



# Winds from massive stars

Joachim Puls

Universitätssternwarte München, Scheinerstr. 1, 81679 München, Germany  
e-mail: uh101aw@usm.uni-muenchen.de

**Abstract.** Mass-loss is a key process which needs to be known *quantitatively* to understand massive star evolution. The standard theory to describe hot, massive star winds is based on radiative line-driving. Basic features of this theory are reviewed, and important scaling relations are provided. We compare theoretical predictions with observational findings, and outline recent results from considering wind inhomogeneities, thought to be related to the intrinsic line-driven instability. Finally, we discuss three potential sites for the acceleration of cosmic rays in massive star winds, (i) wind-embedded shocks, (ii) shocks from colliding winds, and (iii) wind terminal shocks.

**Key words.** Stars: early type – stars: mass loss – stars: winds, outflows – acceleration of particles – shock waves

## 1. Introduction

Massive stars are critical agents in galactic evolution, both in the present and in the early Universe (e.g., re-ionization and first enrichment). Mass loss is a key process, and has to be understood *quantitatively*, since "a change of only a factor of two in the mass-loss rate of massive stars has a dramatic effect on their evolution" (Meynet et al. 1994).

The standard theory to describe hot, massive star winds is based on radiative line-driving, and has been proven to work successfully in most evolutionary phases (OB-stars, A-supergiants, and LBVs in their "quiet" phase). Also for the pivotal Wolf-Rayet (WR) stadium, line-driving is the most promising acceleration mechanism (Sect. 3.)

Pioneering work on this subject was performed by Lucy & Solomon (1970) and Castor et al. (1975a, 'CAK'), where the latter still

builds the theoretical foundation of our present understanding. Improvements with respect to a *quantitative* description and first applications were provided by Friend & Abbott (1986) and Pauldrach et al. (1986), whilst recent reviews on the topic have been published by Kudritzki & Puls (2000) and Puls et al. (2008) (see also Crowther 2007 for a review on Wolf-Rayet stars and their winds).

## 2. Line-driven winds – basics

To be efficient, radiative line-driving requires a large number of photons, i.e., a high luminosity,  $L$ , and the presence of a multitude of spectral lines, with high interaction probabilities, close to flux maximum. The second condition implies a strong metallicity dependence of line-driven mass loss (Sect. 3). Typical mass-loss rates,  $\dot{M}$ , are on the order of  $10^{-7} \dots 10^{-5} M_{\odot} \text{yr}^{-1}$ , with terminal velocities  $v_{\infty} \approx 200 \dots 3,000 \text{ km s}^{-1}$ .

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Send offprint requests to: J. Puls

Radiative line-driving relies on two processes. (i) Momentum is transferred to the wind matter via line absorption/emission processes, mostly resonance scattering, with a net change in *radial* momentum, since the emission process is fore-aft symmetric. Doppler shifts due to the accelerating velocity field are essential for a large radiative acceleration, by enabling a supply of ‘fresh’ continuum photons that can be absorbed. (ii) Because of the huge number of metallic lines, as compared to the few dozens from Hydrogen and Helium, almost only the metal ions are *directly* accelerated, whilst their momentum is transferred to the bulk plasma (H, He) via Coulomb collisions (e.g., Springmann & Pauldrach 1992; Krtička & Kubát 2000; Owocki & Puls 2002). At very low wind-densities, the metallic ions might decouple from the wind, and the acceleration stalls.

The radiative line acceleration can be calculated by summing up the individual contributions from the millions of lines. Alternatively, following CAK, one might replace the summation by appropriate integrals over the line-strength distribution which results from detailed NLTE calculations. The line-strength,  $k$ , of an individual line is its opacity measured in units of the Thomson-scattering opacity. The distribution itself can be fairly well approximated by a power-law,  $dN(k)/dk \propto N_{\text{eff}} k^{\alpha-2}$ , with  $N_{\text{eff}}$  the effective (flux-weighted) number of lines and  $\alpha \approx 0.6 \dots 0.7$  (e.g., Puls et al. 2000). Note that both quantities depend on metallicity and spectral type.

Approximating the radiative transfer in terms of the Sobolev theory (Sobolev 1960), the total line acceleration results in  $g_{\text{rad}}^{\text{lines}} \propto (dv/dr/\rho)^{\alpha}$ , i.e., depends on the *spatial* velocity gradient and on the inverse of the density.

Once  $g_{\text{rad}}^{\text{lines}}$  and continuum acceleration are inserted into the hydrodynamic equations (assuming stationarity), the latter can be solved (almost) analytically, returning the following scaling relations,

$$\dot{M} \propto N_{\text{eff}}^{1/\alpha'} L^{1/\alpha'} \left( M(1 - \Gamma) \right)^{1-1/\alpha'},$$

$$v(r) = v_{\infty} \left( 1 - \frac{R_*}{r} \right)^{\beta}, \quad (1)$$

$$v_{\infty} \approx 2.25 \frac{\alpha}{1 - \alpha} v_{\text{esc}}, \quad v_{\text{esc}} = \left( \frac{2GM(1 - \Gamma)}{R_*} \right)^{\frac{1}{2}},$$

with Eddington-factor  $\Gamma$ , (photospheric) escape velocity  $v_{\text{esc}}$ , and  $\alpha' = \alpha - \delta$ , where  $\delta \approx 0.1$  describes the run of the ionization (Abbott 1982). The velocity-field exponent,  $\beta$ , is on the order of 0.8 (for O-stars) to 2 (for BA-supergiants).

Combining the scaling relations for  $\dot{M}$  and  $v_{\infty}$  yields the so-called modified wind-momentum rate,

$$\dot{M} v_{\infty} \left( \frac{R_*}{R_{\odot}} \right)^{\frac{1}{2}} \propto N_{\text{eff}}^{1/\alpha'} L^{1/\alpha'} \left( M(1 - \Gamma) \right)^{3/2-1/\alpha'}. \quad (2)$$

Accounting now for the fact that  $\alpha' \approx 2/3$ , we obtain the most fundamental prediction for ‘classical’ line-driven winds, the *wind-momentum luminosity relation* (WLR, Kudritzki et al. 1995),

$$\log \dot{M} v_{\infty} \left( \frac{R_*}{R_{\odot}} \right)^{\frac{1}{2}} \approx x \log \frac{L}{L_{\odot}} + D(Z, \text{SpT}), \quad (3)$$

independent of mass and  $\Gamma$ . Both slope  $x = 1/\alpha'$  and offset  $D$  depend on metallicity  $Z$  and spectral type. Though originally it has been proposed to exploit the WLR for measuring extragalactic distances on intermediate scales (up to the Virgo cluster), nowadays the relation is mostly used to test the theory itself, as described in the following.

### 3. Predictions vs. observations

**Results and predictions from hydrodynamic models.** The most frequently cited theoretical wind models (stationary, 1-D, homogeneous) are those from Vink et al. (2000), which base on a Monte-Carlo approach for solving the radiative line transfer. In addition to these models there are many others, which differ with respect to methods and approximations. Based on these models, important results have been published by Pauldrach (1987) and Pauldrach et al. (1994, 2001); by Krtička & Kubát (2000, 2001, 2004, 2009) and Krtička (2006); by Kudritzki (2002, based on Kudritzki

et al. 1989); by Gräfener & Hamann (2005, 2008); by Lucy (2007a,b); and by Müller & Vink (2008).

Most of the various approaches yield similar results, e.g., when comparing the ‘mass-loss recipe’ from Vink et al. (2000) with analogous investigations utilizing different codes (Kudritzki 2002; Pauldrach et al. 2001; Krtička & Kubát 2004). Moreover, the WLR concept is impressively confirmed by the simulations performed by Vink et al.: The predicted modified wind-momenta follow an almost perfect power-law with respect to stellar luminosity alone, *independent of luminosity class*, and, for solar abundances, ‘only’ two distinct relations covering the complete spectral range have been found, one for  $T_{\text{eff}} > 27.5$  kK and the other below this value.

**Observational tests.** In the last decade, various spectroscopic NLTE analyses of hot stars and their winds have been undertaken, in the Galaxy, the Magellanic Clouds, and other, more distant galaxies, in the UV, in the optical, and in a combination of both. Most of this work is based on 1-D, line-blanketed, NLTE, atmosphere/spectrum-synthesis codes allowing for the presence of winds, in particular CMFGEN (Hillier & Miller 1998), WM-BASIC (Pauldrach et al. 2001), and FASTWIND (Puls et al. 2005). The most important mass-loss diagnostics is  $H_{\alpha}$  (e.g., Puls et al. 1996). The central results of these investigations can be summarized as follows.

1. O-stars and BA-supergiants (also extragalactic, e.g. Bresolin & Kudritzki 2004) follow specific WLRs.
2. The predicted scaling  $v_{\infty} \propto v_{\text{esc}}$  is confirmed (Kudritzki & Puls 2000 and references therein).
3. For O- and early B-stars, the theoretically predicted WLR from Vink et al. (2000) is *roughly* met. However, there are certain exceptions and/or problems. (i) Some (all?) low luminosity O-dwarfs display so-called ‘weak winds’, with derived wind-momenta that are much lower (by one to two dex) than predicted. (ii) The observed wind-momenta from O-supergiants with rather dense winds are higher than predicted, by factors about three, which might be explained by wind-clumping effects

(Sect. 4). (iii) B-supergiants display lower wind-momenta than predicted. Regarding (ii) and (iii), see Markova & Puls (2008) and references therein.

**Metallicity dependence.** Also w.r.t. metallicity dependence, the various predictions (in particular, Vink et al. 2001; Kudritzki 2002; Krtička 2006) agree well,

$$\dot{M} \propto (Z/Z_{\odot})^{0.64...0.69}, \quad v_{\infty} \propto (Z/Z_{\odot})^{0.06...0.12}. \quad (4)$$

Within the VLT FLAMES survey of massive stars (Evans et al. 2005, 2006, 2008), Mokiem et al. (2007) confirmed the predictions regarding  $\dot{M}$  observationally, by deriving the *empirical* relation

$$\dot{M} \propto (Z/Z_{\odot})^{0.62 \pm 0.15} \quad (5)$$

from a large sample of Galactic, LMC and SMC OB-stars.

**Mass-loss from WR-stars.** A comparison of OB- and WR-star mass-loss rates (e.g., using the empirical ‘mass-loss recipe’ for WR-stars from Nugis & Lamers 2000) reveals a large difference. Typically,  $\dot{M}$  from WRs are larger, by a factor of 10 and more, at same  $L$ , which cannot be explained by standard wind theory. According to Gräfener & Hamann (2005, 2007, 2008), such dense winds can be modeled by (i) invoking large Eddington-factors,  $\Gamma$ , leading to low effective gravities and a deep lying sonic point at high temperatures, and (ii) by accounting for the Fe ‘opacity bump’ at  $T > 160,000$  K which initiates the mass-loss.<sup>1</sup> Alternative models have been provided by Vink et al. (2011), who point out that for  $\Gamma > 0.7$  line-driven winds become optically thick, favoring a larger  $\dot{M}$  than for optically thin winds.

#### 4. The ‘clumping crisis’

During recent years, overwhelming direct and indirect evidence has been accumulated that massive star winds are not smooth, but consist of small-scale density inhomogeneities, where the wind matter is concentrated in over-dense clumps, and the inter-clump medium is almost void.

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<sup>1</sup> based on an idea by Nugis & Lamers (2002).

Such inhomogeneities are thought to be related to structure formation due to the line-driven ('de-shadowing') instability, a strong instability inherent to radiative line-driving (Lucy & Solomon 1970; Owocki & Rybicki 1984, 1985; Owocki & Puls 1999). *Time-dependent* hydrodynamic models allowing for this instability to operate have been developed by Owocki et al. (1988), Runacres & Owocki (2002, 2005), and by Feldmeier (1995) and Feldmeier (1997). These simulations show that the wind, for  $r \gtrsim 1.3R_*$ , develops extensive structure consisting of strong *reverse* shocks separating slower, dense material from high-speed rarefied regions in between. Within the shocks, the material is heated to a couple of million Kelvin, and subsequently cooled by X-ray emission. Such X-ray emission has indeed been observed by all X-ray observatories, with typical X-ray luminosities  $L_X/L_{\text{bol}} \approx 10^{-7}$  (e.g., Sana et al. 2006).

Interestingly, the spatial/time-averaged structure and mass-loss rate arising from such simulations is very similar to results from *stationary* models. However, the structure seriously affects the radiative transfer, and hence the mass-loss rates *inferred* from observations. Thus, present diagnostic tools (model atmospheres) need to account for such inhomogeneities.

**Micro-clumping.** Until now, most diagnostic methods assume *optically thin* clumps and a void inter-clump matter. In this case, the density inside the clumps can be expressed by  $\rho_{\text{cl}} = f_{\text{cl}} \langle \rho \rangle$ , where  $\langle \rho \rangle$  is the average density resulting from a smooth, stationary wind, and  $f_{\text{cl}}$  the so-called 'clumping factor'. The most important consequence of such optically thin clumps is a *reduction of any  $\dot{M}$*  derived from  $\rho^2$ -dependent diagnostics (e.g., recombination based processes such as  $H_\alpha$ ) assuming smooth models, by a factor of  $\sqrt{f_{\text{cl}}}$ . For O-supergiants, such factors are at least on the order of 2...3 (Puls et al. 2006), and might thus explain corresponding discrepancies outlined in Sect. 3.

**The  $P_V$  problem.** From a mass-loss analysis using the unsaturated FUV resonance line of  $P_V$  for a large sample of O-stars, Fullerton et al. (2006, see also Massa et al. 2003) concluded that the resulting mass-loss rates are *a*

*factor of 10 or more lower* than derived from  $H_\alpha$  and/or radio emission using homogeneous models. Similar results have been found for lower luminosity B-supergiants as well (Prinja et al. 2005). If such large reductions in  $\dot{M}$  were true, the consequences for stellar evolution and feed-back would be enormous. Note that an 'allowed' reduction from evolutionary constraints is at most by a factor of 2 to 4 (Hirschi 2008).

**Optically thick clumps.** Porosity effects (Owocki et al. 2004) might resolve this dilemma. Whenever the clumps are *optically thick* for certain processes, the geometrical distribution of the clumps becomes important (size vs. separation, shape). In this *macro-clumping* approach, the effective opacity becomes reduced and the wind becomes *porous*, because radiation can propagate through the 'holes', and because of saturation effects (clumps hidden behind others become ineffective). Oskinova et al. (2007) used a simple, quasi-analytic treatment of macro-clumping to investigate  $P_V$  in parallel with  $H_\alpha$  from the O-supergiant  $\zeta$  Pup. Whilst macro-clumping had almost no effect on  $H_\alpha$ ,  $P_V$  turned out to be severely affected. Only a moderate reduction of the smooth mass-loss rate (again by factors 2 to 3) was necessary to fit the observations, consistent with the evolutionary constraints from above. Note also that the porosity in velocity space can lead to a lower effective line opacity (Owocki 2008).

Meanwhile, Sundqvist et al. (2010, 2011) conducted a thorough investigation on this matter, by constructing 2-D/3-D winds from hydrodynamic simulations or stochastic models, and performing the radiative transfer directly on top of the structured medium. Also in these detailed models, it turned out that the clumps are optically thick in  $P_V$  (even if the winds become thin), whilst they mostly remain optically thin in  $H_\alpha$ . For the testbed  $\lambda$  Cep (another O-supergiant), Sundqvist et al. (2011) were able to fit  $P_V$  and  $H_\alpha$  in parallel at a mass-loss rate being 'only' a factor of two lower than theoretically predicted, but a factor of six larger than by assuming optically thin clumps for *all* processes. These results are quite promising, but certainly not the last word.

To this end, multi-wavelength studies of many stars need to be conducted.

## 5. Winds and cosmic rays

Most massive stars with winds of significant strength are thermal radio emitters, due to free-free wind emission. However, there are also some non-thermal radio sources (17 WR- and 16 O-stars, see De Becker 2007), suggested to emit synchrotron radiation. In terms of diffuse shock acceleration (DSA), this requires the presence of shocks and magnetic fields.

*Strong* surface magnetic fields with  $B > 100$  G are not common in OB-stars, only for less than 10% such field strengths have been measured (see recent work by Donati et al., Neiner et al., Hubrig et al., and by the MiMeS collaboration – ‘Magnetism in massive stars’, Wade et al.). The surface field strengths required for DSA are on the order of 1-10 Gauss though, below the present detection limit.

Regarding the presence of shocks, three different sites can be invoked, which might accelerate cosmic rays.

**Wind-embedded shocks.** In Sect. 4, we saw that the line-instability can trigger the formation of strong reverse shocks, with very high compression ratios ( $\gg 4$ ) in the intermediate wind (isothermal shocks due to efficient radiative cooling). White (1985) and White & Chen (1992) (among others) have suggested wind-embedded shocks as a region to accelerate electrons and ions to relativistic energies, and pointed out that hot stars are potentially strong emitters of synchrotron and  $\gamma$ -ray radiation (e.g., due to  $\pi^0$ -decay), and/or cosmic rays.

More recently, van Loo (2006) argued that the *observed* non-thermal radio emission cannot be created in this scenario, because (i) radiation created in the shocks from the intermediate wind is absorbed by f-f absorption, and (ii) the shocks in the outer wind are too weak to produce enough synchrotron flux. Edmon (2010) performed 2-D MHD DSA simulations, and showed that wind-embedded shocks are indeed able of accelerating electrons up to 1 MeV and protons up to 1 GeV, with  $f(p) \propto p^{-4}$ . Also he concluded, however, that corre-

sponding radio emission is unlikely, due to f-f absorption.

**Wind-wind collisions.** A second potential site for the acceleration of cosmic rays and non-thermal radio emission are the shocks related to colliding winds from O+O or O+WR binaries. Seminal papers on such wind-wind collisions are from Prilutskii & Usov (1976); Luo et al. (1990); Kallrath (1991); Stevens et al. (1992) and Pittard (2009).

As pointed out by De Becker (2007), almost all non-thermal radio emitters are confirmed binaries, except for three sources. Nowadays, wind-wind collisions are considered as the most likely scenario for non-thermal radio emission. Many recent models and results have been inspired by the observations of WR140 (WC7+O5), which is a long period, highly eccentric binary. Multi-wavelength studies during the recent periastron passage in 2009 have been reviewed by Williams (2011).

Regarding the acceleration/emission of high energy particles/radiation in such wind-collision shocks there is ongoing development, e.g., Pittard & Dougherty (2006), Reimer et al. (2006) and again Edmon (2010) who showed that the strong shocks (with temperatures on the order of  $10^7$  K) are capable of acceleration electrons up to 1 Gev, and protons up to 1 TeV.

**Wind terminal shock.** A third site of strong-shock formation is the wind terminal shock. When a wind expands into the ISM, a ‘wind bubble’ is formed. The evolution and structure of such a wind bubble has been firstly described by Castor et al. (1975b) and Weaver et al. (1977). Briefly, from inside out, there is the stellar wind, then the wind terminal shock, an extended shocked-wind region (with temperatures in the range of  $10^7 \dots 10^6$  K and an almost constant density), a contact discontinuity and an H II ‘shell’, extending until a second shock separates the shell from the ambient, cool gas. The total bubble covers roughly 25 pc after an expansion of 10 Myr. Early studies on the potential acceleration of cosmic rays in wind terminal shocks have been performed by Casse & Paul (1980) and Voelk & Forman (1982). The latter authors conclude that termi-

nal shocks might be the source of very low energy, nuclear cosmic rays.

Actually, the situation can become quite complex, since in parallel with the stellar evolution the bubble is shaped by corresponding winds of different strengths and velocities, which partly interact with each other. For the typical evolution of a 30-40  $M_{\odot}$  star, one finds

O-star (fast wind of intermediate strength) → BA supergiant (intermediate velocity and strength) → red supergiant (slow and dense, dust-driven wind) → WR (fast, dense wind) → SN,

and the SN shock wave interacts with the bubble (or even superbubble, see Montmerle, this volume) which has meanwhile obtained a complex structure (Dwarkadas 2005, and this volume).

## 6. Brief summary

Overall, the winds from hot, massive stars are fairly well understood, including effects from rapid rotation which have not been covered here. However, mass-loss rates (both theoretical and observationally inferred) from OBA-stars are still affected by significant uncertainties, due to wind-clumping. In this respect, the situation for WR-winds seems to be somewhat clearer, because clumping effects are easier to detect. Since, as stressed in the introduction, massive star evolution strongly depends on mass-loss, it is affected by similar uncertainties. Future work will hopefully help to improve this situation.

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