

X-ray monitoring of Classical Novae in the central region of M 31

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Abstract. We review recent results of the first dedicated monitoring programme for supersoft X-ray source (SSS) states of classical novae (CNe) in the central region of the Andromeda galaxy (M 31), performed in high-cadence campaigns with XMM-Newton and *Chandra*. After the first three years we had detected 21 novae in X-rays (17 new), thereby discovering peculiar objects and increasing the number of known M 31 novae with SSS state to 60. This is the largest sample known in any galaxy and we used it to carry out the first statistical analysis of novae in X-rays. We found several correlations between optical and SSS parameters, and carried out a simulation on the completeness of our monitoring as well as the first nova population study in X-rays. This shows that X-ray surveys of CNe populations are a powerful tool to address the open questions connected to these objects.

Key words. Galaxies: individual: M 31 – novae, cataclysmic variables – X-rays: binaries

1. Introduction

1.1. Novae as X-ray sources

The phenomena of classical nova (CN) explosions occur on the surface of white dwarfs (WD) in cataclysmic binary systems.

Hydrogen-rich material is accreted from a close companion, main-sequence star or red giant, and deposited on the WD surface until temperature and pressure become sufficiently large to initiate hydrogen-fusion in the degenerate matter and trigger a thermonuclear runaway. This process happens explosively, leading to the ejection of the hot, accreted shell at high velocities. This causes the brightness of the star to increase by ~ 12 magnitudes within hours to days (see e.g. Bode & Evans 2008).

However, not all accreted hydrogen is expelled during the outburst; a fraction of it stays in steady burning on the surface of the WD (Starrfield et al. 1974). This powers a supersoft X-ray source (SSS) which becomes observable at the *SSS turn-on time* when the opacity of the expanding shell reduces sufficiently (Starrfield 1989; Krautter 2002). The life time of the SSS state is determined by the amount of hydrogen left after the outburst and the *SSS turn-off time* indicates the end of the hydrogen burning on the WD surface. These two time scales, that are only accessible through X-ray observations, therefore allow estimates on the masses ejected and burned during the CN outburst (e.g. Sala & Hernanz 2005). X-ray observables are sensitive to the WD mass and might provide a way to answer the question which fraction of CNe ultimately become type Ia supernovae.

1.2. M 31 - the ideal target

The Andromeda galaxy M 31 is the nearest big spiral galaxy. This means essentially two things: First, its large stellar mass implies a high nova rate: a total nova rate of $\sim 65 \text{ yr}^{-1}$ has been estimated by Darnley et al. (2006). Second, its proximity at only 780 kpc distance (Holland 1998; Stanek & Garnavich 1998) was the main reason for M 31 having been the earliest target for extragalactic nova surveys, thereby creating the most extensive data base of CNe available today.

The first survey for novae in M 31 was carried out by Hubble (1929) during his pioneering observations of M 31, reporting 85 objects on photo plates obtained at Mt. Wilson observatory in 1909-1927. The earliest nova detections listed in Hubble (1929) were made

by Ritchey (1917) on photo plates taken in 1909, which means that today our catalogues extend over more than a century. The survey of Hubble (1929) was followed-up during the following 60–70 years mainly by Arp (1956); Rosino (1964, 1973); Rosino et al. (1989); Börngen (1968); Sharov & Alksnis (1991, 1994, 1996); Henze et al. (2008b) utilising large telescopes in long photographic campaigns.

Although these campaigns were very successful they only covered limited time spans, from months to years at a time. However, with the advent of CCD detectors and their wide availability also for low-cost observatories a quasi-continuous monitoring of M 31 has become feasible during the last 20 years. In this time, thanks to multi-epoch surveys (e.g. Shafter & Irby 2001; Darnley et al. 2004; Shafter et al. 2011), long-term monitoring programmes (e.g. Pietsch 2010; Kasliwal et al. 2011) and the dedicated work of various amateur observers the number of nova candidates known in M 31 doubled to more than 880 as of September 2011. The increasing rate of discoveries can be seen in the yearly frequency of optical nova candidates found in M 31 over the last 30 years, shown in Fig. 1 together with the X-ray frequency. We provide online regularly updated catalogues of novae in M 31 and other Local Group galaxies¹.

This impressive data base of nova discoveries dwarfs even the number of novae known in the Galaxy (~ 410) by more than a factor of two. But this is not the only difference between the two galaxies that argues in favour of using M 31 for nova population studies. Owing to their proximity, Galactic CNe can be observed in great detail. In the X-ray regime, high-resolution spectra can be obtained with XMM-Newton or *Chandra* (e.g. Ness et al. 2007, 2011) and detailed light curves can be recorded with *Swift* (e.g. Osborne et al. 2011). However, those objects have to be observed separately, whereas the field of view of the X-ray telescopes contains many novae in M 31 at the same time. All of these novae are ef-

¹ www.mpe.mpg.de/~m31novae/opt/m31/index.php

fectively at the same distance, whereas determining the distance to a Galactic nova is not trivial. Furthermore, a relatively low Galactic foreground extinction of $N_{\text{H}} \sim 6.7 \times 10^{20} \text{ cm}^{-2}$ (Stark et al. 1992) favours the detection of soft X-rays in particular. Therefore, while Galactic novae are the best probes for detailed analyses of individual objects, M 31 is the ideal target for studying the nova populations of a large spiral galaxy.

2. The X-ray monitoring

Pietsch et al. (2005) correlated optical nova catalogues of M 31 with X-ray catalogues from ROSAT, XMM-Newton, and *Chandra* and found that optical novae represent the major class of SSSs in this galaxy. In a follow-up paper, Pietsch et al. (2007) searched for X-ray counterparts of CNe in the central region of M 31 based on archival *Chandra* and XMM-Newton observations and concluded that the number of optical novae detected as SSSs is much higher than previously estimated.

These results motivated a dedicated, still on-going monitoring project aimed at SSS states of CNe in the central region of M 31. Up to now, three observation campaigns with the X-ray telescopes XMM-Newton and *Chandra* between 2006 and 2009 have already been published (Henze et al. 2010, 2011, hereafter HPH2010/11). While in the first campaign (June 2006 - March 2007) the individual observations were separated by about 30 days, the following two campaigns had a higher cadence of only 10 days. The monitoring strategy was changed in order to account for a significant number of fast SSS nova counterparts discovered by Pietsch et al. (2007).

The X-ray observations are supplemented by an optical monitoring of the M 31 central area carried out in collaboration with several observatories using small optical telescopes (35 – 130 cm). The field of view covered in the optical observations is comparable to the field of view of the X-ray monitoring, which has $\sim 30'$ in diameter.

In three years of optical monitoring we discovered 25 nova candidates which were announced in 24 issues of the *Astronomer's*

*Telegram*². For 17 of these candidates optical spectra could be obtained, thereby allowing to confirm them as novae and to classify them within the system of Williams (1992). A highlight discovery was M31N 2008-08b: the first recurrent nova in M 31 with sub-arcsecond position agreement between the two outbursts. Following our discovery alert of the nova candidate (see Henze et al. 2008a), Di Mille et al. (2008) were able to confirm spectroscopically that M31N 2008-08b is a He/N nova in M 31.

The X-ray monitoring led to the detection of 21 nova counterparts, only four of which were known before. The new novae include several interesting individual objects, the most remarkable of which were the first two SSSs in M 31 globular clusters (Henze et al. 2009) and the first two novae with a periodically variable X-ray light curve in M 31 (HPH2010, Pietsch et al. 2011).

Even more important, with these newly discovered objects we could increase the total number of novae with SSS counterpart in M 31 to 60. This is by far the largest population known for any galaxy, including the Galaxy where the number is smaller by about a factor of two. The impact that our dedicated monitoring had on the detection frequency of SSS nova counterparts in M 31 can be seen in Fig. 1 where we show the nova discovery rates in optical and X-rays. But not only the size of the sample was significantly improved, also its quality increased considerably because our high-cadence monitoring enabled more accurate estimates of various nova parameters.

Both factors allowed us to perform source statistics and population studies on a homogeneous sample of novae with SSS counterpart for the first time. The results of this analysis were presented in detail in HPH2011 and will be summarised in the following chapters.

3. Correlations between nova parameters

A thorough analysis of the optical and X-ray parameters of the 60 M 31 novae with SSS counterpart revealed four correlations. Two of

² <http://www.astronomerstelegam.org>

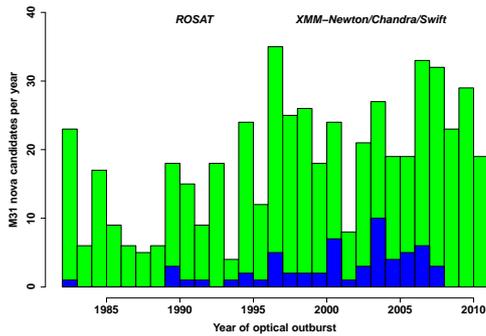


Fig. 1. Yearly number of nova candidates discovered in M 31 in the optical (green) and later in X-rays (blue) from 1982 till August 2011.

those relations connect X-ray parameters of the post-nova SSS: turn-on time vs turn-off time and turn-off time vs blackbody temperature (kT). The other two correlations show dependencies between the X-ray and optical properties: SSS turn-on time vs decay time of the optical R band light curve ($t_{2,R}$) and SSS turn-on time vs expansion velocity of the ejected envelope (determined from optical spectra).

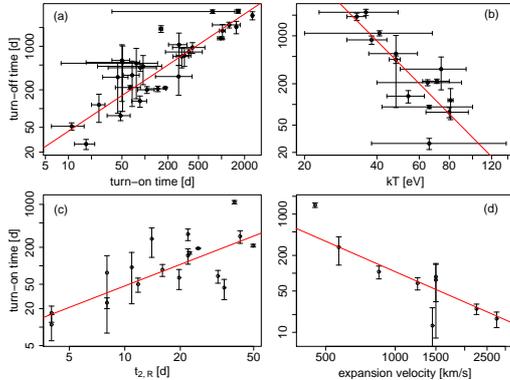


Fig. 2. Correlations between nova parameters, including best-fit powerlaw models (red). Plots are double-logarithmic. Credit: HPH2011, reproduced with permission ©ESO.

All correlations are shown in Fig. 2 together with best-fit models based on simple

powerlaw approximations. These results paint a coherent picture: optically fast novae with short decay times and large expansion velocities exhibit short SSS turn-on times, which are connected to a faster SSS turn off and higher blackbody temperatures of the post-nova X-ray source. A possible interpretation of these effects would be that the properties of a nova in outburst are most strongly determined by the mass of the underlying WD.

In the optical this dependence on the WD mass manifests itself in the maximum-magnitude rate-of-decline relation (MMRD) as was pointed out by Livio (1992); Della Valle & Livio (1995); Della Valle (2002). The fact that the mass of the WD is the dominant influence on the X-ray parameters of the nova has been noticed in theoretical work (see e.g. Sala & Hernanz 2005; Hachisu & Kato 2006). However, these models stress that nova parameters also depend on other properties, such as accretion rate or chemical composition of WD and accreted material. Therefore, while the WD mass might have the most important influence on the overall behaviour of the nova it should not be considered as the only significant parameter.

4. The completeness of the X-ray monitoring

A puzzling result of our monitoring is that even with high-cadence observations the rate of M 31 novae found as SSSs is still as low as $\sim 20\%$. This is surprising, because from theoretical models all novae are expected to go through a SSS stage (see e.g. Hachisu & Kato 2006, 2010). The reason why only a small fraction of novae are detected as SSSs could either be (a) an intrinsic effect, like self-absorption as discussed for persistent SSSs (e.g. Bøje Nielsen et al. 2011), or (b) a consequence of the inevitably incomplete observational coverage. Benefiting from having assembled the largest sample of novae with SSS counterpart in any galaxy and the large number of homogeneous monitoring observations we were able to test possibility (b) for the first time. To achieve this goal we compared the results of our obser-

vations to predictions from theoretical models, as will be described in the following.

The starting point was a simple estimate of the observed WD mass distribution based on the theoretical work of Truran & Livio (1986). The crucial point of their model is that this distribution is strongly dominated by novae with high-mass WDs, because although these systems have a lower absolute frequency (see e.g. Catalán et al. 2008, and references therein) their short recurrence time scales lead to a much higher rate of outbursts than for low-mass WDs. The mass distribution was converted into a SSS turn-on time distribution. For this step we used the models of Hachisu & Kato (2006). From this we derived a distribution of SSS durations, using the correlation between turn-on and turn-off time we computed from our data (see Fig. 2).

We simulated expected nova detections based on this distribution using a Monte Carlo Markov chain approach: SSS durations were randomly assigned to actual optical novae found in the M 31 region between 1995 and 2009. For those ~ 200 objects it was then checked if they would be detected in our X-ray monitoring. The result was that the intrinsic fraction of novae with SSS state that would lead to a detected rate of $\sim 20\%$ is very close to 100%. Therefore, we concluded that our observations do not rule out the possibility that the low detection fraction is caused by an observational bias. Considering future monitoring observations and more sophisticated population synthesis models might however dramatically change this picture. After our first study has shown that such an analysis is possible for M 31 novae, an extension of the simulation has to be the subject of future work.

5. Nova populations in X-rays

For the first time, we were able to investigate nova populations based on the X-ray properties of the objects. Currently, such an approach is only possible for M 31 novae, where large data bases of optical and X-ray observations together with a high intrinsic nova rate led to the most extensive sample of novae with SSS emission existing today. While optical classi-

fication methods, like decay time and spectral type, are mainly based on the properties of the ejected envelope, X-ray observations allow a more direct access to the parameters of the WD. We are therefore optimistic that X-ray parameters can provide a different and potentially more powerful view on possible influences of the underlying stellar population on the properties of the nova system.

Our investigation method involved two different approaches, illustrated in Fig. 3. First, we classified our sample into bulge and disk novae, according to their coordinates. This classification assumed a M 31 bulge with a projected ellipticity of 0.5, semi-major axis of $700''$, and position angle of $\sim 50^\circ$ (Beaton et al. 2007), shown as a white ellipse in Fig. 3. This figure also displays the result of this purely geometrical classification. We compared the X-ray properties of the two sub samples and found significantly different distributions for the blackbody temperatures. In Fig. 4(a) we give the two distributions which are different on the 88% confidence level.

The second approach defines two different sub samples of novae with high- and low-mass WDs and looks at their distances to the M 31 centre. For this, we made use of the models by Hachisu & Kato (2006) and used the observed SSS turn-on (t_{on}) times to estimate the WD mass. Here, high-mass WDs are defined as having $t_{on} \lesssim 100\text{d}$, corresponding to $M_{WD} \gtrsim 1.2M_\odot$, and low-mass WDs show $t_{on} \gtrsim 500\text{d}$, corresponding to $M_{WD} \lesssim 0.7M_\odot$. The positions for both mass ranges are indicated in Fig. 3. In Fig. 4(a) we compare their M 31 centre distances, which have been corrected for projection. A Kolmogorov-Smirnov-test shows that both distributions are significantly different on the 95% confidence level. Note, that extinction within the M 31 disk might influence this result.

Although both methods suffer from projection effects caused by the high inclination of M 31 (77.5° ; e.g. Beaton et al. 2007) they use two different X-ray parameters and come to a similar conclusion: there is evidence for two nova populations connected to the old (young) stellar populations in the M 31 bulge (disk). However, the low confidence levels in-

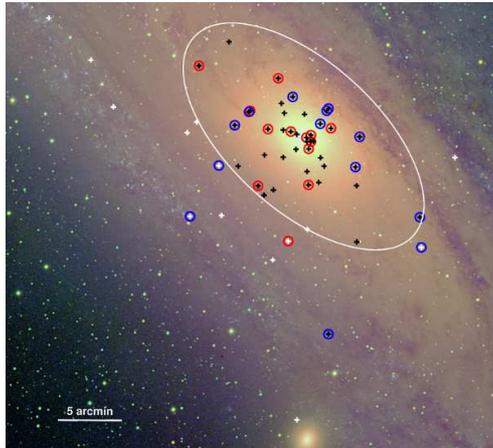


Fig. 3. M 31 colour composite (based on *UBV* photo plates, courtesy of TLS Tautenburg) with overlaid positions of bulge (white) and disk (black) novae with SSS counterpart. The galaxy bulge is marked by a white ellipse, and novae with high (low) mass WDs by blue (red) circles. East is left, north is up.

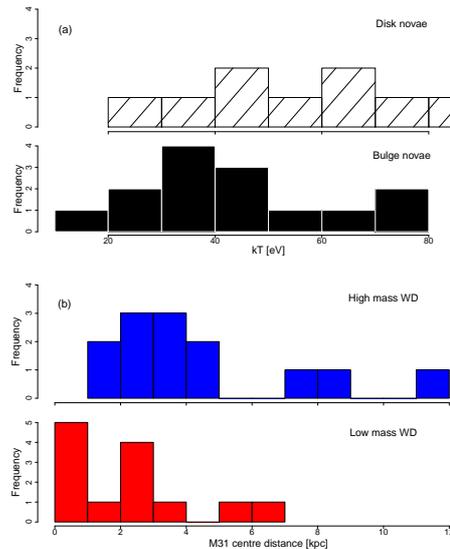


Fig. 4. Distributions of (a) blackbody temperature for disk and bulge novae and (b) distance to the M 31 centre (de-projected) for novae with high and low mass WDs. Colours as for Fig. 3. Credit: HPH2011, reproduced with permission ©ESO.

involved mean that this result should only be seen as a first hint towards the potential discovery space for X-ray observations of novae in M 31. Future observations, in particular of the relatively neglected M 31 disk, will be crucial for testing and validating our initial results with a larger data base.

With the knowledge about the two possible nova populations we re-analysed the correlations discussed in Sect. 3. For the correlation between SSS turn-on and turn-off time we indeed found a different best-fit powerlaw models for bulge and disk novae. In Fig. 5 we show the two sub samples in a plot similar to Fig. 2(a). Such a behaviour is not predicted by current theoretical models (e.g. Hachisu & Kato 2006). However, the statistical significance of this result is low (1σ). This might be due to the still relatively large error bars and the small sample of disk novae. Again, additional observations will be vital for re-analysing this hint with higher significance.

6. Summary

We have reviewed the set-up, discoveries and initial results of the first dedicated X-ray monitoring project aimed at CNe in M 31. A multi-campaign, high-cadence monitoring with the X-ray telescopes XMM-Newton and *Chandra* helped to build the most extensive, homogeneous data base of novae with SSS counterpart in any galaxy. An essential factor for the success of this program also was the support by a quasi-continuous optical monitoring carried out in collaboration with several observatories using small telescopes. Based on the nova data base we compiled a homogeneous sample and discovered rare objects and fast SSSs, proving our monitoring strategy successful. For the first time we were able to carry out a statistical study which resulted in (a) the discovery of correlations between (optical and X-ray) nova parameters, (b) an estimated on the detection completeness of the monitoring,

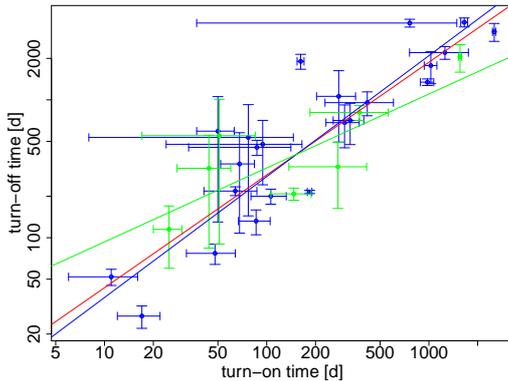


Fig. 5. Same as Fig. 2(a) for bulge (blue) and disk (green) novae separately. Solid lines show best-fit powerlaws, with red indicating the overall best fit. Credit: HPH2011, reproduced with permission ©ESO.

and (c) the first population study of M 31 novae based on X-ray measurements.

7. Outlook

Many of the results reviewed here only provide a first glimpse into the discovery space of X-ray monitoring observations of novae, owing to still relatively small sample sizes. However, the future looks promising. As of today, two additional monitoring campaigns have been obtained in autumn/winter of 2009/10 and 2010/11, resulting in the detection of 16 novae in total. Of those objects, nine are new discoveries, for seven SSS spectra could be obtained, and six shown short SSS turn-on times of less than 100 d. A comprehensive description of these results is currently in preparation but they certainly promise enhanced statistics. Finally, at least one more monitoring campaign will be obtained in 2011/12. Continuing support by optical surveys is guaranteed and we hope that the X-ray monitoring will be extended in the future, thereby allowing the full potential of the unique target M 31 to be exploited.

8. DISCUSSION

GIORA SHAVIV's Comment: Since you mention history I want to give a small amus-

ing historical note. Duncan 1918 discovered 9 novae in M 31 but that was not sufficient to eliminate the famous great debate. Hubble did not trust novae as standard candles and he did not know about the Eddington luminosity which was found by Milne in 1932. Even the Belanowsky nova in M 87 1923, was not sufficient for Hubble, who suspected M 87 was extragalactic - but did not dare to say so. So he took the wrong P-L relation by Shapley.

ŞÖLEN BALMAN: You mentioned extinction problems on the disk, basically you are experiencing absorption effects on the disk which will affect your SSS spectrum. Basically due to the absorption in the galactic disk you are only detecting harder and more hot SSSs and thus only the more massive WDs.

MARTIN HENZE: This is a possible explanation. However, it does not explain why there are fewer hot SSSs detected in the (practically extinction-free) bulge of M 31 than in the disk. Nevertheless, we hope to include extinction effects in future work based on an extended data base.

KENJI TANABE: Why don't you choose M 33?

MARTIN HENZE: M 33 has a much lower observed nova rate (1-2 per year) than M 31 (30-40 per year) and therefore would require an infeasibly large amount of observing time to build up a nova sample of similar size.

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