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The mass and radius evolution of globular clusters in tidal fields

Mark Gieles

Department of Physics – University of Surrey, Guildford GU2 7XH , UK
e-mail: m.gieles@surrey.ac.uk

Abstract. We present a simple theory for the evolution of initially compact clusters in a tidal field. The fundamental ingredient of the model is that a cluster conducts a constant fraction of its own energy through the half-mass radius by two-body interactions every half-mass relaxation time. This energy is produced in a self-regulative way in the core by an (unspecified) energy source. We find that the half-mass radius increases during the first part (roughly half) of the evolution and decreases in the second half, while the escape rate is constant and set by the tidal field. We present evolutionary tracks and isochrones for clusters in terms of cluster half-mass density, cluster mass and galacto-centric radius. We find substantial agreement between model isochrones and Milky Way globular cluster parameters, which suggests that there is a balance between the flow of energy and the central energy production for almost all globular clusters. We also find that the majority of the globular clusters are still expanding towards their tidal radius. Finally, a fast code for cluster evolution is presented.

Key words. Galaxy: globular clusters – Stars: kinematics and dynamics

1. Introduction

Capturing cluster evolution in equations is complex because several processes, including two-body relaxation, interactions with binary stars, escape across the tidal boundary, and the internal evolution and mass-loss of single and binary stars are all at work at the same time. However, it is desirable to have a simple parameterisation of the evolution of some fundamental cluster parameters such as mass and radius (e.g. Prieto & Gnedin 2008).

Here we provide a physically motivated and simple prescription for the behaviour of the half-mass radius and tidal radius (i.e. mass). With this we construct evolutionary tracks and isochrones for clusters (i.e. not for the stars within them!) evolving in a tidal field. This forms the theoretical framework to explain empirically established correlations between structural parameters and their environment as found for Milky Way globular clusters (e.g. Djorgovski 1995; McLaughlin 2000) and for extra-galactic globular cluster systems (e.g. Jordán et al. 2005; McLaughlin et al. 2008; Harris et al. 2010). We do not aim to explain the shape and dependence on environment of the globular cluster mass function.

The other principal exclusion is the evolution of the core parameters (i.e. the core mass and radius). Although the core is the place where the energy is produced, the assumption of excluding it in the model is justified by

Send offprint requests to: M. Gieles@surrey.ac.uk

the discovery of Hénon (1975) that the *rate of flow of energy is controlled by the system as a whole, and not by the core.* In Hénon's picture the mechanism of energy generation in the core is self-regulatory and so we can assume that the core produces the right amount of energy required by the system as a whole. This is comparable to the self-regulative energy production in stellar cores, which was first realised by Eddington. The application of this idea to stellar dynamics was a breakthrough allowing modellers to overcome the core collapse phase.

From N-body simulations of the long term (post-collapse) evolution of single-mass clusters, i.e. where stars have the same mass, it was found that binary stars act as the energy source (Giersz & Heggie 1994; Baumgardt et al. 2002). In models with more realistic initial conditions these binaries are usually considered to be primordial. Other mechanisms of energy generation have been considered, including the action of a central intermediate-mass black hole (Baumgardt et al. 2004; Heggie et al. 2007)

Mass-loss from stellar evolution can also provide the energy for the dynamical evolution of clusters (Gieles et al. 2010). Typically most of the mass is lost from the most massive stars in the cluster, that reside in the cluster core as the result of mass segregation. The resulting energy production works together with binaries in driving an expansion on relaxation time-scales (Gieles et al. 2010). Indeed, it may even dominate, as in the specific example of 47 Tuc, a high-concentration cluster in which the evolution of the core- and half-mass radii appear to be little affected by the primordial binary population (Giersz & Heggie 2011).

These self-regulatory mechanisms of energy generation take time to establish the balance between the energy generated in the core and the energy requirements of the overall evolution of the cluster. We refer to the subsequent evolution as 'balanced'. In much research on cluster dynamics this kind of evolution is usually associated with 'post-collapse' evolution, but this term is ambiguous; the phrase 'post-collapse' might be used for the entire evolution after the end of mass segregation of massive stars ($\gtrsim 10\,\mathrm{M}_\odot$), whilst others use it to refer to the evolution following the decrease in the core radius after several Gyrs. For this reason we prefer the term 'balanced evolution'.

In Section 2 we present the model that unifies two models of Hénon. The evolution in the initial phase resembles that of the model for the isolated cluster which expands with little loss of stars (Hénon 1961, hereafter H61) and near final dissolution the cluster resembles the 'homologous cluster' (Hénon 1965, hereafter H65), which contracts at a constant density. Here the model is described in its simplest form and we refer to (Gieles, Heggie, & Zhao 2011, hereafter G11) for more details. In Section 3 the model is compared to parameters of the Milky Way globular clusters. In Section 4 a fast code for cluster evolution based on the theory presented in this work is presented.

2. Model

Our attempt to unify the evolution of mass and radius of the two models of Hénon begins with one of the physical properties which the two models have in common, i.e. a flux of energy at the half-mass radius which is fed by an energy source in the core. We constrain ourselves to the energy flow at this radius, because we can then construct a relatively simple set of relations for the behaviour of the bulk properties of the cluster. We adopt Hénon's idealisation of systems in which all stars have the same mass m. We estimate the total energy of the cluster as usual by

$$E = -\alpha\frac{GM^2}{r_{\mathrm{h}}}, \tag{1}$$

where M is the mass of the cluster, r_{h} is the half-mass radius, and $\alpha \simeq 0.2$ is a 'form factor'.

In constructing a unified approximate model which includes the transition from nearly isolated evolution to tidally limited evolution, we assume that in both phases there is an energy flow due to two-body relaxation of magnitude

$$\frac{\dot{E}}{|E|} = \frac{\zeta}{\tau_{\mathrm{rh}}}. \tag{2}$$

Here $\tau_{\rm rh}$ is the half-mass relaxation time-scale and ζ is a constant that can be interpreted as the efficiency of energy conduction. In Hénon's models $\zeta \simeq 0.08$[1] and from numerical simulations Alexander & Gieles (2012) find $\zeta \simeq 0.1$. We approximate the expression for $\tau_{\rm rh}$ by assuming that the Coulomb logarithm is constant, such that (Spitzer 1987)

$$\tau_{\rm rh} \propto N\tau_{\rm cr}. \tag{3}$$

Here $N = M/m$ is the total number of stars and $\tau_{\rm cr}$ is the crossing time of stars in the cluster at the half-mass radius. We define $\tau_{\rm cr}$ as

$$\tau_{\rm cr} \equiv (G\rho_{\rm h})^{-1/2}\,, \tag{4}$$

with $\rho_{\rm h} \equiv 3M/(8\pi r_{\rm h}^3)$ the cluster density within the half-mass radius.

Before proceeding further, we shall change the variables in which the total energy E is expressed, because this will facilitate the further development of our model. Instead of using the half-mass radius, $r_{\rm h}$, we shall use $\tau_{\rm cr}$ such that $E \propto -M^{5/3}\tau_{\rm cr}^{-2/3}$. Putting this together with our assumption about the energy flux (equation 2) we find

$$-\frac{5}{3}\frac{\dot{M}}{M} + \frac{2}{3}\frac{\dot{\tau}_{\rm cr}}{\tau_{\rm cr}} = \frac{\zeta}{\tau_{\rm rh}}. \tag{5}$$

We recall that $\tau_{\rm rh} \propto M\tau_{\rm cr}$ and thus equation (5) has two variables: M and $\tau_{\rm cr}$. The differential equation can be solved by relating $\dot{M}$ to $\tau_{\rm cr}$. Because $\dot{M}$ depends only on the orbit and is, to good approximation, independent of the cluster mass and radius (Lee & Ostriker 1987; Gieles & Baumgardt 2008) we can write for the dimensionless escape rate $(\dot{M}/M)\tau_{\rm rh} = -(3/5)\zeta\tau_{\rm cr}/\tau_{\rm cr1}$. Here $\tau_{\rm cr1}$ is the maximum $\tau_{\rm cr}$ which depends on the tidal field: if the tides are weak, the cluster can expand to larger $\tau_{\rm cr}$. Combining this dimensionless escape rate with equation (5) we find the dimensionless expansion rate $(\dot{\tau}_{\rm cr}/\tau_{\rm cr})\tau_{\rm rh} = (3/2)\zeta(1 - \tau_{\rm cr}/\tau_{\rm cr1})$. Dividing $\dot{M}$ by $\dot{\tau}_{\rm cr}$ we find the surprisingly simple relation between M and $\tau_{\rm cr}$

$$\frac{{\rm d}M}{{\rm d}\tau_{\rm cr}} = \frac{2}{5}\frac{M}{\tau_{\rm cr} - \tau_{\rm cr1}}. \tag{6}$$

[1] In G11 the values for the isolated and homologous cluster are derived from Hénon's papers.

Integration gives an expression for $\tau_{\rm cr}(M, \tau_{\rm cr1})$, i.e. the isochrones, and this, combined with equation (5), can be used to get the time-dependent solutions $M(t)$ and $\tau_{\rm cr}(t)$, i.e. the evolutionary tracks. We do not give the functional forms here, but instead refer the reader to G11. In the next section we proceed with a direct comparison between cluster isochrones and Milky Way globular clusters parameters.

3. Milky Way globular clusters

3.1. Are clusters still expanding?

In order to see if these types of predictions are relevant for real globular clusters we compare our results to the globular clusters of the Milky Way. We use the 2003 version of the Harris (1996) catalogue which contains entries for 150 globular clusters, and for 141 of them a luminosity, half-light radius and galacto-centric radius determination are available. To convert luminosity to mass we adopt a mass-to-light ratio of 2 (McLaughlin & van der Marel 2005) and we multiply the projected half-light radius by 4/3 to correct for the effect of projection (Spitzer 1987) and get an estimate for $r_{\rm h}$.

The first thing we determine from the data is the fraction of globular clusters that are in the expansion dominated phase. We define the end of the expansion phase as the moment where the time derivative of $\tau_{\rm rh}$ is zero, which is when the fractional change in M has the same magnitude as the fractional change in $\tau_{\rm cr}$, i.e. ${\rm d}\ln M/{\rm d}\ln\tau_{\rm cr} = -1$ (equation 3). Combined with equation (6) we then find that this happens when $M/M_0 = (2/7)^{2/5} \simeq 0.6$. In time this is when the cluster has evolved for 40% of its total life, because of the linear decrease of M. The end of the expansion phase therefore depends on M and the mass-loss rate $\dot{M}$, which depends on the orbit.

We express $\dot{M}$ in terms of the galactocentric radius $R_{\rm G}$. We assume that the Milky Way halo is an isothermal sphere and approximate the $R_{\rm G}$ dependent mass-loss rate by

$$\dot{M}R_{\rm G} \simeq -20\,{\rm M}_{\odot}\,{\rm Myr}^{-1}\,{\rm kpc}, \tag{7}$$

which is in reasonable agreement with the evaporation rates found in both N-body and

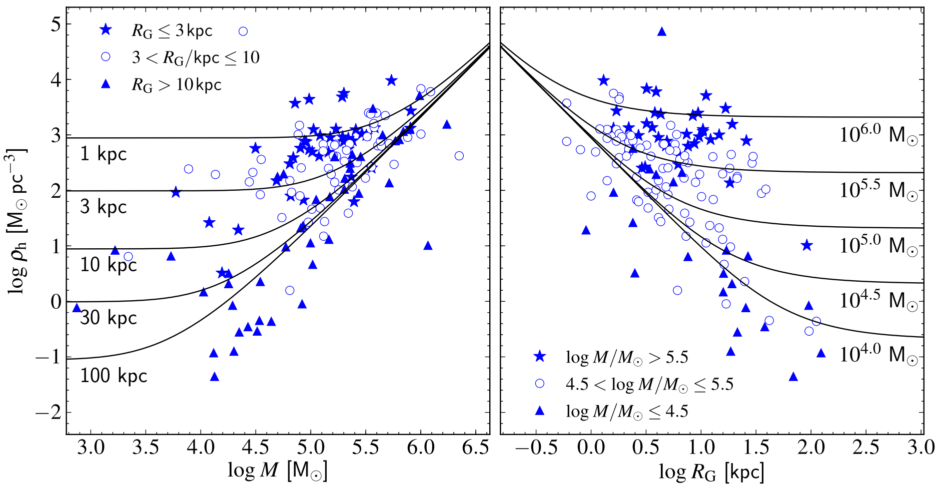

Fig. 1. Isochrones based on the balanced evolution model of Section 2. In the left panel 5 isochrones for different R_G values are shown and in the right panel 5 isochrones for different M. Both panels show the 141 globular clusters in the Harris (1996) catalogue from which M, r_h and R_G are available. In both panels different symbols are used for data in different R_G and M regimes.

Fokker-Planck models with a globular cluster type stellar mass function (G11). We adopt an age of 13 Gyr for all clusters. Clusters that are now at the end of the expansion phase have 60% of their evolution, or 1.5×13 Gyr, ahead of them. The remaining life-time, or life expectancy, is defined as $M/\dot{M}$ such that with equation (7) we find that clusters with a mass

$$M \gtrsim 10^5\,\mathrm{M}_\odot \left(\frac{4\,\mathrm{kpc}}{R_G}\right) \quad (8)$$

are still in the expansion-dominated phase. This relation is satisfied by 93 of the 141 clusters (i.e. roughly 2/3). It follows that the remaining 48 clusters (roughly 1/3) have expanded to the tidal boundary and are in the evaporation-dominated phase. This perhaps surprising result has some interesting consequences. The most important one is that the present day densities of the majority of the globular clusters follow (roughly) from the self-similar expansion model for isolated clusters: $\tau_{rh} \propto M\tau_{cr} \simeq$ constant, i.e. $\rho_h \propto M^2$. This scaling relation is caused by internal two-body relaxation and is independent of the tidal field; therefore a similar scaling, with the same proportionality, should also hold for extra-galactic clusters. Moreover, in extra-galactic cluster samples the fraction of clusters in the expansion-dominated phase is probably larger; they are easier to detect because they have (on average) higher mass (equation 8).

The prediction that a $\rho_h^{1/2} \propto M$ scaling must hold for the majority of the Milky Way globular clusters is one of the main results of this work.

3.2. Isochrones

Because all globular clusters have roughly the same age we focus on isochrones with an age of 13 Gyr, rather than the evolutionary tracks. For a given age the isochrones can be expressed as $\rho_h(M, R_G)$ and the detailed results are given in the appendices of G11. In Fig. 1 we show the isochrones $\rho_h(M)$ for several values of R_G (left) and $\rho_h(R_G)$ for several values of M (right) diagrams together with the 141 globular clusters for which data are available in the Harris catalogue.

In the left panel isochrones for clusters at different R_G between 1 kpc and 100 kpc are shown. The isochrones roughly encompass the data. The 100 kpc isochrone clearly shows the asymptotic $\rho_h^{1/2} \propto M$ behaviour following from expansion, which roughly follows the lower envelope of data points. In the outer halo the tidal field is so weak that all clusters with $M \gtrsim 10^4\,M_\odot$ are still expanding towards their tidal boundary. In the right panel the densities are shown as a function of R_G together with five isochrones for different masses. These isochrones also roughly encompass the data. The asymptotic behaviour of the isochrones in both diagrams is given by labels in the two diagrams.

4. A fast code for cluster evolution

The model presented here makes several approximations in order to facilitate simple analytical results. An improved version of the model, which includes the escape of stars in the isolated regime, the small N dependence in the Coulomb logarithm and the delayed escape of stars due to the anisotropic tidal field (Fukushige & Heggie 2000) is presented in Alexander & Gieles (2012). Here the differential equations for $\dot{M}$ and $\dot{r}_h$ are solved numerically with a Runge-Kutta solver, which gives near instantaneous results for $M(t)$ and $r_h(t)$ as a function of the tidal field strength and the initial cluster parameters which can be specified on the command line. The code accurately reproduces the results of N-body integrations of single-mass clusters (Alexander & Gieles 2012) and is publicly available on `https://github.com/emacss/emacss`. Future versions of the code will be able to reproduce the evolution of more realistic stellar clusters, including a stellar mass function and the effects of stellar evolution.

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