is the obvious consequence of the finite age of the Galactic disk. The origin of this deficit of cool stars is clear: white dwarfs have not had time enough to cool down beyond this luminosity. Since the cooling process of white dwarfs is sensitive to $\dot{G}/G$ it is rather evident that in the case of a secularly varying G the position of the cut-off should be different. Such an analysis still remains to be done, as it requires the calculation of an extensive set of cooling sequences for different values of $\dot{G}/G$ and the initial value of G, which requires considerable efforts, but it is one of our priorities for the next future.

In summary, we have demonstrated that due to their relative structural simplicity, to the fact that the gravothermal cooling process that governs their evolution is well understood, to the well determined individual and ensemble properties, and to the sensitivity of their properties to the value of $\dot{G}/G$, white dwarf stars can be used to constrain alternative theories of gravitation, and that future efforts, both on the observational and on the theoretical sides, can result in improved upper bounds on the rate of change of the gravitational constant.

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