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White dwarfs and fundamental physics

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Abstract. The evolution of white dwarfs (WDs) stars is essentially a cooling process that is easier to compute compared to other evolutionary phases. However, sizeable discrepancies are still present between the current theoretical models. In fact, the present knowledge of the physical behavior of matter under the high-density regimes typical of the WD interiors is still affected by significative uncertainties. In this contribute, I will discuss some of the main uncertainties in the input physics adopted in the computation of WD cooling evolution.

Key words. Stars: white dwarfs – Stars: evolution – Stars: interiors – Stars: luminosity function – Galaxy: globular clusters – Dense matter – Conduction

1. Introduction

In this paper, I will describe some basic features of white dwarf (WD) structure and evolution. The aim of the present contribution is twofold: firstly to provide a sort of introduction to the detailed discussion by Leandro Althaus in this book and secondly to illustrate the dependence of the current models on some of the physical assumptions adopted.

The literature on WDs is quite large and heterogeneous, but the interested reader can obtain a first and useful guide in several reviews published over the last two decades (D'Antona & Mazzitelli 1990; Koester & Chanmugam 1990; Fontaine, Brassard & Bergeron 2001; Koester 2002; Hansen & Liebert 2003; Hansen 2004; Moehler & Bono 2008).

WDs represent the final evolutionary phase of low and intermediate-mass stars, namely, stars with initial mass less than M_{up}, defined as the minimum stellar mass for which a carbon ignition occurs before the onset of the asymptotic giant branch (AGB). Current stellar models show that the value of M_{up} varies between 6.5 and 7.5 $M_\odot$, for metallicity ranging between Z=0 and 0.02 (for recent computations Cassisi, Castellani & Tornambé 1996; Dominguez et al. 1999; Castellani et al. 2003). Since the initial mass function (IMF) of low and intermediate-mass stars can be reasonably described by a power-law with a quite steep exponent (Salpeter 1955, $dN/dm \sim m^{-2.35}$ for the well-known Salpeter's IMF), this means that the WD stage represents the most common, by far, final destiny in stellar evolution. Such a situation by itself makes WDs objects well worth studying: they provide a sort of "cosmic archaeology". Furthermore, the extreme physical conditions which characterize WD interiors allow us to test the accuracy of the current physical theories describing the behavior of matter in high pressure regimes. It was recognized several decades ago that WDs can also be used as cosmic chronome-

Send offprint requests to: P. G. Prada Moroni ters (Schmidt 1959) and standard candles (Fusi Pecci & Renzini 1979; Renzini et al. 1996), but their actual application as cosmic tools has been hampered until very recently by their faintness, especially for WDs belonging to globular clusters (GCs). Now, thanks to the huge progress in the available instrumentation, this situation is quickly changing, thus triggering new interest in WDs. Thanks to the HST and the very-large ground based telescopes, data for WDs are available and growing. Particularly intriguing has been the detection of WDs in globular (Elson et al. 1995; Paresce, De Marchi & Romaniello 1995; Cool, Piotto & King 1996; Renzini et al. 1996; Richer et al. 1997) and open clusters (Von Hippel, Gilmore & Jones 1995; Von Hippel & Gilmore 2000) and the subsequent investigation down to very faint luminosities (Hansen et al. 2002; De Marchi et al. 2004; Hansen et al. 2004; Monelli et al. 2005; Hansen et al. 2007; Bedin et al. 2008; Calamida et al. 2008).

The reliability and precision of the age estimates obtained by using WDs depends on the accuracy of the models. That is why it would be wise to check the reliability of the theoretical models before adopting WDs as cosmic clocks. In fact, in spite of the improvements in the computation of their structure and evolution (Benvenuto & Althaus 1999; Hansen 1999; Chabrier et al. 2000; Salaris et al. 2000; Corsico et al. 2001; Prada Moroni & Straniero 2002; Serenelli et al. 2002; Panei et al. 2007), there still remains a not negligible uncertainty in the prediction of the cooling times of the old WDs (Montgomery et al. 1999; Salaris et al. 2000; Fontaine, Brassard & Bergeron 2001; Prada Moroni & Straniero 2002, 2007).

After a brief historical introduction, I shall describe the basic characteristics of the WD evolution, and then I shall discuss two of the major sources of uncertainty in the present generation of cooling models: the still poorly constrained C-O abundances in the core and the conductive opacity.

2. Historical overview

In 1844, the famous astronomer and mathematician Friedrich Wilhelm Bessel (1784-1846) in a letter to Herschel, reported the results of several years of long and difficult visual astrometric observations of the Sirius proper motion. The brightest star of the sky describes on the heavenly vault a wave rather than a straight line. As Bessell immediately understood, this behavior can be easily explained if Sirius belongs to a binary system with a faint and undetected companion.

The hunt for this faint companion went on until 1864, when the American telescope constructor Alvan Clark (1804-1887) finally succeeded in observing it, Sirius B. When in 1915, the American astronomer Walter Adams (1876-1956) succeeded in the very hard task of observing the spectrum of Sirius B, the situation appeared quite puzzling: Sirius B's spectral type is A0 like Sirius A (Adams 1915), contrary to every expectation the faint Sirius B is white.

In 1924, Sir Arthur Eddington (1882-1944), one of founders of modern theoretical astrophysics, named, with his usual sense of humor, this new class of objects "white dwarfs". It has been, in fact, immediately clear that this name sounded as an astronomical oxymoron. In those early years, a lot of dwarf, faint stars were already known, but they were all red. In those early studies, the astronomers clearly understood that an incredible result would be obtained if one associated to this strange object the effective temperature of a main sequence star belonging to the same spectral type. In fact, from the well-known relation

$$L = 4\pi R^2 \sigma T_{eff}^4$$

which relates the luminosity L of a star with its effective temperature T_{eff} and radius R, it follows that two stars with about the same effective temperature and very different luminosities must have a large difference in their radii, the fainter being the smaller. Sirius B is almost ten magnitudes fainter than Sirius A, thus it must be a very compact object. Today, we know that the spectral type of standard WDs doesn't indicate the temperature but is essentially an effect of the peculiar chemical composition of the atmosphere of these stars, which are constituted by nearly pure hydrogen, for the most common DA (see e.g. the contribution by

Koester in this book). The effective temperature of Sirius B is about 25000 K while that of Sirius A 9900 K, so that the conclusion that the faint Sirius' companion must be a very compact object is still valid. Moreover, this is the main characteristic of WDs: objects with a solar mass, packed in a volume of the order of that of the Earth. As a consequence, WDs are characterized by very high densities (10^6 - 10^7 g/cm^3) and surface gravities (10^8 -10^9 cm/s^2). Eddington showed that such a high density is not absurd, provided the atoms are fully ionized. On the other hand, he pointed out that this compact star would eventually be in an embarrassing situation. Indeed, as he stated in his famous and beautiful book "The internal constitution of the stars" (Eddington 1926): *"the star will need energy in order to cool"*. However, he clearly understood that, behind the apparently peculiarity of WDs, there must have been something profound, as we can infer from his prophetic statement: *"Strange objects, which persist in showing a type of spectrum entirely out of keeping with their luminosity, may ultimately teach us more than a host which radiates according to rule"* (Eddington 1922).

The pioneering studies on stellar structure of the second half of XIX and beginning of the XX century by, among others, Lane, Ritter, Kelvin and Emden, showed that stars are the result of the balance between the forces of gravity and pressure. In other words, stars are massive gaseous balls in hydrostatic equilibrium. Since those early studies, we have known that the stability of stars is governed by an important global constraint, the virial theorem. I will follow the treatment and notations of the classic Kippenhahn & Weigert (1994) book. The virial theorem for systems in hydrostatic equilibrium and with vanishing surface pressure can be written as:

$$E_g + 3\int_0^M \frac{P}{\rho} dm = 0 \qquad (1)$$

where E_g is, as it is well known, the gravitational energy of the star, P and ρ the pressure and density, respectively. We can further simplify the relation by assuming a monoatomic ideal gas, yielding

$$E_g + 2E_i = 0 \qquad (2)$$

where E_i is the total internal energy of the star. In order to obtain a more general result, which holds for any equation of state (EOS), it is usually introduced the quantity ζ, defined as:

$$\zeta u \equiv 3\frac{P}{\rho} \qquad (3)$$

In the case of the classical ideal gas, it is straightforward to obtain $\zeta = 3(\gamma - 1)$, that for a monoatomic gas gives ζ=2. A generalized version of the eq. 2 can be written by assuming ζ constant throughout the star

$$E_g + \zeta E_i = 0. \qquad (4)$$

If we define the total energy of the star as $W = E_g + E_i$, from the generalized version of the virial theorem (eq. 4) follows

$$W = (1 - \zeta)E_i = \frac{\zeta - 1}{\zeta} E_g \qquad (5)$$

In the absence of thermonuclear energy sources, from the principle of energy conservation and eq. 5 it follows that

$$L = -\dot{W} = (\zeta - 1)\dot{E}_i = -\frac{\zeta - 1}{\zeta}\dot{E}_g \qquad (6)$$

where L is the stellar luminosity. Since stars are not isolated systems, but are embedded in a huge and very cold environment, they must lose their energy. In other words, L must be a positive quantity. Therefore, normal stars without energy sources are forced by the eq. 6 to shrink ($\dot{E}_g < 0$) and increase their internal energy ($\dot{E}_i > 0$), then, getting hotter and hotter (Lane's law) in response to the energy loss. At variance with the familiar objects in our everyday life, stars react to the inevitable energy loss, rather counter-intuitively, by raising their internal temperature. A fascinating way to describe such a behavior is saying that normal stars have a negative heat capacity. In spite of its simplicity, this is a profound result, which plays a key role in the evolution of stars and that, in the pioneering studies of stellar structure in the second half of the XIX century, was considered rather surprising.

Now, given such a result, how can a star ever cool? By paraphrasing Lynden-Bell's words concerning his studies on gravothermal catastrophe (Lynden-Bell & Wood 1968), the rate of energy loss in a self-gravitating system can never be sufficiently high to force it to cool. As it is well known, the solution of the puzzle is in the quantum behavior of electrons under condition of high density/low temperatures. The Pauli exclusion principle forbids particles with half-integer spin from occupying precisely the same quantum state. A gas of fermions can still exert a pressure even at T=0, as the pressure is not the consequence of thermal agitation. Just a few months after the discovery of the quantum statistic by Fermi and Dirac, Ralph Fowler understood that the new statistical behavior could solve the Eddington's paradox. In fact, he realized that WDs are supported by the pressure of degenerate electrons (Fowler 1926). The classic Boltzmann statistic holds only in the limit of rarefied gas, where the average distance between particles is much larger than the de Broglie wavelength. We can write this condition as $a_e >> \Lambda_e$, where $a_e = (4/3\pi n_e)^{-1/3}$ is the average distance between electrons and

$$\Lambda_e = \left(\frac{2\pi\hbar^2}{m_e k_B T}\right)^{1/2}$$

is the thermal de Broglie wavelength. In the very dense interiors of WDs, this condition is not fulfilled. As an example, for a mean density and temperature of about 10^6 g/cm^3 and 10^6 K, respectively, $\Lambda_e/a_e >>100$. Fowler understood that the Fermi-Dirac quantum statistic can prevent the gravothermal catastrophe, as it breaks the tight dependence of the internal energy on temperature. This leads to a fundamental difference between normal stars and WDs.

Completely degenerate stars can be in hydrostatic equilibrium without shining, at variance with normal stars, which must be hot in order to provide the pressure gradient required to balance the gravitational force, and thus they must radiate (Cox & Giuli 1968). In WDs, the mechanical and thermal properties are largely independent of each other. In order to understand their behavior, we should remember that there is no equipartition between the kinetic energy of ions and electrons. The virial theorem holds for every system in hydrostatic equilibrium, regardless of the degree of degeneracy. So the equation 2 is still valid for WDs and writing the ionic and electronic contribution to the pressure separately (P=P_i+P_e), it is easy to see that in highly degenerate regimes, where $P_e >> P_i$, the ζ quantity previously defined is equals to 1 in the relativistic case and 2 in the non-relativistic one. The virial theorem can be written as

$$E_g + \bar{\zeta} E_i = 0. \tag{7}$$

where $\bar{\zeta}$ is the average value of ζ throughout the star and $1 < \bar{\zeta} < 2$. As in the previous case, from the conservation of energy follows

$$L = (\bar{\zeta} - 1)\dot{E}_i = -\frac{\bar{\zeta} - 1}{\bar{\zeta}}\dot{E}_g \tag{8}$$

which has the same form of the eq. 6. Partially degenerate stars shrink and increase their internal energy in response to energy loss, exactly as normal stars do. This is an obvious, but too often misunderstood, result (see e.g. the discussion in Koester 1978)

But unlike normal stars, WDs do not heat up. As I already mentioned, the crucial point is that in partially degenerate matter, there is no equipartition of energy between electron and ions. For the sake of simplicity, I will assume a gas consisting of ideal non-degenerate ions and degenerate non-relativistic electrons. In this case, $\bar{\zeta}$=2 and L=-1/2 $\dot{E}_g$. As the gravitational energy $E_g \sim -\rho^{1/3}$, we get that

$$\frac{\dot{E}_g}{E_g} = \frac{1}{3}\frac{\dot{\rho}}{\rho}$$

As the WD shrinks the internal energy of the electron gas E_e increases, since the Fermi energy is forced to rise, $E_e \approx E_F \sim \rho^{2/3}$ and

$$\frac{\dot{E}_e}{E_e} = \frac{2}{3}\frac{\dot{\rho}}{\rho}$$

In highly electron-degenerate regimes, the main contribution to the internal energy is given by the kinetic energy of the degenerate electrons, since $E_e >> E_{ion}$. We can say that the gravitational energy released by the contraction is almost entirely converted into kinetic energy of the degenerate electrons ($\dot{E}_e \approx$

$-\dot{E}_g = 2L$). Since the rate of increase in the kinetic energy of the degenerate electrons $\dot{E}_e$ is nearly equal to the rate of change of the gravitational energy $\dot{E}_g$, the luminosity of a partially degenerate object is essentially provided by the rate of decrease of the internal energy of the ion gas $\dot{E_{ion}}$. Thus, a WD cools as it loses energy, contrary to normal stars, which get hotter and hotter, which means that WDs have a positive heat capacity. Since their luminosity is largely supplied by their ion thermal content, WD evolution is usually referred to be a cooling process.

From this brief sketch emerges a simple picture of WD structure where degenerate electrons provide almost all the pressure which supports the star, while their contribution to the heat capacity is negligible. Indeed, only those electrons near the surface of the Fermi sea can contribute, while the others are already in their lowest energy state. For non-relativistic degenerate electrons, the specific heat for unit mass is:

$$c_v = \frac{3}{2}\frac{k_B}{Am_p}\frac{\pi^2}{3}Z\frac{T}{T_F}$$

where the Fermi temperature, T_F, is typically of the order of 10^9 K. On the other hand, ions are non-degenerate and do not contribute significantly to the pressure, but they provide almost the entire thermal energy of the WD.

As it was early shown by Marshak (1940), heat conduction is very efficient in the highly degenerate regimes typical of WD interiors, contrary to normal stars, where it is negligible. In fact, the mean free path of electrons gets longer and longer as the degeneracy degree increases, because of the corresponding decrease of the free state density in the phase space. While the thermal energy is stored in the ion component, it is transferred outward by the degenerate electrons. Electron conductivity is efficient enough to keep the degenerate core nearly isothermal. In contrast, in the outer layers, electrons are only either partially or not degenerate, thus the electron conduction is inefficient. Therefore, the thin outer layers, whose mass is lower than about 1% of the total mass, represent the most opaque region of the WD, a true thermal bottleneck which regulates the heat loss rate.

The structure of a typical C-O WD can be roughly envisioned as constituted by:

- a highly degenerate core, nearly isothermal, whose mass is about 98-99% of the total mass, which constitutes the thermal energy reservoir of the star;
- a thin, non-degenerate envelope which plays the role of a thermal insulator.

We could say that a WD behaves as an hot iron ball wrapped in an insulating blanket immersed in an extremely cold environment, whose evolution can be roughly described as a cooling process where the luminosity is largely supplied by the heat content of the internal matter. Therefore, the evolution results from the balance between the thermal energy stored in the C-O ions constituting the core (more than 98% of the WD mass), and the energy transport through the He-rich mantel and the H-rich envelope, the most opaque region of the star (for a detailed description of WD evolutionary properties see e. g. the contribution by Althaus in this book).

3. Input physics and model uncertainties

From a numerical point of view, the evolution of WDs is much easier than other stellar evolutionary phases, such as thermal-pulsing AGB (see e.g. the contribution by Wood in this book). As Icko Iben said, *"Any fool can make a WD!"* (as quoted by Winget & Van Horn 1987). However appearances may deceive us. Figure 1 shows the comparison between some of the most recent cooling curves in the literature of a DA CO WD of M=0.6 $M_\odot$. As can be easily seen, there is still a substantial disagreement in the predicted evolutions, in particular at the faint end (log $L/L_\odot \approx$ -5.5), where the discrepancy in the cooling ages reaches 4 Gyr.

While the computation of WD models is not too difficult, since it does not require sophisticated alghoritms, detailed and accurate descriptions of the physical behavior of matter under condition typical of WD interior are a

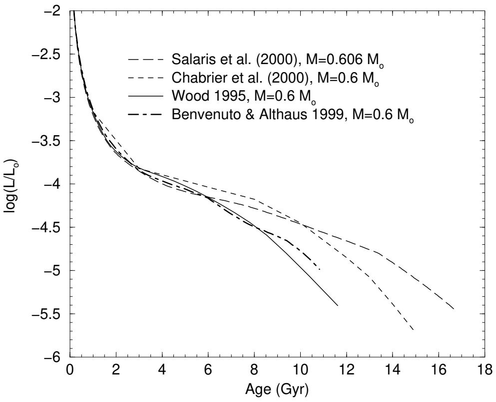

Fig. 1. Comparison between some of the most recent theoretical cooling sequences for a DA WD of M=0.6 $M_{\odot}$: Wood (1995, solid line), Benvenuto & Althaus (1999, dot-dashed line), Chabrier et al. (2000, dashed line), Salaris et al. (2000, long dashed line). All these sequences, except the one by Chabrier et al. (2000), have been computed without including the effect of the chemical segregation occurring upon crystallization and by using a full evolutionary code. The Chabrier et al. sequence has been derived from static WD models and includes chemical segregation.

very hard task. Furthermore, the pre-WD evolution is affected by some still poorly understood processes, like convection and mass loss. In the next subsections, I will analyze the effect on the predicted cooling times of the present uncertainty in the chemical abundance profiles of the core and of the different assumptions on the efficiency of the heat conduction, mainly in the partially degenerate regimes characterizing the thin outer layers.

3.1. C-O abundances

As I previously mentioned, WDs radiate essentially at the expenses of the internal thermal energy stored in the ions of the core. Since carbon has a higher specific heat per unit mass than oxygen, the relative amounts of carbon and oxygen present in the core have a great influence upon the WD cooling rate (D'Antona & Mazzitelli 1990; Salaris et al. 1997; Prada Moroni & Straniero 2002). Unfortunately, the core chemical profile of a CO WD is still

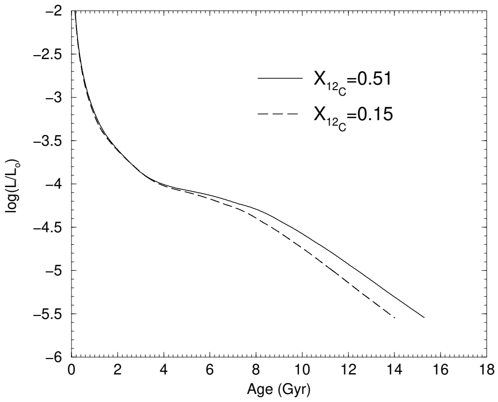

Fig. 2. Theoretical cooling sequences for a C-O DA WD of 0.6 $M_{\odot}$ obtained for the two extreme central Carbon abundances, namely $X_{^{12}C}$ =0.15 (dashed line) and $X_{^{12}C}$ =0.51 (solid line).

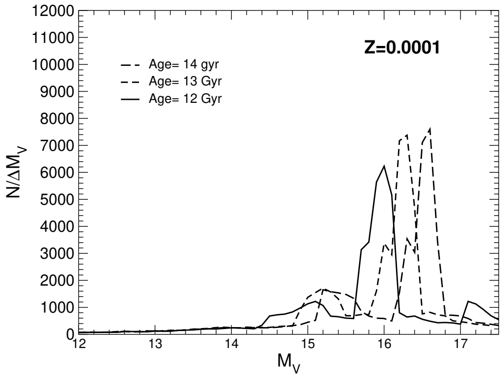

Fig. 3. Theoretical LFs for 12 (solid line), 13 (dashed line) and 14 Gyr (long-dashed line). Each magnitude bin is ΔM_V=0.1 mag. The total number of simulated WDs is 10000.

poorly constrained. It is the fossil of the previous He-burning evolutionary phase, both in the center and in the shell, the result of the competition of the 3α, the carbon producer, and the $^{12}C(\alpha,\gamma)^{16}O$, the carbon consumer. While the reaction rate of 3α at the energy appropriate for He-burning ($\approx$ 300 KeV) is known with an uncertainty of about 10%, that of the $^{12}C(\alpha,\gamma)$ is much more uncertain (Buchmann 1996, 1997; Kunz et al. 2001, 2002). Furthermore, the chemical stratification of the core depends also on the efficiency of the convective mixing during He-burning phase, which is still

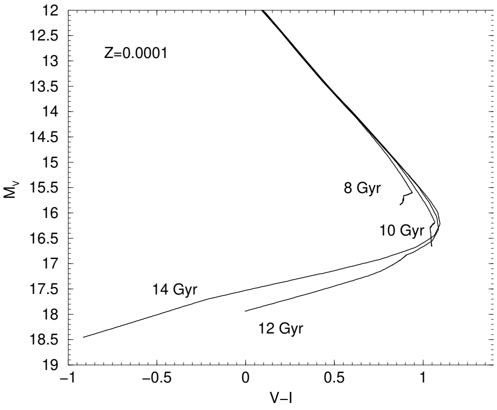

Fig. 4. WD isochrones for ages in the interval 8-14 Gyr. We adopted the HST/ACS transmission curves *F*606*W* (broad *V*) and *F*814*W* (broad *I*).

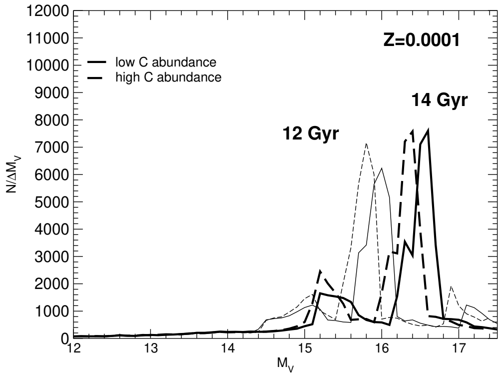

Fig. 5. Theoretical LFs for 12 (thin lines) and 14 Gyr (thick lines) for WDs with low (solid lines) and high (dashed lines) C abundance.

poorly known because of the persistent lack of a satisfactory and self-consistent theory of convection. In particular, any mechanism that increases the size of the mixed region during the final part of the He-burning (mechanical overshoot, semiconvection, breathing pulses or rotational induced mixing), where the largest fraction of carbon is converted into oxygen, leads to a decrease of the carbon abundance left in the core of WDs (Imbriani et al. 2001; Straniero et al. 2003).

As a consequence, the central carbon abundance in mass is currently quite uncertain, ranging between 0.15 and 0.5 in mass (Prada Moroni & Straniero 2002; Straniero et al. 2003). Figure 2 shows the comparison between the cooling curves of a DA CO WD of 0.6 $M_\odot$ with high (solid line) and low (dashed line) carbon abundance, respectively. As expected, owing to the lower heat capacity of the carbon-poor material, the cooling time is shorter in these models compared to the cooling time of the carbon-rich models. The global uncertainty in the cooling evolution, due to the present uncertainty of the C-O profile, caused by convection plus nuclear reaction rate, is at least of 9% at the faint end (Prada Moroni & Straniero 2002). Note that this is probably a lower limit of the uncertainty, since we did not take into account chemical segregation occurring at crystallization (Isern et al. 1997; Montgomery et al. 1999). According to Montgomery's theoretical estimates (see e.g. their table 2), element segregation should increase the age difference at the faint end to 13-14%.

This uncertainty severely limits one of the main applications of WDs, the cosmochronology (Schmidt 1959). Figure 3 shows the theoretical WD luminosity functions (LFs) for simple stellar populations of 12, 13, 14 Gyr. These LFs have been computed by means of Monte Carlo simulations that distribute 10000 WDs along the quoted isochrones. Data plotted in the figure 3 shows that, in the typical range of ages of the galactic GCs, the peak of the WD LF shifts by about 0.3 mag per Gyr in the V band (De Marchi et al. 2004; Prada Moroni & Straniero 2007). Such a considerable dependence of the position of the peak on the age is the reason why the WD LF is, in principle, a very robust age indicator. As it is well known, the current main source of error in dating GCs is the measure of the distance modulus, thus the weaker the dependence of the luminosity of an age indicator on the age of the system, the larger the uncertainty on the estimated age due to the poorly constrained distance of the GC. As an example, the main sequence turn-off luminosity, the traditional cluster clock, shifts only by about 0.1 mag per Gyr. Thus, the uncertainty on the distance modulus of a GC translates in an error in the inferred age by means of the turn-off luminosity about three times larger than that obtained by the luminosity of the WD LF peak. For the same reason, the blue-turn of the WD isochrone, it is not the best WD clock pointer. In fact, the absolute magnitude of the blue-turn significantly depends on the adopted passbands. For stellar clusters older than 10 Gyr, the blue-turn is almost insensitive to the age in the *V* vs. *V*-*I* diagram, commonly used in observations of cool WDs (Prada Moroni & Straniero 2007). Such a behavior is the consequence of the collision induced absorption (CIA) of H_2-H_2 molecules (Bergeron, Saumon & Wesemael 1995) in the atmosphere of cool (T_e < 5000 K) DA WDs, which causes a strong reduction of the infra-red flux in the emergent spectrum and a shift toward bluer colors of cold WDs. The extent of this blue shift depends on the wavelength of the transmission curves of the filter adopted in the observations. In particular, since the CIA of H_2-H_2 molecules is the main opacity source in the infra-red, the blue shift will be much larger in the *V*-*I* than in the *B*-*V*. Furthermore, such a blue shift, which is essentially only an atmospheric effect, depends on the effective temperature and not on the age. This is the reason why in the *V* vs. *V*-*I* plane the blue-hook luminosity of old WD isochrones is essentially insensitive to the age and the blue-tail of the 14 Gyr WD isochrone is brighter than the 12 Gyr one, as shown in figure 4. Such a degeneracy of the position of the blue-turn with age severely limits the effectiveness of the WD isochrones to date old stellar systems such as the galactic GCs.

These arguments strongly support the usefulness of the WD LF peak as cosmic chronometer. However, the theoretical uncertainty in the cooling times leads to an uncertainty in the predicted position of the WD LF peak. Figure 5 shows the 12 (thin lines) and 14 Gyr (thick lines) theoretical LFs for WD models with low (solid lines) and high (dashed lines) carbon abundance. As described above, owing to the faster cooling of the carbon-poor models as compared to the carbon-rich ones, the corresponding WD LF peak occurs at lower luminosity. The effect of the present uncertainty in the chemical profile in the core on the position of the peak is quite large, causing an uncertainty of about 0.6 Gyr in the inferred GC age (Prada Moroni & Straniero 2007).

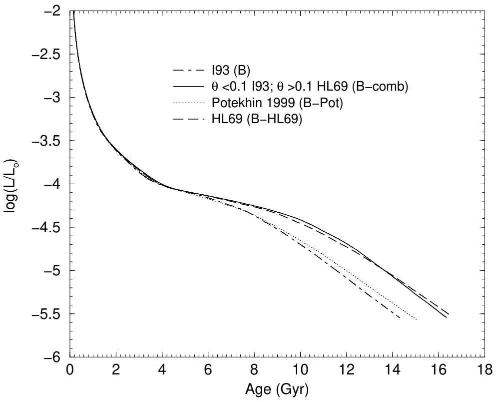

Fig. 6. Theoretical cooling sequences under different prescriptions for the conductive opacity: I93, Mitake et al. (1984) for the fluid phase and Itoh et al. (1983) for the solid phase (dot-dashed line); HL69, Hubbard & Lampe (1969) (dashed line); I93 for the fully degenerate regime ($\theta \leq 0.1$) and HL69 for the partially degenerate regime (solid line); Potekhin et al. (1999) (dotted line).

As I previously mentioned, this is probably a lower limit, since in these models the process of chemical segregation which occurs at crystallization (Isern et al. 1997; Montgomery et al. 1999), with the concomitant energy release, has not been implemented.

3.2. Conductive opacity

Since the pioneering study by Marshak (1940), it has been known that in highly degenerate regimes, such as those present in the WD interiors, the electron thermal conduction is the most efficient process of energy transfer. On the contrary, in the He-rich mantel and in the H-rich envelope the electrons are only partially or not degenerate, so that the thermal conduction is less efficient. The thin envelope is the most opaque region of the WD, a kind of insulating layer which regulates the temperature decrease of the core. Since nearly all the temperature rise occurs in the outer zone of the core and in the non-degenerate envelope, the computed temperature profile, and thus the cooling evolution, is extremely sensitive to both the conductive and radiative opacity. Among the most frequently

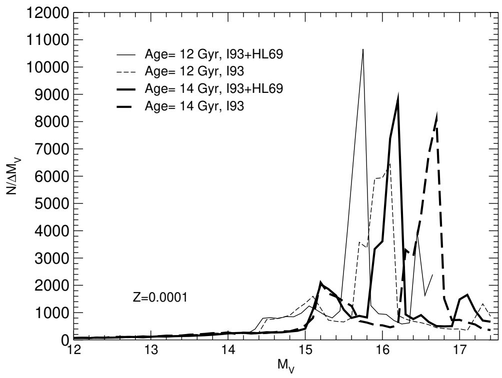

Fig. 7. Theoretical LFs for 12 (thin lines) and 14 Gyr (thick lines) for WD models computed adopting the conductive opacity of Itoh and coworkers (I93) in the fully degenerate regime ($\theta = T_F/T < 0.1$) and the Hubbard & Lampe (1969) (HL69) in the partially degenerate one ($\theta > 0.1$) (I93+HL69, solid lines) and the I93 in the whole structure (I93, dashed lines).

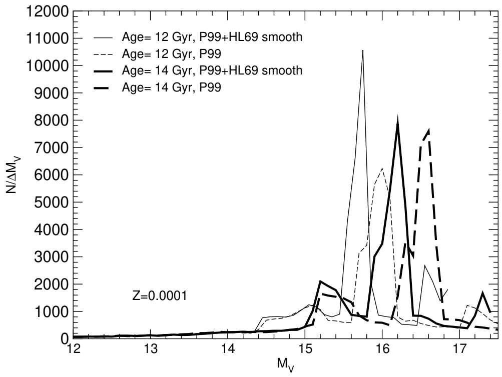

Fig. 8. Theoretical LFs for 12 (thin lines) and 14 Gyr (thick lines) for WD models computed adopting the conductive opacity of Potekhin (P99) and HL69 in the regions where, respectively, $\theta < 0.1$ and $\theta > 1$, and a linear interpolation in the transition zone (P99+HL69 smooth, solid lines) and the ones computed adopting the P99 conductive opacity in the whole structure (dashed lines).

adopted conductive opacities in the computation of stellar models, we can list the pioneering study by Hubbard & Lampe (1969) (hereafter HL69), the extensive contributions by Itoh and coworkers (hereafter I93, Itoh et al. 1983; Mitake et al. 1984; Itoh et al. 1983) and the recent computations by Potekhin (hereafter P99, Potekhin 1999; Potekhin et al. 1999; Cassisi et al. 2007).

The conductive opacities provided by Itoh and coworkers are strictly valid only in the complete degenerate regimes ($T_F/T < 0.1$, where T_F is the electronic Fermi temperature), in fact they underestimate the contribution of the electron-electron interactions which is not negligible in the partially degenerate regimes. On the other hand, HL69, which take into account the contribution of electron-electron scattering and is, in principle, appropriate to cover the partially degenerate regime, should not be used in region where $\Gamma > 10$ ($\Gamma = (Ze)^2/k_B T a$ and a is the interionic distance), because of an outdated treatment of the liquid-to-solid phase transition. The Potekhin (1999) is, as far as I know, the only set of computations covering the whole WD structure during the cooling evolution.

Figure 6 shows the comparison between the cooling curves of a 0.6 $M_\odot$ WD computed with different prescriptions for the conductive opacity. The dot-dashed line represents the model computed adopting the Itoh opacity (I93) in the whole WD structure ; the dotted-line the model computed adopting in the whole WD the Potekhin opacity (P99); the dashed line those adopting in the whole WD the Hubbard & Lampe opacity (HL69); and finally the solid line shows the model computed adopting I93 in the fully degenerate regions ($T/T_F < 0.1$) and HL69 in the partially degenerate zones. The cooling time is extremely sensitive to the chosen conductive opacity. In particular, note the huge difference between the dot-dashed line and the solid line, about 16% at low luminosity (Prada Moroni & Straniero 2002). Such a difference is quite surprising, as these two models differ only in the conductive opacity adopted in the partially degenerate regime, roughly corresponding to the very thin external layer, whose mass is less than 1% of the WD mass. In partially degenerate regimes, typical of the core/envelope boundary, the electron-electron scattering processes, usually neglected in denser regions, may significantly reduce the conductivity. The calculation of the conductive opacities, especially in a partially degenerate regime, is crucial for a reliable calibration of the WD cooling time. To this regard, the very recent sets of conductive opacities suitable for the WD interiors by Cassisi et al. (2007) and Itoh et al. (2008) are particularly important.

In order to check the effect of the adopted conductive opacity on the inferred age of GCs, we computed several sets of cooling tracks with masses in the range 0.5-0.9 $M_\odot$ and the related set of isochrones and LFs for different prescription regarding the conductive opacity. I will only show a couple of examples (for a detailed and exhaustive description see Prada Moroni & Straniero 2007). Figure 7 shows the comparison between two sets of models: one computed adopting in the whole WD structure the I93 opacity and the other adopting a combination of I93 in the fully degenerate regions ($T/T_F < 0.1$) and HL69 in the partially degenerate zones. Note that, even though the two sets differ only in the treatment of the conductive efficiency in the partially degenerate regime, the effect is remarkable, causing a shift in the position of the LF peak of about 0.3 mag for 12 Gyr and 0.5 mag for 14 Gyr, which means roughly a difference in age of 1 Gyr and 1.6 Gyr, respectively.

Figure 8 shows a similar prescription where the I93 opacities have been substituted by those of P99. Once again, the effect is quite large: a shift of about 0.4 mag, roughly corresponding to 1.3 Gyr (Prada Moroni & Straniero 2007).

These results demonstrate the extreme sensitivity of the computed cooling times and thus of the theoretical WD LFs on the treatment of conductive opacity in the very thin (M_{env}/M_{WD} <0.01) envelope, where electrons are only partially degenerate. Once again in order to provide firmer predictions on WD cooling times more precise treatments of electron conduction for partially degenerate regimes are needed. At present, the choice of the prescription adopted for the conductive opacity affects the age estimate of GCs at the level of 10%.

4. Conclusions

The vast majority of stars will die quietly as a cooling compact degenerate object, a WD. The evolution of this very common phase is rather simple to compute, as complex algorithms are not needed. However, the current generation of theoretical models is still poorly constrained, as shown by the significative discrepancy between the cooling times of old and cold WDs. Probably the main reason for the quoted disagreement lies in the difficulty of obtaining accurate input physics suitable for the high pressure regimes. I showed that two of the main uncertainty sources in the cooling models are the chemical profiles of the C-O core and the treatment of the conductive opacity, mainly in the partially degenerate zone characterizing the outer layers. The former introduces an uncertainty in the predicted cooling time of a DA CO WD of M= 0.6 $M_\odot$ at low luminosity ($\log L/L_\odot$=-5.5) of about 9%, but which can reach 13-14% if the effect of chemical segregation is taken into account. The latter is even trickier, as the chosen conductive opacity can determine a change in the predicted cooling age a low luminosity of about 16%. In order to get firmer evolutionary times, we need improved inputs physics, in particular a much more precise cross section of the $^{12}C(\alpha, \gamma)$ and more refined conductive opacity, particularly in the partially degenerate regime. These two uncertainties alone affect the age determination of galactic GCs by means of WDs at the level of 5% and 10%, respectively. Probably in the near future this situation will get significantly better, since in the last years, several experimental and theoretical efforts have been focused on the $^{12}C(\alpha, \gamma)$ reaction, providing a progressively more accurate astrophysical factor at the energy of astrophysical interest (see e.g. Hammer et al. 2005).

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