Deep mixing at variable speed in stars: is thermohaline diffusion sufficient?

M. Busso, S. Palmerini, and E. Maiorca

Dipartimento di Fisica, Università di Perugia, and INFN, Sezione di Perugia via A. Pascoli, 06123 Perugia, e-mail: busso@fisica.unipg.it

Abstract. We study the circulation of matter in red giants above the H-burning shell, which is known to yield the appearance at the stellar surface of p-capture isotopes like $^7$Li, $^{13}$C, $^{17}$O and the unstable $^{26}$Al. These isotopes were observed (either in presolar grains of circumstellar origin or in the photospheres of evolved stars) to display abundance ratios to other nuclei that cannot be accounted for by canonical stellar models. Slow mixing below the convective envelope is the usual explanation invoked for their abundance. Diffusion generated by an inversion in the molecular weight is today the most commonly assumed driving mechanism for it. We argue that slow transport reaching moderate temperatures ($T < 4 \times 10^7$ K), like the one achievable by diffusive processes induced by a $\mu$ inversion can account for some, but not all the observational constraints. In particular the production of Li after the first dredge up and of $^{26}$Al in the final evolutionary stages both call for substantially different mechanisms. We show how the buoyancy of magnetic instabilities from toroidal flux tubes can offer a paradigm where not only the production and destruction of Li, but also the appearance of all the other known abundance peculiarities, including the presence of $^{26}$Al, can be accounted for. Even in the case that thermohaline mixing were recognized as a necessary byproduct of shell-H burning, it would be nevertheless clear that evolved stars must also host other transport processes, whose effects would therefore combine in the production of the isotopes of light and intermediate elements.

Key words. Nucleosynthesis – Stars: low-mass – Magnetohydrodynamics

1. Introduction

Evolved stars below ~ 2 - 3 M$_{\odot}$ show isotopic abundance anomalies at their surface not explained by current stellar models. These composition peculiarities are found early after the bump of the luminosity function along the red giant branch (RGB) and then continue through the asymptotic giant branch (AGB) stages. The observational database on these problems has grown since the early nineties, for various elements between lithium and oxygen (Gilroy & Brown 1991; Pilachowsky et al. 1993; Gratton et al. 2000). Other connected effects include the anti-correlations shown by higher mass elements up to Mg, appearing already on the Main Sequence in low-mass stars of globular clusters (Gratton et al. 2001). They might offer an indirect evidence for the mentioned abundance peculiarities in red giants, as they were probably inherited by previous generations of low-mass stars.

Non-convective circulation or diffusion of matter that underwent some p-captures is gen-
generally invoked to explain the above complex composition patterns (Charbonnel & Do
Nascimento 1998; Denissenkov et al. 1998). Such a phenomenon is often called deep or
extended mixing, or even cool bottom processing (CBP; Wasserburg et al. 1995). Some
form of non-convective mixing must also involve the He- and C-rich zones during the
dredge-up phases of the AGB (Iben & Renzini 1982; Herwig et al. 2003). This is so be-
cause $\alpha$-processing in AGB stars requires that $^{12}\text{C}$ is produced in the He-intershell from $p$-
captures on the abundant $^{12}\text{C}$ (Gallino et al. 1988; Arlandini et al. 1999) and this can be
obtained if some protons are transported downward from the envelope at dredge-up.

In the past, among the processes proposed to account for the mass transport, it was com-
mon to find several phenomena induced by rotation (Zahn 1992; Denissenkov et al. 1998).
Frequently, the mixing processes were modelled through diffusion treatments, adopting the
diffusion coefficient and the amount of mass mixed as free parameters (Denissenkov
& Tout 2000). Such approaches were common also in modelling radiatively stratified mas-
sive stars (Meynet et al. 2004). However, the idea of a purely rotationally-induced mixing
subsequently underwent difficulties (Goriely & Siess 2004; Palacios et al. 2006). More
recently, it was noticed by Eggleton et al. (2006, 2008) that partial mixing should be in-
duced naturally in stars, thanks to the molecular weight inversion generated by the fusion
reaction $^4\text{He} + ^4\text{He} \rightarrow ^8\text{He} + 2\text{p}$. This was recognized as being a form of thermohaline mixing (Charbonnel & Zahn 2007).

However, when the star experiences extra mixing of whatever nature on the RGB, its
$^3\text{He}$ reservoir in the envelope is consumed. The remaining small abundance, integrated by the
$^3\text{He}$ supply from shell-H burning, was recently found to be insufficient to drive the mixing
(Denissenkov et al. 2009). This would suggest that thermohaline mixing alone is not suitable
to explain the whole set of abundance anomalies, including those created on the AGB. As
measuring $^3\text{He}$ in stellar atmospheres is not possible, it becomes crucial to explore whether
there exist constraints from observable nuclei pointing in this same direction; they would of-
fer the proof that other mechanisms must be at play.

In this paper we intend to perform such an analysis and to show that in fact we need mix-
ing phenomena involving both faster transport and hotter layers than expected in a mecha-
nism driven by the molecular weight inversion due to $^3\text{He}$ burning. We shall also show that
magnetic buoyancy can provide a general scenario, reaching down to hot layers where $^{26}\text{Al}$
is produced, and allowing for a variable mixing speed that can either produce or destroy Li.
In section 2 we shall recall some general features of the mixing induced by magnetic buoyancy
that are useful for the numerical calculations. Then we shall first consider (section 3) the
type of constraints that can be accounted for by slow transport phenomena, either magnetic
or not, in order to verify whether they give us information on the physical mechanisms at
play. Subsequently, we shall consider (section 4) those nuclei (like $^7\text{Li}$ and $^{26}\text{Al}$) whose pro-
duction poses more stringent constraints on either the mixing velocity or the temperature, in
order to derive from them clear indications on the physical processes at play in the transport.
Some tentative conclusions will be drawn in section 5.

2. Magnetic buoyancy as a driver of mixing

We can notice that a situation similar to the one driving thermohaline mixing can be obtained
in other ways. Indeed, for a given value of $T$, the density $\rho$ of a perfect gas linearly depends
not only on the molecular weight $\mu$, but also on the pressure $P$. One can therefore create an im-
balance in $\rho$ not only by inducing a variation in $\mu$, but also by introducing mechanisms act-
ing on $P$. This is exactly what happens if the star contains magnetized regions, as there the
pressure equilibrium with the environment is established by making use also of the magnetic
pressure $B^2/8\pi$, so that gas pressure is smaller than in non-magnetized zones. Magnetic bub-
bles are therefore lighter and undergo a buoy-
ancy effect. Recently, Busso et al. (2007) sug-
gested that this fact might induce extra mixing.
in RGB and AGB stars. Maintaining suitable magnetic fields below the envelope requires the action of a stable stellar dynamo; while this possibility has recently been the object of some controversy, evidence for it has been obtained on observational grounds (Herpin et al. 2006).

We further notice that the stellar zones where the occurrence of extra mixing is needed in evolved stages (the radiative layers below the envelope after dredge-up phenomena), have a homogeneous composition, and are therefore unstable to any mixing perturbation. For those zones, Nordhaus et al. (2007) found that a dynamo can exist if differential rotation is maintained against the action of the magnetic field that carries out the angular momentum. This is the well-known A effect, also occurring in the Sun. The above authors proved that in the relevant regions of evolved stars the required supply of energy from the envelope needed to maintain the differential rotation is minimal, hence the dynamo can easily occur for magnetic field values similar to those suggested by Busso et al. (2007).

On this basis, we assume that magnetic buoyancy is in fact active below the convective border of the envelope in the studied evolutionary stages. As clarified by Denissenkov et al. (2009), thermal exchanges with the environment will occur during this buoyancy. The effect is proportional to the surface of the magnetized bubbles, so that the velocity of large magnetized domains would be very small. On the basis of the treatment by Parker (1974) one can verify that small structures (km-size) would travel at speed in excess of 0.1-1 km/sec, while large magnetized zones (100 km-size) would decrease their speed so much (down to the 0.01-0.1 cm/sec level) that they would mimic the effects of other slow mechanisms of circulation or diffusion.

In what follows we therefore assume two different schemes for magnetically-induced mixing. In particular, we shall define as “model A” a case adopting the buoyancy velocity $v_0$ for the magnetized flow from the simple assumption that:

$$M = 4\pi r^2 \int_0^r f \rho v_b \, \tau$$

where $f$ is the fractional area covered by magnetic structures and $M$ is the circulation rate, for which see Nollett et al. (2003). For $f \sim 0.5$, this corresponds to average speeds below 1 cm/sec, and can therefore be assumed as representative of the slow motion (with heat exchanges) of large magnetized structures. As a second possibility we define “model B” a case in which the buoyancy velocity is taken, from Busso et al. (2007), to be:

$$v(r) = \frac{1}{2} \left( \frac{\rho(r)}{\rho_0} \right) \frac{1}{\tau} \left( \frac{r}{r_0} \right)^2 \left( \frac{g_0 a_0}{C_D} \right) \frac{B_0}{\sqrt{T_r}} \right)$$

where $B_0$ is the magnetic field somewhere near the H-burning shell, $a_0$ is the initial radius of flux tubes in the same position, $C_D$ is the aerodynamic drag coefficient, and the other quantities are given by the stellar structure.

We assume that the mixing starts on the RGB, at the luminosity bump and is re-established on the AGB. On this basis we shall present results for low-mass stars ($M = 1.3 - 2 M_\odot$) with metallicity $Z = Z_\odot/2$, computed with FRANEC by Straniero et al. (2003). We adopt the prescriptions by Nollett et al. (2003) for $T_P$, i.e. $\Delta \log T = \log T_H - \log T_P \geq 0.1$ with respect to the H shell temperature. We remember that this choice for $T_P$ was motivated by the need of not adding a significant amount of nuclear energy from proton captures occurring during the transport.

Concerning the circulation rate $M$, it is left as a free parameter, which in our case implies assuming, as free parameters, the field strengths in the buoyant structures, following equations (2), (7a) and (8) in Busso et al. (2007). The occurrence of the third dredge-up (TDU) after each thermal instability of the He-shell (or thermal pulse) is then included. The timing and mixed mass of each dredge-up episode are taken from the original stellar models and their analytical approximations (Straniero et al. 2003) and the composition of the dredged-up material is derived from Wasserburg et al. (2006) and Busso et al. (2003).

3. Nucleosynthesis in slow mixing phenomena

In our calculations we adopted, as a starting choice, the NACRE compilation of reaction
rates (Angulo et al. 1999). Initial abundances were taken from Asplund et al. (2009), except for N and O, for which the choices discussed by Caffau et al. (2009) were adopted.

The results we present derive from the assumption that extra mixing starts occurring at the bump of the luminosity function on the RGB and then operates again on the AGB in the interpulse periods, wherever envelope penetration has reestablished uniformity of the radiative region.

In this paragraph we start by dealing with the results of our "Model A" (slow mixing). For this model, Fig. 1 (left panel) illustrates the trends of some relevant isotopic ratios in AGB evolutionary phases (the occurrence of similarly slow extra-mixing processes on the previous RGB stages is taken into account). The stepwise variations of abundances is due to the superpositions of third dredge-up episodes over the smooth trend induced by extra mixing. This last operates over the abundance distributions established in the radiative layers by H burning. A typical such distribution is shown in the right panel of the same figure. The main effect of dredge-up for light and intermediate elements is to enrich the envelope with fresh $^{12}$C and $^{26}$Ne from the He shell.

For the same slow-mixing process ("Model A") Fig. 2 shows the trend of oxygen and aluminum isotopic ratios, which are relevant for the comparison with measurements in presolar oxide grains: in particular grains of group 1, 2 and 3 are shown (Nittler et al. 1997). Grains of group 2 (open circles) are in general attributed to extra mixing due to the large destruction of $^{18}$O. The left panel, presenting the oxygen isotopic ratios, shows that most grains of group 1 cluster around the bold line defining first dredge-up compositions for various stellar masses: Dotted lines refer to extra-mixing models in RGB and AGB phases for stars with initial masses 1.3, 1.5 and 1.7 M$_\odot$, computed with different values of $T_P$ (the displacement from the first dredge-up curve is larger for larger values of $T_P$). It is clear that only through models of circulation below the envelope can the abundances of group 2 grains be explained. The right panel of the figure integrates the above evidence by including the aluminum isotopic ratio. Model curves refer to different $T_P$ values (as indicated) and different $M$ values (largest $^{18}$O depletion factors corresponding to larger $M$ values). Here extra mixing is computed only on the AGB, as $^{26}$Al is essentially absent in RGB phases, where the H-burning shell temperature is too low. Again, the grid of model curves covers satisfactorily the area where most data are present.

In general, the isotopic and elemental abundances obtained here are very close to those found by Nollett et al. (2003). In particular, it is found that extra mixing reduces the $^{12}$C/$^{13}$C and the $^{18}$O/$^{16}$O ratios. The $^{14}$N/$^{15}$N ratio instead increases remarkably and, once on the AGB, the unstable nucleus $^{26}$Al is produced abundantly. The results arise from the interplay of $^{12}$C production in TDU, its (incomplete) conversion into $^{13}$C and $^{14}$N in the CBP burning material, and the enrichment of the envelope with H-burning ashes at a rate $\dot{M}$. This can explain the relatively low $^{12}$C/$^{13}$C ratios observed in some S and C stars, as being due to CBP (Abia et al. 2002).

It is also found that stars below 1.8-1.9 M$_\odot$ do not become C-rich, ending their life as S stars; they have a high N/O ratio, as indeed observed in these sources (Lambert et al. 1986). Reaching a ratio C/O < 1 while CBP is active is possible for stars of at least $M = 2$ M$_\odot$, where TDU is more effective than CBP. In lower masses one should maintain CBP at low efficiencies to allow for the formation of a C star.

We confirm that the aluminium isotopic ratio is essentially a thermometer, $^{26}$Al being primarily sensitive to the $T_P$ value (and growing with it), and depending only weakly on $M$, as already found in Nollett et al. (2003). $^{26}$Al is then also affected by dredge-up; this is due to the fact that the large amount of $^{26}$Al engulfed in the He shell during shell H-burning, a small amount (10%) survives the neutron captures occurring in thermal pulses and can be returned to the envelope (together with its remaining abundance in the deepest layers of the H shell).

For a slow transport the physical model is not very relevant. From this point of view, the specific hypothesis of a mixing driven by mag-
Fig. 1. Left: Evolution of the envelope isotopic ratios along the AGB in our 2M\textsubscript{\odot} model. The chemical composition of the stellar envelope is modified by both extra mixing and TDU. The discrete occurrence of the latter produces a saw-tooth structure, mainly in the trends of \textsuperscript{12}C, \textsuperscript{22}Ne and \textsuperscript{26}Al. Right: the internal distribution of abundances in the radiative zone above the H-burning shell, during the AGB phases of our 2M\textsubscript{\odot} model.

Fig. 2. Left: Oxygen isotopes as displayed by presolar oxide grains of groups 1 (heavy dots), 2 (open circles) and 3 (diamonds), as compared to models (dotted lines) for extra mixing in different stellar masses. See text for explanations. Right panel: The destruction of \textsuperscript{18}O and the production of \textsuperscript{26}Al plotted one against the other. There is no direct correlation, as \textsuperscript{18}O consumption mainly depends on \dot{M}, while \textsuperscript{26}Al production is mainly related to the maximum \textsuperscript{T}{\textsubscript{P}} value. The grid of extra-mixing models satisfactorily covers the area of the data.
netic buoyancy plays no direct role, if we limit ourselves to our “Model A”. Any slow mechanism reaching down to sufficiently hot layers to produce $^{26}$Al in sufficient quantities would provide the same results.

4. Nucleosynthesis of Li

In a slow mass flow there would be ample time for CBP to occur, as typical crossing times of the radiative layers would reach up to thousands of years. During this period, not only the photospheric $^7$Li, but also any $^3$He remaining in the envelope has time to be destroyed (for $^3$He this occurs by either $^3$He + $^4$He $\rightarrow$ $^4$He + 2p or $^3$He + $^4$He $\rightarrow$ $^7$Be + γ). Then $^7$Be can be re-synthesized in the circulation. Its addition is however very slow; in the finite duration of the RGB phase (about 40 million years) and even more during the short TP-AGB stage (1-2 Myr) destruction of Li prevails over its synthesis if $\dot{M}$ is not very large. Any slow circulation, either from magnetic buoyancy or from thermohaline diffusion, is therefore bound to destroy Li, or to produce it only slightly and on long time scales. There is however another way for producing Li in mixing processes induced by magnetic fields, if a fast but intermittent process of bubble release, under the form of magnetic instabilities, occurs in the absence of a slow downflow. This case is the one we have previously called “Model B”. It essentially mimics the so-called Cameron-Fowler mechanism: $^7$Be must in this case be produced by equilibrium radiative burning itself, and magnetic instabilities must occur rarely enough to give it time to reform. We would in this case have upflows and downflows occurring erratically and at a very fast speed, as observed in many cases in the Sun [Briand & Solanki 1998]. Then $^7$Be would be simply “sampled” at high T and delivered into the envelope without further processing, thus increasing the Li abundance there. Depending on the value of $T_p$ we can either have a process in which only Li is contributed or obtain also a decrease in the $^{12}\text{C}/^{13}\text{C}$ ratio and other effects.

As many Li-rich giants exist already during the first ascent of the Hayashi track, one has to assume that fast mixing must occur rather early, close in time to the bump of the luminosity function. This might correspond to a phase in which magnetic fields are still rather small ($< 10^5$ G) and only occasional, fast instabilities reach the condition for buoyancy. Due to their small size, they will travel fast, virtually at the Alfvén velocity (0.1 - 1 km/sec) and will carry fresh Li to the surface, crossing the radiative layers in few days (a time shorter than the lifetime of $^7$Be). An example of this is presented in Fig. 3, which shows the effects of a specific choice in the velocities of magnetic mixing, suitable to interpret the observational evidence in normal RGB stars. The hypotheses adopted there assume that: i) at the luminosity bump on the RGB we find the mentioned phase characterized by small toroidal magnetic fields; small unstable “bubbles” can however travel through the radiative zone (our “Model B”), hence a rapid rise in the surface Li oc-

![Fig. 3. A possible scheme of extra-mixing occurrence explaining the observations of Li in first-ascent red giants. At the bump in the luminosity function fast magnetic instabilities carry $^7$Be to the surface, where it decays to Li (dashed line). The subsequent increase in the field strength leads to the buoyancy of large magnetized zones, which slow down and lead to a destruction of Li at rates depending on the rate of mass circulation. Dots refer to observations (see Guandalini et al. 2009).](attachment:image.png)
curs, up to levels typical of moderately Li-rich red giants \((\log(\epsilon) = 2)\); ii) if, as expected, the stellar core increases its spinning rate with time, moving as a rigid body, then magnetic fields will grow, thus promoting the buoyancy of larger magnetized domains or even of entire flux tubes. These will exchange heat with the environment thus establishing a slower circulation (as in our “Model A”). Such a slow circulation gives rise to a decrease in the surface Li, as shown in the figure. Stars in the early-AGB phase and in advanced AGB stages, showing moderate Li enrichment, will therefore simply lay on the extension of the model lines plotted in Fig. 3.

5. Conclusions

From the results discussed in the previous sections we can deduce that thermohaline mixing and in general all slow diffusion processes, can explain only in part the evidence provided by the chemical composition of evolved low-mass stars and of presolar grains originating in their envelopes. In particular the production of enough Li to explain Li-rich red giants cannot be obtained in this way.

One has further to notice that the occurrence of deep mixing on the RGB consumes \(^3\)He extensively, so that its remaining abundance becomes very small on the AGB. In the short time available it is not reformed efficiently, so that no noticeable \(\mu\) inversion is generated by its burning and thermohaline mixing disappears. It has also to be underlined that even in case \(^3\)He were preserved up to the AGB phases (e.g. because no deep mixing occurs on the RGB), it would drive diffusion only down to the relatively shallow layers, contrary to the evidence provided by the presence of abundant \(^{26}\)Al in presolar grains. Indeed, producing this nucleus requires high temperatures.

From both the above considerations we can conclude that other mechanisms must be active, as an alternative or in addition to thermohaline diffusion.

In this paper we have shown that, in particular, the buoyancy of magnetized structures of different size and speed has all the characteristics required to explain the inventory of constraints coming from both stellar observations and presolar grain isotopic measurements.

References

Busso, M., Gallino, R., & Wasserburg, G. J. 2003, PASA, 20, 356
Guandalini, R., Palmerini, S., Busso, M., & Utenthaler, S. 2009, PASA, 26, 168
Straniero, O., Domínguez, I., Cristallo, S. & Gallino, R. 2003, PASA, 20, 389