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Astronomical relativistic reference systems and their application for astrometry

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Abstract. The status of the relativistic model for microarcsecond astrometry is reviewed. Theoretical foundations of the model and its main components are elucidated. Difficulties of increasing the accuracy of the model from micro- to nanoarcseconds are discussed. Each effect included in the model can be used to test the theory predicting this effect. A brief overview of the possible relativistic tests with astrometric data is given. It is stressed that systematic errors in the observational data and in the auxiliary input parameters (e.g. orbit of the observer) must be taken very seriously when conceiving tests of fundamental physics with astrometry.

1. Introduction

In the last 20 years, astrometry, being the oldest branch of astronomy, has made a stunning progress. Within the next 5–10 years the ESA cornerstone mission Gaia is expected to reach an accuracy of up to several microarcseconds for about one billion celestial objects. This will be the final step for astrometric methods to become an important and unique source of physical and astrophysical information.

Gaia as well as Hipparcos and most of other planned instruments of space astrometry are not intended to observe close to the Sun where relativistic effects are especially large. For example, Gaia is indented to observe further than 45° from the Sun. However, it is easy to estimate that even for observations so far from the Sun relativistic effects may reach about 40 milliarcseconds for individual observations. This means that relativistic effects are 4 orders of magnitude larger than the goal accuracy of the mission and, therefore, 5 orders of magnitude larger than the required accuracy of the model. At this level one cannot expect to successfully cope with general relativity in a form of small additional corrections. On the contrary, each aspect of data processing and data modelling must be formulated in the language of general relativity, each parameter used in the data processing or estimated as a result of the data processing must be defined in the framework of general relativity. Given the wide variety of input parameters and the broad community providing those parameters it becomes clear that it is this general-relativistic consistency throughout the whole data processing that represents the main challenge of the relativistic modelling for astrometry missions like Gaia.

2. Relativistic astronomical reference systems

A good way to ensure the basic consistency is to define some standard relativistic reference

Send offprint requests to: S.A. Klioner systems and consequently use them for each components of the data modelling. Standard relativistic reference systems – the BCRS and GCRS – have been constructed by a number of authors and officially adopted by the International Astronomical Union (Soffel *et al.* 2003). Both reference systems are defined by their metric tensors in harmonic gauge. Both reference systems are constructed for a "central body" embedded in a background of external gravitational fields. The external gravitational field in these reference system are represented by tidal potentials and the gravitational field of the "central body" is the same as if the body were isolated when the tidal external potentials are neglected. The existence of such reference systems in general relativity is guaranteed by the Einstein Equivalence Principle. Both reference system are physically adequate to describe physical processes in certain vicinity of the central body. These reference systems represent direct relativistic generalization of the Newtonian concept of quasi-inertial reference systems.

In the case of BCRS, the whole Solar system is considered as the compound "central body". The external gravitational potentials in the BCRS metric are usually neglected, but can be also retained to give the tidal potentials due to external bodies: the stars and other bodies of the Galaxy, other Galaxies and the Universe as a whole (Klioner & Soffel 2004). Usually these tidal gravitational fields are considered as fully negligible for practical applications. This is also the case for space astrometry at microarcsecond level. The Earth alone is considered as a central body for the GCRS, the metric tensor of which also contains the tidal potentials of the other solar system bodies. The same technique can be applied to construct a physically adequate reference system of an observer: the mass-less observer should be chosen as the central body and the gravitational field of all massive bodies are given by tidal potentials. Klioner (2004) has shown that in such a local reference system of an observer the tangents to the coordinate lines at the origin coincide with the vectors of coordinate-induced tetrad. This property is used to formulate observable quantities directly from the coordinate quantities defined in the observer's local reference frame.

All these reference systems were originally constructed in the first post-Newtonian approximation. The extension to include the limited number of post-post-Newtonian effects needed for microarcsecond astrometry is straightforward (see below). To reach higher accuracy a full post-post-Newtonian formulation is required. Some work towards the complete post-post-Newtonian formulation has been already done (Klioner *et al.* 2012).

3. Relativistic model for astrometry

The relativistic reference systems can be used to model any kinds of astronomical observations. Generic scheme for relativistic modelling of high-accuracy astronomical observations is given e.g. by Klioner (2003). The application of this scheme to astrometric observation is straightforward and also described in Klioner (2003, 2004). The main components of the model are:

(1) The model for astrometric parameters in the BCRS: position, proper motion, parallax, light travel-time effects for stars and quasars, and orbital parameters (e.g. initial conditions or osculating elements) for solar system objects. This model represent the standard astrometric model of a source in BCRS coordinates.

(2) The model for the motion of the observer (e.g. Gaia satellite) in the BCRS. This model usually consists of the EIH-like relativistic equations of motion of a test body in the BCRS.

(3) The model for light propagation in the BCRS coordinates from a source at the moment of light emission to the observer at the moment of observation. This is the most complicated part of the model and should include monopole light deflection due to a large number of bodies (see Fig. 1 and Table 1), smaller effects due to translational motion of the bodies and due to quadrupole gravitational fields of the giant planets of the solar system. The monopole effects must include also the so-called enhanced post-post-Newtonian effects (see below) that may reach 16 μas in the Gaia data.

(4) The relativistic model of aberration, that is, of the difference between the observed direction as seen by the real (moving) observer and a fictitious observer co-located with the real observer at the moment of observation, but not moving in BCRS coordinates. Note that this definition of aberrational effects is valid even in the exact General Relativity and is not related to any approximation schema.

The model used specifically for Gaia is called GREM and described in a number of publications (e.g. Klioner 2003, 2004). We will not go into details of the model here, but only briefly discuss the enhanced post-post-Newtonian effects. It is perfectly true that the post-Newtonian metric is sufficient to describe the astrometry with an accuracy of 1 μas provided that observations closer than $\sim$3.3 angular radii of the Sun are excluded. However, the standard post-Newtonian solution for light propagation is not sufficient to reach the accuracy of 1 μas. The largest post-post-Newtonian effects may reach 16 μas even if observations close to the Sun are not considered. Klioner & Zschocke (2010) have demonstrated that some formally post-post-Newtonian terms in the light propagation formulas, called enhanced post-post-Newtonian terms, may become large and should be taken into account if the accuracy of 1 μas is desired. Since the complete post-post-Newtonian formulas are fairly complicated one needs an optimized formula for the light propagation that contain only those terms that are really necessary to reach the accuracy of 1 μas. Note that the idea of *numerical accuracy* and not the *analytical order of smallness* of various terms was not widely used in the field of general relativity. Finally, the optimized formulas have been derived by Klioner & Zschocke (2010) and Zschocke & Klioner (2010):

$$\begin{aligned} \boldsymbol{n} &= \boldsymbol{\sigma} + \boldsymbol{d}_\sigma\, Q\,(1 + Q\,|\boldsymbol{x}_{\rm obs}|)\,, \\ Q &= -(1+\gamma)\,\frac{m}{|\boldsymbol{d}_\sigma|^2}\left(1 + \frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}_{\rm obs}}{|\boldsymbol{x}_{\rm obs}|}\right), \end{aligned} \qquad (1)$$

$$\begin{aligned} \boldsymbol{n} &= \boldsymbol{k} + \boldsymbol{d}\,P\,(1 + P\,|\boldsymbol{x}_{\rm obs}|)\,, \\ P &= -(1+\gamma)\,\frac{m}{|\boldsymbol{d}|^2}\left(\frac{|\boldsymbol{x}_{\rm source}| - |\boldsymbol{x}_{\rm obs}|}{|\boldsymbol{R}|} + \frac{\boldsymbol{k}\cdot\boldsymbol{x}_{\rm obs}}{|\boldsymbol{x}_{\rm obs}|}\right) \end{aligned} \qquad (2)$$

where $\boldsymbol{R} = \boldsymbol{x}_{source} - \boldsymbol{x}_{obs}$, $\boldsymbol{k} = \boldsymbol{R}/|\boldsymbol{R}|$, $\boldsymbol{d} = \boldsymbol{k} \times (\boldsymbol{x}_{obs} \times \boldsymbol{k})$, $\boldsymbol{d}_\sigma = \boldsymbol{\sigma} \times (\boldsymbol{x}_{obs} \times \boldsymbol{\sigma})$, γ is the PPN parameter equal to 1 in general relativity, $m = GM/c^2$ is the Schwarzschild radius of the deflecting body, $\boldsymbol{x}_{source}$ is the BCRS position of the source at the moment of emission, $\boldsymbol{x}_{obs}$ is the position of the observer at the moment of observation, $\boldsymbol{\sigma}$ is the direction of light propagation at past null infinity and characterizes the remote source (star or quasar), $\boldsymbol{n}$ is the direction of light observed by a fictitious observer located at the same position as the real observer and being at rest with respect to the BCRS. Eq. (1) is suitable for remote sources (stars and quasars) and Eq. (2) is useful for solar system objects. Both equations give numerical accuracy of 1 μas for an observer situated in the Solar system provided that observations within 3.3 angular radii from the Sun are excluded.

The model is not only complete at the accuracy level of 1 μas but also highly optimized. For example, the quadrupole deflection defined by a complicated formula is only computed when its magnitude really exceed the requested accuracy level. To this end a set of highly efficient upper estimates of the quadrupole light deflection has been found by Zschocke & Klioner (2011). These estimates allows one to decide if the quadrupole deflection is large enough using only a few float-point operations. Besides this the model can be automatically simplified if lower accuracy is requested. Lower accuracy can be used at early stages of data processing when the ultimate accuracy is not expected and/or for faint sources having lower observational accuracy. Table 2 gives the performance of an implementation of the model on a typical CPU as available in the year of 2011.

4. Is nanoarcsecond astrometry possible?

After 20 years of technical and scientific development that was clearly influenced by financial constraints, the accuracy of Gaia is now fixed at the level of some μas for the best ("Gaia-optimal") stars. The actual performance depends on the magnitude and color of the source

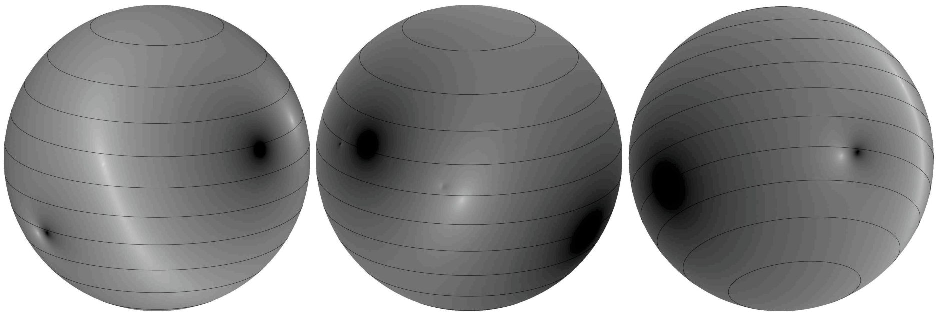

Fig. 1. Magnitude of light deflection due to solar system planets at some fixed moment of time as distributed on the celestial sphere shown from three sides. The larger the deflection the darker is the area. Although the deflection from each planet monotonically falls off with the angular distance from the planet, vectorial character of the deflection leads to a complicated distribution of the magnitude when several planets come into play. The solid lines are lines of constant declination.

and on its position on the sky (ESA 2011). It is clear that with this accuracy Gaia is expected to deliver a lot of important scientific results. Nevertheless it is interesting to understand what sort of science can be achieved with even higher astrometric accuracy than that of Gaia. Some impressive answers are given in Perryman (2000) and Unwin (2009). These reports were compiled for the original design of Gaia and for SIM, that is, for an accuracy 4–8 times better than that of the current Gaia. It seems that the number of important astrophysical applications rapidly grows with increasing accuracy. Among the applications one can find the search for Earth-mass exoplanets in the habitable zones of nearby stars, further investigation of the distribution of dark matter in the Milky Way and Local Group, measuring masses of compact Galactic objects with microlensing, investigation of compact binary systems with black holes and neutron stars, and others. Further increase of accuracy, towards 1–10 nanoarcseconds would open research opportunities that looks as science fiction nowadays. To give an example, it would be possible to directly measure trigonometric parallaxes of cosmologically relevant objects and investigate possible non-homogeneity of the Hubble expansion.

Increasing astrometric accuracy from microarcseconds to nanoarcseconds is a tremendous task which will require many components of current astrometric machinery to be substantially improved. Currently, it is not clear if a sort of nanoarcsecond astrometry is possible and even makes sense. Many questions arise in this respect. What sort of technology would allow nanoarcsecond accuracy technically? Do natural sources (celestial objects) that are sufficiently stable (in the astrometrical sense) at the level of 1 or 10 nanoarcsecond exist? (And if yes, in which waveband – optical, IR, radio, etc. – one should observe them?) Is it possible to compute time-dependent part of the effects of interstellar and interplanetary medium on light propagation with that accuracy? Finally, closer to the subject of this paper, is it possible to trace a light ray observed in the solar system back to the source with an accuracy of a nanoarcsecond? The background of this last question is the fact that starting from some level of accuracy, light deflection in the gravitational field of the bodies of our Galaxy and those in the solar system become chaotic (similar to the chaotic atmospheric refraction leading to the blurring and twinkling of images seen through the atmosphere). Moreover, even forgetting about possible chaotic effects, several key elements of a relativistic model for nanoarcsecond astrometry are currently missing. It seems that to arrive at nanoarcsecond accuracy one needs to derive

- the post-post-Minkowskian or post-post-Newtonian metric tensor of a system of *N* arbitrarily moving bodies with full multipole structure, and
- the post-post Minkowskian or at least post-post-Newtonian analytical light-propagation laws in the field of *N* arbitrary moving bodies with full multipole structure.

Depending on how close to the Sun or other stellar-mass objects one expects to observe one may need to take into account also higher-order effects. Let us also note that if analytical solutions for light propagation are not feasible, one could, in some cases, resort to numerical solutions of geodetic equations.

5. Testing fundamental physics with astrometry

Clearly, any effect used in an observational model can be used to test the theory predicting this effect. This is also true for astrometric observations with Gaia or other high-accuracy astrometry projects. Let us briefly list what is planned for Gaia:

- Tests of overall gravitational light deflection expressed by the PPN parameter γ. This is the most precise test dominated by the monopole light deflection due to the Sun.
- Tests of Local Lorentz Invariance from aberration (Klioner *et al.* 2010).
- Tests of subtle structure of the light deflection close to giant planets: translational gravitomagnetic effect and quadrupole deflection.
- Tests of possible non-Einsteinian effects in the motion of solar system bodies (additional perihelion precession, Nordtvedt effects, time-dependence of the Newtonian gravitational constant, etc.).
- Attempts to measure the masses of invisible components of compact binaries from the corresponding astrometric wobble of visible components (e.g., the masses of black hole candidate in Cyg X1).
- Various tests related to pattern matching in the proper motions and/or residuals of individual observations: acceleration of the solar system relative to quasars, primordial ultra-low frequency gravitational waves, higher-frequency gravitational waves.

The main challenge for these tests is to cope with possible systematic errors in the observational data and/or auxiliary input parameters used in the data processing. The fight against systematic errors should go along three lines:

1. The role of systematic errors should be clearly understood theoretically. One should clearly see which systematic errors are able to bias the estimates of physically relevant parameter (e.g. the PPN parameter γ).
2. One should attempt to design the instrument in such a way that these systematic errors are suppressed as much as possible or at least can be calibrated independently.
3. The presence of these systematic errors in observations should be analyzed statistically.

In principle, the problem of systematic errors is fairly standard. A good example for astrometry with scanning satellites with two fields of view like Hipparcos and Gaia is the degeneracy between the parallax zero point and certain periodic change in the angle between two fields of view (basic angle). It is because of this degeneracy that the basic angle stability or measurability over the times shorter than about one rotational period of a satellite is a scientific requirements for scanning satellites and represents an important engineering challenge.

One can demonstrate that in principle, for scanning satellites with two fields of view the PPN γ is degenerated with a certain other change of basic angle and, independently, with a certain error in the barycentric velocity of the observer. In these circumstances statistical analysis should be used to investigate the reliability of the estimate of γ.

It is also important to consider the correlations between γ and other parameters since large correlations reduce the accuracy and reliability of parameter estimations. It is well known that γ is substantially correlated with the parallax zero point. Using Eqs. (3.11) of

Table 1. Maximal light deflection due to individual bodies of the solar system as expected in Gaia data. Index '*' means that the body itself can, in principle, be observed by Gaia and the shown deflection is that for grazing rays. For bodies shown in parentheses (Mercury and the Moon) Gaia scanning law prohibits observations close to them and the expected maximal light deflection is below 1 μas. $\psi_{\max}$ is the maximal angle at which the deflection attains still 1 μas(irrespective of how close to that body Gaia can observe).

body	Sun	(Mercury)	Venus*	Earth	(Moon)	Mars*	Jupiter*	Saturn*	Uranus*	Neptune*
deflection (μas)	9900	0.3	493	5	0.14	116	16270	5780	2080	2530
$\psi_{\max}$	180°	9′	4.5°	125°	5°	25′	90°	17°	71′	51′

Table 2. Performance of an implementation of the relativistic model for Gaia on a typical CPU.

target accuracy	for stars/quasars	for Solar system objects
0.1 μas	3.1 μs	5.3 μs
10.0 μas	1.7 μs	2.1 μs
1000.0 μs	1.6 μs	2.0 μs

Mignard (2001) one can derive analytical formula for this correlation in one field of view:

$$\rho_{\gamma\rho} = -\sqrt{\frac{2}{1+\sec\chi}}\,, \tag{3}$$

where χ is the solar aspect angle: the angle between the rotational axis of the satellite and the direction to the Sun. Considering the fact that the attitude of the satellite should be also determined from the same observations one concludes that only the difference of the light deflections in two fields of view can be used to estimate γ (Lindegren 2011) (This is also true for all other parameters not related to the attitude). The correlation between the differences, in two fields of view, of the signals coming from γ and from parallaxes can also be computed analytically:

$$\rho^{\rm diff}_{\gamma\rho} = -\frac{2\sqrt{\cos\chi}}{1+\cos\chi}\sqrt{\sin^2\frac{\Gamma}{2}+\cos^2\frac{\Gamma}{2}\cos^2\chi}\,, \tag{4}$$

where Γ is the angle between two fields of view (basic angle). For Gaia one has $\chi = 45°$ and $\Gamma = 106.5°$ which leads to $\rho_{\gamma\rho} = -0.910$ and a slightly smaller correlation for the differences: $\rho^{\rm diff}_{\gamma\rho} = -0.893$. In reality this correlation should be even lower because of the differential effects within each field of view and the fact that the light deflection due to the planets is not correlated with parallaxes.

6. Concluding remarks

Relativistic model for microarcsecond astrometry at an angular distances of more than a few degrees from the Sun is well understood and represents neither theoretical nor practical difficulties. The formulation of such a model is sufficiently compact to allow massive calculations (up to 10^{12} individual observations and a total of up to 10^{15} applications of the model are expected for Gaia). Significant theoretical efforts are needed to construct the relativistic model for the accuracy of 1–10 nanoarcseconds – the accuracy that promises to provide a truly new insight to the Universe as a whole.

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