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Cosmological distance indicators by coalescing binaries

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Abstract. Gravitational waves detected from well-localized inspiraling binaries would allow to determine, directly and independently, both binary luminosity and redshift. In this case, such systems could behave as "standard candles" providing an excellent probe of cosmic distances up to $z < 0.1$ and thus complementing other indicators of cosmological distance ladder.

Key words. gravitational waves – standard candles – cosmological distances

1. Introduction

Coalescing binaries systems are usually considered strong emitter of gravitational waves (GW), ripples of space-time due to the presence of accelerated masses in analogy with the electromagnetic waves, due to accelerated charged. The coalescence of astrophysical systems containing relativistic objects as neutron stars (NS), white dwarves (WD) and black holes (BH) constitute very standard GW sources which could be extremely useful for cosmological distance ladder if physical features of GW emission are well determined.

These binaries systems, have components that are gradually inspiralling one over the other as the result of energy and angular momentum loss due to (also) gravitational radiation. As a consequence the GW frequency is increasing and, if observed, could constitute a "signature" for the whole system dynamics. The coalescence of a compact binary system is usually classified in three stages, which are not very well delimited one from another, namely the *inspiral phase*, the *merger phase* and the *ring-down phase*.

Temporal interval between the inspiral phase and the merger one is called *coalescing time*, interesting for detectors as the American LIGO (Laser Interferometer Gravitational-Wave Observatory) (Abramovici et al. 1992) and French/Italian VIRGO (Caron et al. 1997). A remarkable fact about binary coalescence is that it can provide an *absolute measurement* of the source distance: this is an extremely important event in Astronomy (Capozziello et al. 2010). In fact, here we want to show that such systems could be used as reliable standard candles.

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2. Coalescing binaries as standard candles

The fact that the binary coalescence can provide an absolute measurement of the distance to the source, can be understood looking at the waveform of an inspiraling binary; as long as the system is not at cosmological distances (so that we can neglect the expansion of the Universe during the propagation of the wave from the source to the observer) the waveform of the GW, to lowest order in v/c is (see (Maggiore 2007) for a detailed exposition of GW theory)

$$h_+(t) = \mathcal{A}\frac{1}{r}\left(\frac{\pi f(t_R)}{c}\right)^{2/3}\left(\frac{1+\cos^2 i}{2}\right)\cos\left[\Phi(t_R)\right] \quad (1)$$

$$h_\times(t) = \mathcal{A}\frac{1}{r}\left(\frac{\pi f(t_R)}{c}\right)^{2/3}\cos i\sin\left[\Phi(t_R)\right] \quad (2)$$

where $\mathcal{A} = 4\left(\frac{GM_C}{c^2}\right)^{5/3}$, h_+ and $h_\times$ are the amplitudes for the two polarizations of the GW, and i is the inclination of the orbit with respect to the line of sight,

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \quad (3)$$

is a combination of the masses of the two stars known as the chirp mass, and r is the distance to the source; f is the frequency of the GW, which evolves in time according to

$$\dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_C}{c^3}\right)^{5/3} f^{11/3}, \quad (4)$$

t is retarded time, and the phase Φ is given by

$$\Phi(t) = 2\pi\int_{t_0}^{t} dt'\, f(t'). \quad (5)$$

For a binary at a cosmological distance, i.e. at redshift z, taking into account the propagation in a Friedmann-Robertson-Walker Universe, these equations are modified in a very simple way:

1. The frequency that appears in the above formulae is the frequency measured by the observer, $f_{\rm obs}$, which is red-shifted with respect to the source frequency f_s, i.e. $f_{\rm obs} = f_s/(1+z)$, and similarly t and t_{ret} are measured with the observer's clock.
2. The chirp mass M_c must be replaced by $\mathcal{M}_c = (1+z)M_c$.
3. The distance r to the source must be replaced by the luminosity distance $d_L(z)$.

Then, the signal received by the observed from a binary inspiral at redshift z, when expressed in terms of the observer time t , is given by

$$h_+(t) = h_c(t_R)\frac{1+\cos^2 i}{2}\cos\left[\Phi(t_R)\right], \quad (6)$$

$$h_\times(t) = h_c(t_R)\cos i\sin\left[\Phi(t_R)\right]. \quad (7)$$

where

$$h_c(t) = \frac{4}{d_L(z)}\left(\frac{G\mathcal{M}_C(z)}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}, \quad (8)$$

Let us recall that the luminosity distance d_L of a source is defined by

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}, \quad (9)$$

where $\mathcal{F}$ is the flux (energy per unit time per unit area) measured by the observer, and $\mathcal{L}$ is the absolute luminosity of the source, i.e. the power that it radiates in its rest frame. For small redshifts, d_L is related to the present value of the Hubble parameter H_0 and to the deceleration parameter q_0 by

$$\frac{H_0 d_L}{c} = z + \frac{1}{2}(1-q_0)z^2 + \dots. \quad (10)$$

The first term of this expansion give just the Hubble law $z \simeq (H_0/c)d_L$, which states that redshifts are proportional to distances. The term $O(z^2)$ is the correction to the linear law for moderate redshifts. For large redshifts, the Taylor series is no longer appropriate, and the whole expansion history of the Universe is encoded in a function $d_L(z)$. As an example, for a spatially flat Universe, one finds

$$d_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')}, \quad (11)$$

where $H(z)$ is the value of the Hubble parameter at redshift z. Knowing $d_L(z)$ we can therefore obtain $H(z)$. This shows that the luminosity distance function $d_L(z)$ is an extremely important quantity, which encodes the whole expansion history of the Universe.

Now we can understand why coalescing binaries are standard candles. Suppose that we can measure the amplitudes of both polarizations $h_+, h_\times$, as well as $\dot{f}_{\rm obs}$ (for ground-based interferometers, this actually requires correlations between different detectors). The amplitude of h_+ is $h_c(1+\cos^2\iota)/2$, while the amplitude of $h_\times$ is $h_c\cos\iota$. From their ratio we can therefore obtain the value of $\cos\iota$, that is, the inclination of the orbit with respect to the line of sight. On the other hand, (4) (with the replacement $M_c \to \mathcal{M}_c$ mentioned abobe) shows that if we measure the value of $\dot{f}_{\rm obs}$ corresponding to a given value of f_{obs}, we get $\mathcal{M}_c$. Now in the expression for h_+ and $h_\times$ all parameters have been fixed, except $d_L(z)$.[1] This means that, from the measured value of h_+ (or of $h_\times$) we can now read d_L. If, at the same time, we can measure the redshift z of the source, we have found a gravitational standard candle, and we can use it to measure the Hubble constant and, more generally, the evolution of the Universe (Schutz 1986). The difference between gravitational standard candles and the "traditional" standard candles is that the luminosity distance is directly linked to the GW polarization and there is no theoretical uncertainty on its determination a part the redshift evaluation. Several possibilities have been proposed. Among these there is the possibility to see an optical counterpart. In fact, it can be shown that observations of the GWs emitted by inspiralling binary compact systems can be a powerful probe at cosmological scales. In particular, short GRBs appear related to such systems and quite promising as potential GW standard sirens (**?**). On the other hand, the redshift of the binary system can be associated to the barycenter of the host galaxy or the galaxy cluster as we are going to do here.

3. Numerical results

We have simulated several coalescing binary systems at redshifts $z < 0.1$ In this analysis, we do not consider systematic errors and errors on redshifts to obviate the absence of a complete catalogue of such systems. The choice of low redshifts is due to the observational limits of ground-based interferometers like VIRGO or LIGO. Some improvements are achieved, if we take into account the future generation of these interferometers as Advanced VIRGO[2] and Advanced LIGO[3]. Advanced VIRGO is a major upgrade, with the goal of increasing the sensitivity by about one order of magnitude with respect to VIRGO in the whole detection band. Such a detector, with Advanced LIGO, is expected to see many events every year (from 10s to 100s events/year). In the simulation presented here, sources are slightly out of LIGO-VIRGO band but observable, in principle, with future interferometers.

Table 1. Redshifts, characteristic amplitudes, frequency range for NS-NS systems.

Object	**z**	h_c	**Freq.** (Hz)
NGC 5128	0.0011	10^{-22}	$0 \div 10$
NGC 1023 Gr.	0.0015	10^{-22}	$0 \div 10$
NGC 2997	0.0018	10^{-22}	$5 \div 15$
NGC 5457	0.0019	10^{-22}	$10 \div 20$
NGC 5033	0.0037	10^{-22}	$25 \div 35$
Virgo Cl.	0.0042	10^{-22}	$30 \div 40$
Fornax Cl.	0.0044	10^{-22}	$35 \div 45$
NGC 7582	0.0050	10^{-22}	$45 \div 55$
Ursa Major Gr.	0.0057	10^{-22}	$50 \div 60$
Eridanus Cl.	0.0066	10^{-22}	$55 \div 65$

Here, we have used the redshifts taken by NED[4] (Abell 1989), and we have fixed the redshift using z at the barycenter of the host galaxy/cluster, and the binary *chirp mass* M_C, typically measured, from the Newtonian part of the signal at upward frequency sweep, to $\sim$ 0.04% for a NS/NS binary Cutler et al.

[1] It is important that the ellipticity of the orbit does not enter; it can in fact be shown that, by the time that the stars approach the coalescence stage, angular momentum losses have circularized the orbit to great accuracy.

[2] `http://www.cascina.virgo.infn.it/advirgo/`

[3] `http://www.ligo.caltech.edu/advLIGO/`

[4] NASA/IPAC EXTRAGALACTIC DATABASE

Table 2. For each cluster we indicate redshifts, characteristic amplitudes, frequency range for BH-BH systems.

Object	**z**	h_c	**Freq.** (Hz)
Pavo-Indus	0.015	10^{-21}	$65 \div 70$
Abell 569	0.019	10^{-21}	$75 \div 80$
Coma	0.023	10^{-21}	$100 \div 105$
Abell 634	0.025	10^{-21}	$110 \div 115$
Ophiuchus	0.028	10^{-21}	$130 \div 135$
Columba	0.034	10^{-21}	$200 \div 205$
Hercules	0.037	10^{-21}	$205 \div 210$
Sculptor	0.054	10^{-21}	$340 \div 345$
Pisces-Cetus	0.063	10^{-21}	$420 \div 425$
Horologium	0.067	10^{-21}	$450 \div 455$

(1994); Capozziello et al. (2010). The distance to the binary d_L ("luminosity distance" at cosmological distances) can be inferred, from the observed waveforms, to a precision $\sim 3/\rho \lesssim 30\%$, where $\rho = S/N$ is the amplitude signal-to-noise ratio in the total LIGO network (which must exceed about 8 in order that the false alarm rate be less than the threshold for detection). In this way, we have fixed the characteristic amplitude of GWs, and frequencies are tuned in a range compatible with such a fixed amplitude, then the error on distance luminosity is calculated by the error on the chirp mass with standard error propagation.

The systems considered are NS-NS and BH-BH. For each of them, a particular frequency range and a characteristic amplitude (beside the chirp mass) are fixed. We start with the analysis of NS-NS systems ($M_C = 1.22 M_\odot$) with characteristic amplitude fixed to the value 10^{-22}. In Table 1, we report the redshift, the value of h_C and the frequency range of systems analyzed.In Fig. 1, the derived Hubble relation is reported.

The Hubble constant value is 72 ± 1 $km/sMpc$ in agreement with the recent WMAP estimation (Larson, D., et.al. 2011).

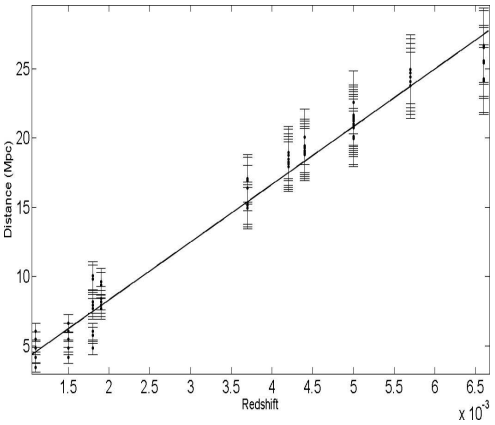

Fig. 1. Luminosity distance vs redshift for simulated NS-NS systems.

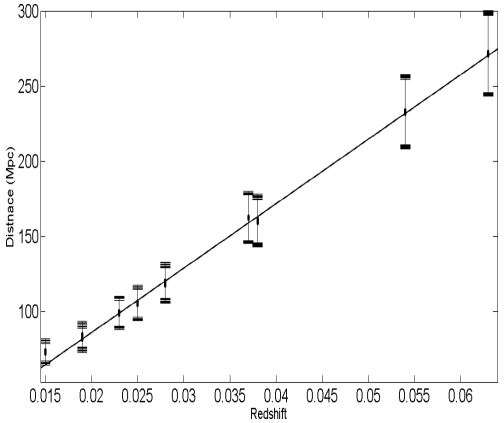

Fig. 2. Luminosity distance vs redshift for simulated BH-BH systems.

The same procedure is adopted for BH-BH systems ($M_C = 8.67 M_\odot$, $h_C = 10^{-21}$). In Tables 2 we report the redshift, the value of h_C and the frequency range. The simulations are reported in Fig. 2, and the Hubble constant value computed by these systems is 69 ± 2 $km/sMpc$.

4. Conclusions

We have considered simulated binary systems whose redshifts can be estimated considering the barycenter of the host astrophysical system as galaxy, group of galaxies or clus-

ter of galaxies. In such a way, the standard methods adopted to evaluate the cosmic distances (e.g. Tully-Fisher or Faber-Jackson relations) can be considered as "priors" to fit the Hubble relation. We have simulated, for example, NS-NS, and BH-BH binary systems. Clearly, the leading parameter is the chirp mass M_c, or its red-shifted counter-part $\mathcal{M}_c$, which is directly related to the GW amplitude. The adopted redshifts are in a well-tested range of scales and the Hubble constant value is in good agreement with WMAP estimation. The Hubble-luminosity-distance diagrams of the above simulations show the possibility to use the coalescing binary systems as distance indicators and, possibly, as standard candles. The limits of the method are, essentially, the measure of GW polarizations and redshifts. Besides, in order to improve the approach, a suitable catalogue of observed coalescing binary-systems is needed. This is the main difficulty of the method since, being the coalescence a transient phenomenon, it is very hard to detect and analyze the luminosity curves of these systems. Furthermore, a few simulated sources are out of the LIGO-VIRGO band.

Next generation of interferometer (as LISA[5] or Advanced-VIRGO and LIGO) could play a decisive role to detect GWs from these systems. At the advanced level, one expects to detect at least tens NS-NS coalescing events per year, up to distances of order $2Gpc$, measuring the chirp mass with a precision better than 0.1%. The masses of NSs are typically of order $1.4M_\odot$. The most important issue that can be addressed with a measure of $d_L(z)$ is to understand "dark energy", the quite mysterious component of the energy budget of the Universe that manifests itself through an acceleration of the expansion of the Universe at high redshift. This has been observed, at $z < 1.7$, using Type Ia supernovae as standard candles (Riess et al. 1998; Perlmutter et al. 1999). A possible concern in these determinations is the absence of a solid theoretical understanding of the source. After all, supernovae are complicated phenomena. In particular, one can be concerned about the possibility of an evolution of the supernovae brightness with redshift, and of interstellar extinction in the host galaxy leading to unknown systematics. GW standard candles could lead to completely independent determinations, and complement and increase the confidence of other standard candles, (Holz et al. 2005), as well as extending the result to higher redshifts. In the future, the problem of the redshift could be obviate finding an electromagnetic counterpart to the coalescence and short GRBs could play this role.

In summary, this new type of cosmic distance indicators could be considered complementary to the traditional standard candles opening the doors to a self-consistent *gravitational astronomy*.

[5] http://www.lisa-science.org

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