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Evolutionary properties of white dwarf stars

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Abstract. White dwarfs are the most common end-point of stellar evolution. As such, they have potential applications to different fields of astrophysics. In particular, the use of white dwarfs as independent reliable cosmic clocks provides valuable constraints to fundamental parameters of a wide variety of stellar populations. This is possible in part thanks to the relative simplicity of their evolution. Here, I describe a simple cooling model that captures the essentials of the physics of white dwarf evolution. Also, I comment on the results given by full evolutionary calculations of these stars, with a particular emphasis on the different energy sources and processes responsible for chemical abundance changes that occur in the course of their evolution.

Key words. Stars: white dwarfs – Stars: evolution – Stars: interiors

1. Introduction

White dwarf (WD) stars constitute the final stage of stellar evolution for the vast majority of stars. Indeed, more than 95% of all stars, including our Sun, are expected to end their lives as WDs. They are very old objects, therefore the present population of WDs contain valuable information about the evolution of individual stars from birth to death and the star formation rate throughout the history of our Galaxy.

The study of WDs has thus potential applications to different fields of astrophysics (see e.g. Winget & Kepler 2008, for a recent review). These applications include the use of WDs as independent cosmic clocks to infer the age of a wide variety of stellar populations, like the galactic disk (Hernanz et al. 1994) and several globular and open clusters (Richer et al. 1997; Von Hippel & Gilmore 2000; Von Hippel et al. 2006; see also contributions by Bono and Moehler in this volume). In addition, the existence of old and faint WDs belonging to the Halo has been reported (Liebert et al. 1989). A possible detection of a faint population in the Halo allows to provide a robust lower limit to the age of our Galaxy.

WDs usually play a role in some interacting binaries, such as cataclysmic variables (Ritter, this volume), where mass transfer from a companion star onto the WD gives rise to very strong energetic events. In addition, WDs are candidates for progenitors of Supernovae of type Ia, which are thought to be the result of the explosion of a WD exceeding the Chandrasekhar limit due to accretion from a mass–losing companion star. An understanding of the evolutionary and structural properties of WDs may help to improve our knowledge of the supernova events, in particular, the variation in brightness with redshift, with the important underlying implications for cosmol-

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Send offprint requests to: L. G. Althaus ogy. Also, the presence of WDs in binary systems with millisecond pulsar companions provides the opportunity to constrain the age of millisecond pulsars and thereby the timescale for magnetic field decay (van Kerkwijk et al. 2005 for a review).

Finally, the high densities and temperatures that characterize WDs turn these objects into cosmic laboratories to study numerous physical processes under extreme conditions that cannot be achieved in terrestrial laboratories. Thanks to the fact that many WDs exhibit pulsational instabilities, these stars constitute powerful tools for applications beyond stellar astrophysics. In this sense, WDs can be used to constrain fundamental properties of elementary particles, such as axion and neutrinos, and to study problems related to the variation of fundamental constants.

These potential applications of WDs have led to a renewed interest in the computation of full evolutionary models for these stars. Here, I will discuss the essentials of the physics of WD evolution, in order to understand the reasons that make these stars excellent chronometers and potential tools with a wide variety of applications. Observational aspects of WDs are briefly touched upon; the reader is referred to review articles by Koester (2002) and Hansen & Liebert (2003) for further details.

2. Basic properties of white dwarf stars

A remarkable characteristic of WDs is that they are located well below the main sequence on the HR diagram. This means that these objects are characterized by a very small radius, comparable to the size of the earth. By 1910, it was possible to determine the mass of a WD for the first time: the WD companion to Sirius, with a stellar mass of about 1 $M_{\odot}$. Combined with the small radius, this implies very large average densities. With the discovery of other WDs, it was soon realized that these stars were indeed a new class of stars, quite different from ordinary stars. In fact, the existence of WDs provided one of the first test of the quantum theory of matter and a demonstration of the Pauli exclusion principle for electrons. Also, the detection of the gravitational redshift by 1925, resulting from the strong gravity of the WD companion to Sirius, provides an independent test of the high densities in WDs and a confirmation of general relativity.

More than 10,000 spectroscopically identified WDs are known. Spectroscopic inferences, coupled with theoretical stellar atmospheres models, allow to determine their surface gravity, on average log g=8, which considering the radii, yields average masses of $M \approx 0.6 M_{\odot}$. Indeed, typical WD mass distributions show that the mass values of most WDs are around this value (see Kepler et al. 2007), with a tail extending towards high stellar mass — with several WDs with spectroscopically determined masses within the interval 1.0-1.3 $M_{\odot}$ and most probably being the result from either carbon burning in semidegenerate conditions during the evolution of progenitor star or mergers of individual WDs of standard masses. There is also an important population of low–mass WDs, which is probably the result of binary evolution.

Spectroscopic observations also reveal that the atmospheric constituents of most WDs consist almost entirely of hydrogen (H) with at most traces of other elements. These are the DA WDs and they comprise about 80 % of all WDs. Observations also reveal the existence of WDs with helium (He)–rich atmospheres (DB), which make up almost 20% of the total. There are also WDs with hybrid atmospheres, or those with peculiar abundances. It is accepted that post–AGB stars evolve into two main channels: those with H–rich atmospheres, and those with H–deficient composition. The ratio of DA to non-DA changes with effective temperature. In fact, there is evidence that the surface composition of a given WD may change as it evolves, as a result of the interplay of competing processes, such as convection, mass–loss episodes, accretion and gravitational settling. Individual WDs are believed to undergo spectral evolution.

Values of $T_{\rm eff}$ for WDs range from about 150,000 K for the hottest members to 4000 K for the coolest degenerate dwarfs. The stellar luminosity ranges from $10^2 - 10^3$ to about 10^{-5} $L_{\odot}$ for the faintest observed WDs. The majority of known WDs have temperatures higher than the Sun, and hence the "white" in their name. Because the intrinsic faintness of most WDs, quantitative studies of these stars, traditionally based on photometric and spectroscopic observations, are restricted to nearby objects. Other observational techniques have revealed the presence of WD populations located well beyond our own neighborhood, such as in distant open and globular clusters and in the galactic halo.

3. Simple treatment of white dwarf evolution

As mentioned above, many applications involving WDs require a knowledge of the evolutionary properties of WDs. A key feature to understand the evolution of WDs is that, in degenerate configurations, the mechanical and thermal structures are more or less decoupled from each other and can be treated separately. Thermal properties ($T \neq 0$) are responsible for radiation and evolution of WDs. For the mechanical structure, the limit $T = 0$ is usually assumed. This property will allow us to derive a simple picture of how WD evolve.

3.1. Zero-temperature approximation: Chandrasekhar's theory

In this approximation, the structure of a WD can be described in terms of a zero temperature degenerate electron gas. At zero temperature, the structure of a spherically symmetric star is determined by the equations of hydrostatic equilibrium and mass conservation. Chandrasekhar (1939) showed that the problem can be reformulated into a second order differential equation for a dimensionless function.

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\Phi}{d\eta}\right) = -\left(\Phi^2 - \frac{1}{y_o^2}\right)^{3/2} \qquad (1)$$

This equation is solved numerically with appropriate boundary conditions. For a fixed value of the molecular weight per electron, μ_e, a one-parameter family of models is thus obtained, defining implicitly a relation between mass and radius, such that the more massive the star, the smaller is its size. In the limit $\rho \rightarrow \infty$, the relativistic "softening" of the equation of state causes the radius R to become zero, while the mass approaches a limiting value $5.826/\mu_e^2[M_\odot]$, the Chandrasekhar limiting mass. According to our present understanding of stellar evolution, a carbon and oxygen core composition is expected for most WDs. In this case, the limiting mass becomes 1.4 $M_\odot$. For larger masses, gravitational forces overwhelm the electron pressure support, and no stable WD can exist. The existence of a limiting mass has strong implications, not only for the WD themselves, but also for stellar evolution in general. This is particularly true regarding the occurrence of mass-loss episodes: stars must lose a large fraction of their mass at some point in their evolution in order to end up as WDs with masses smaller than 1.4 $M_\odot$.

3.2. Mestel's model of white dwarf evolution

WD evolution can be understood in terms of two main problems:

1) The total energy content E of the star.

2) The rate at which this energy is radiated away.

The first problem involves a detailed knowledge of the thermodynamics of the degenerate core of the WD, which contains more than 99% of the mass. Because the efficiency of degenerate electron in transporting energy, the core is essentially isothermic, a fact that strongly simplifies the evaluation of E.

The second problem involves the solution of the energy transfer through the non-degenerate envelope above the core. In the envelope, energy is transferred by radiation and/or convection, which are less efficient than electron conduction. Because of this, the envelope controls the rate at which energy is transferred from the core into the empty space.

A simple approach to determine the structure of the envelope can be derived by making the following assumptions: 1) An ideal equation of state, i. e., a non-degenerate gas, 2) an abrupt transition between the envelope and the degenerate core and 3) energy is transported only by radiation, for which a Kramers' law is adopted for the radiative opacity ($\kappa = \kappa_0\, \varrho\, T^{-3.5}$, where κ_0 depends on the envelope's chemical composition). Under these assumptions, the thermal structure of the envelope is specified by

$$\frac{dP}{dT} = \frac{16\pi ac}{3\kappa_0} \frac{GMT^{6.5}}{\varrho L}, \tag{2}$$

where no energy sources or sinks are assumed in the envelope (L is the surface luminosity). By using the equation of state for a non-degenerate gas ($\varrho = P\mu/\Re T$), we arrived at a differential equation which can be integrated from the surface (zero temperature and pressure) down to the base of the envelope — the transition layer that separates the envelope from the degenerate core. The integration yields

$$\varrho_{\rm tr} = \left(\frac{32\pi ac}{8.5\ 3\kappa_0} \frac{\mu}{\Re} \frac{GM}{L} \right)^{1/2} T_{\rm tr}^{3.25}. \tag{3}$$

Here, T_{tr} and ϱ_{tr} are the temperature and density at the transition layer. Another relation between density and temperature at the transition layer is given by the fact that at this layer the degenerate electron pressure is equal to the ideal gas pressure (pressure is continuous across the transition layer). Finally, because the core is isothermal, we can replace T_{tr} by T_c (central temperature). We reached a simple power law relation between the surface luminosity, stellar mass and central temperature of the WD (see Shapiro & Teukolsky 1983 for details):

$$L = 5.7 \times 10^5\ \frac{\mu}{\mu_e^2}\ \frac{1}{Z(1+X)}\ \frac{M}{M_\odot}\ T_{\rm c}^{3.5}, \tag{4}$$

where Z and X are the envelope metallicity and H abundance by mass respectively. Some important inferences can be drawn from this simple treatment: for typical compositions and $M = M_\odot$ we find that for $L = 10 L_\odot$ and $L = 10^{-4} L_\odot$, the central temperature is of the order of $T_c = 100 \times 10^6 K$ and $T_c = 4 \times 10^6 K$, respectively. The high central temperatures expected in high-luminosity WDs exclude the presence of H or He in the degenerate core. Indeed, at the high densities of the core, stable core nuclear burning is not possible — the WD would be destroyed by a thermal runaway (see Mestel 1952).

If there is no stable nuclear burning, then which energy sources are involved when a normal WD loses energy by radiation ? An important source of energy is gravitational contraction. Although WDs cannot contract appreciably because of electron degeneracy, the residual contraction — resulting from the gradual decreasing ion pressure — is very important. This is because WDs are compact objects, so that a small change of the radius releases an important amount of gravitational energy. If we neglect nuclear reactions and neutrino losses, the total energy results of contributions from ions, electrons and gravitational energy, i. e., $E = E_{\rm ion} + E_{\rm elec} + E_{\rm grav}$. The luminosity is given by the temporal decrease of the total energy E

$$L = -\frac{dE}{dt} = -\frac{d}{dt}(E_{\rm ion} + E_{\rm elec} + E_{\rm grav}). \tag{5}$$

It can be shown from the virial theorem for degenerate configurations that the release of gravitational energy is used to increase the Fermi energy of the electrons (about the density-dependent contribution of the electronic energy, see Koester 1978 and also Prada Moroni, in this volume). Thus, only the changes in the *thermal* contribution to E must be considered. This gives rise to an elementary treatment of WD evolution:

$$L = -\frac{\partial}{\partial T_c}(E^T_{\rm ion} + E^T_{\rm elec})\ \frac{dT_c}{dt}, \tag{6}$$

where we have assumed that the core is isothermal. In terms of the specific heat of ions and electrons, the luminosity equation becomes

$$L = - < C_V > M \frac{dT_c}{dt}, \tag{7}$$

where $< C_V >= C_V^{\rm elec} + C_V^{\rm ion}$ and M is the WD mass. Thus, we have a simple relation between the luminosity and the rate of change of the central temperature. Clearly, the source of luminosity of a WD is essentially the decrease in the thermal energy of ions and electrons. Because of this, WD evolution is described as a cooling process. But at low temperatures, $C_V^{\rm elec} \ll C_V^{\rm ion}$, since strongly degenerate electrons barely contribute to the specific heat. Thus, *the source of luminosity of a cool WD is basically the change of the internal energy stored in the ions*.

By using the relation between the luminosity and central temperature given by the envelope integration ($L = CMT_c^{3.5}$, see Eq. 4), and assuming an ideal ion gas ($C_V^{\rm ion} = 3\Re/2A$, A is the atomic weight), Eq. 7 can be rewritten as $dT_c/dt \propto T_c^{3.5}$. By integrating this differential equation from an initial time t_0 to the present time t and introducing the *cooling time* as $\tau = t - t_0$, i.e., the age from the start of cooling at high effective temperatures, we obtain

$$\tau \approx \frac{10^8}{A}\left(\frac{M/M_\odot}{L/L_\odot}\right)^{5/7} \text{ years}, \tag{8}$$

where it has been assumed that the central temperature at the beginning of WD formation is much larger than the present central temperature of the WD. This is the Mestel's model of WD evolution (Mestel 1952). For $A = 12$ (carbon interior), typical cooling ages for the faintest observed WDs ($L \sim 10^{-4.5} L_\odot$) of 10^{10} yr are derived. The time spent in the WD phase thus turns out to be a major phase in stellar life. WDs are very old objects, and thus are potential candidates to date stellar populations.

Although the Mestel's model is the simplest picture of WD evolve, it captures the essentials of the physics of WD evolution: It shows that the cooling time of a WD is related essentially to its core chemical composition, its mass and its luminosity. The following features emerge: a) cooling times depend on the core chemical composition (A) of the WD: oxygen WDs are expected to cool faster than carbon WDs, because the specific heat per gram of oxygen is smaller than that of carbon — there are more ions in a gram of carbon than in a gram of oxygen. b) Because of their larger thermal content, massive WDs are characterized by long cooling times and c) cooling time increases as luminosity decreases.

3.3. Improvements to the Mestel's model

The increase in cooling times with decreasing luminosity predicted by the Mestel model implies that more and more WDs should be observed at fainter luminosities. This is indeed the case. However, there is an abrupt and steep drop-off in the numbers of WDs at very low luminosities ($L \sim 10^{-4.5} L_\odot$) that cannot be explained by this model. This discrepancy between theory and observation motivated the exploration of physical effects which might be responsible for shortening the cooling times at very low luminosities.

The Mestel model is a reasonable description of the behavior of real WDs only at intermediate luminosities, where the assumptions of ideal gas, isothermal core, and non-degenerate radiative envelope are more or less valid. However, it requires major improvements if we want to use cool WDs as reliable cosmic clocks . The main improvements to be considered are:

Coulomb interactions and Debye cooling: Coulomb interactions modify the specific heat of ions and the cooling times of WDs. The strength of Coulomb interactions — relative to the thermal kinetic energy — is determined by the Coulomb coupling parameter $\Gamma = (Ze)^2/aKT = 2.26 \times 10^5 Z^{5/3} \rho^{1/3}/T$, where a is the inter-ionic separation. For very low Γ values, the coulomb forces are of minor importance (relative to thermal motions) and the ions behave like an ideal non-interacting gas. But, once Γ becomes of order unity, ions begin to undergo short-range correlations, eventually behaving like a liquid. For large enough Γ (~ 200) the ions form a lattice structure, experiencing a first-order phase transition in the process and the release of latent heat, of about $\sim KT$ per ion. This results in a new source of energy, which introduces an extra delay in the WD cooling. In addition, there is an increase in the specific heat capacity in the crystallized region (Dulong-Petit law) due to the extra degree of freedom associated with lattice vibration. But, most importantly, as the WD cools further, fewer modes of the lattice are excited, and the heat capacity drops according to the Debye relation. This reduction starts to manifest itself once the temperature drops below the Debye temperature ($\theta_{\mathrm{D}} = 4 \times 10^3 \rho^{1/2}$), reaching fast cooling ($C_V \propto T^3$) once $\theta_{\mathrm{D}}/T > 15$. As a result, the thermal content of the WD gets smaller and smaller. This fast cooling is expected to take place in cool WDs, i. e. those characterized by very low surface luminosities.

We can estimate the importance of Debye effect on the cooling of WDs as follows: from Eqs. 4 and 7 we can write $CMT_c^{3.5} = - < C_V^{\mathrm{ion}} > MdT_c/dt$. We approximate the specific heat of ions during the Debye regime by $C_V^{\mathrm{ion}} \sim 3.2\pi^4 (T_c/\theta_D)^3 \Re/A$. In a first approximation we get

$$C \int_{t_o}^{t} dt' = -\frac{16\pi^4}{5} \frac{\Re}{A} \frac{1}{\theta_D^3} \int_{T_o}^{T} {T_c}^{-1/2}\, dT_c, \tag{9}$$

where T_o is the initial temperature from which Debye cooling begins ($T_o \sim \theta_D$) and $T < T_o$. The integration yields the cooling time *during the Debye phase*

$$\tau_D \sim \frac{32\pi^4}{5} \left(\frac{T}{\theta_D}\right)^3 \left[\left(\frac{T_o}{T}\right)^{1/2} - 1\right] \frac{TM}{L} \frac{\Re}{A}. \tag{10}$$

Note that the cooling time during the Debye phase is shorter than the classical result of Mestel (last factor), whenever T is less than about $0.1\theta_D$ (if $T = 0.05\theta_D$ then τ_D is a factor of 4 smaller than τ_{Mestel}). It was hoped that this reduction in the cooling times would explain the deficit of observed WDs below $\sim 10^{-4.5} L_{\odot}$. However, *it is difficult to attribute the deficiency of WDs observed at low luminosity to the onset of Debye cooling*. We can see this by computing, for a given WD mass, the luminosity at which Debye cooling becomes relevant from $L = CMT_c^{3.5}$, where T_c is the central temperature at the onset of Debye cooling (of about $0.1\theta_D$, as we have recently shown). We use the Chandrasekhar's model to estime the central density to compute θ_D. Results are shown in Table 1, which lists, for a given stellar mass, the Debye temperature $\theta_D(K)$ and the luminosity for the onset of Debye cooling. Note that, for a typical WD, fast Debye cooling occurs at too much low luminosity, and thus no observable consequences are expected in this case. Debye cooling is relevant at observable luminosities only in very massive WDs, which are much less abundant.

Table 1. Onset of Debye cooling

$M(M_{\odot})$	$\theta_D(K)$	$L(L_{\odot})$
0.6	7.0×10^6	10^{-7}
1.0	2.2×10^7	$10^{-5.1}$
1.2	4.0×10^7	$10^{-4.1}$

From these simple estimates, it can be concluded that the decrease in the number of dim WDs is difficult to explain in terms of a rapid cooling. The lack of WD at very low luminosity suggests instead that the Galaxy is not sufficiently old to contain cooler (and thus older) WDs, i.e. it is a result of the finite age of the Galaxy. Hence, the cooling age of oldest WDs provides an estimate for the age of the galactic disk (Winget et al. 1987).

Convection: At low temperatures, the assumption of purely radiative transfer is not valid: transport of energy in the outer layers of cool WDs can be due to convection, which results from the recombination of the main atmospheric constituents. With decreasing temperature, the base of the convection zone gradually moves deeper into the star, following the region of partial ionization. As long as convection does not reach the degenerate core, the convergence to the radiative temperature gradient causes the central temperature to be independent of the outer boundary conditions, i.e. chemical stratification or treatment of convection. The heat flow depends in this case on the opacity at the edge of the degenerate core, as assumed in the Mestel treatment.

But at low luminosities, convection reaches the degenerate core and the radiative gradient convergence is lost. The thermal profile

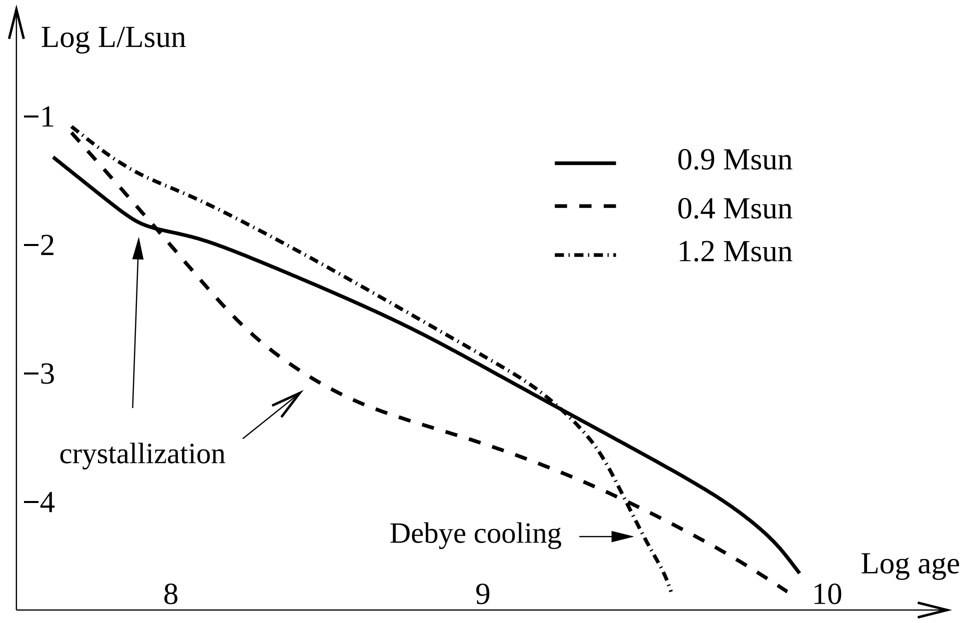

Fig. 1. Surface luminosity as a function of cooling time for carbon-core WDs with different stellar masses. The onset of crystallization and Debye cooling are indicated.

of an *old* WD becomes strongly constrained[1] with the consequence that changes in the atmospheric parameters are reflected in changes in core temperature. This *convective coupling* modifies the $L - T_c$ relationship, and thus the rate of cooling of cool WDs. In particular, the core temperature becomes lower — convection is much more efficient than radiation in transporting energy — as compared with the predictions of purely radiative envelopes. Because of the lower central temperature, the star has initially an excess of thermal energy which must be eliminated, thus causing a delay in the cooling times at low luminosities. The convective coupling occurs at different stages in the WD evolution depending on the outer layer chemical stratification. This will be critical to understand the different cooling behavior of cool WDs with different envelope compositions.

Finally, convection, via mixing episodes, plays a role in the interpretation of the spectral evolution (the observed change in surface composition as WD evolves) of these stars. This is particularly true if the WD is formed with a thin H envelope, and convection penetrates beyond the surface H layers.

Specific heat of electrons: The Mestel model assumes that electrons are completely degenerate and so $C_V^{\rm elec} \ll C_V^{\rm ion}$. This is true for low-luminosity (low T_c) WDs, where the specific heat of strongly degenerate electron is $C_V^{\rm elec} \propto KT/\varepsilon_F \rightarrow 0$. But at high luminosities, and particularly in low-mass WDs, the energy of electrons is not completely independent of temperature; so the specific heat of electrons must be considered in the computation of the thermal energy content of the WD. For instance, for a 0.5 $M_\odot$ carbon-rich WD at $T_c \sim 10^7$ K ($L \sim 10^{-3} L_\odot$), $C_V^{\rm elec} \sim 0.25 C_V^{\rm ion}$.

These improvements alter the cooling times of WDs as given by the simple Mestel's cooling law. The resulting cooling times for

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[1] In fact, the degenerate core is isothermal, and convection — which extends from the atmosphere to the core — is essentially adiabatic.

different stellar masses are depicted in Fig. 1. The main results are:

- The dependence of cooling times on the stellar mass is different from that predicted by the Mestel model ($\tau \propto M^{5/7}$). This is true at high luminosities, where the contribution of electrons to the specific heat increases the cooling times of low-mass WDs — because of their lower degree of electron degeneracy — and at low luminosities where massive WDs cool faster as a result of Debye cooling.
- The impact of crystallization (increase in the ion heat capacity) on the cooling times is strongly dependent on the stellar mass and core composition. In particular, massive WDs crystallize at very high luminosities because of their higher densities.
- Cooling times strongly depend on core chemical composition, as predicted by the Mestel's model ($\tau \propto 1/A$).
- At observable luminosities, Debye cooling is relevant only for $M > 1 M_\odot$.

4. Detailed models of white dwarf evolution

The Mestel's model, with the improvements we have described, provides a reasonable description of WD evolution. However, if we want to use WDs as reliable cosmic clocks to infer the age of stellar populations, we need to consider a much more elaborated treatment that includes all potential energy sources and the way energy is transported within the star. The main points to be considered are:

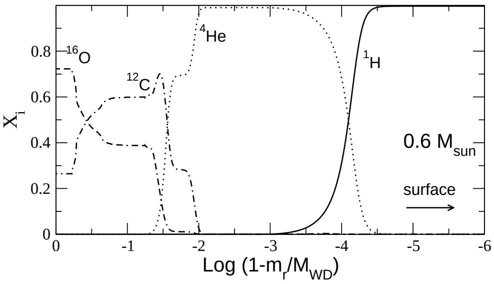

Fig. 2. ^{1}H, ^{4}He, ^{12}C, and ^{16}O distribution within a typical WD in terms of the outer mass fraction $1 - m_r/m_{\rm WD}$.

- The thermal and hydrostatic evolution have to be treated simultaneously. In fact, the structure of a WD changes with time in response to the energy radiated. This means that pressure is a function of density *and temperature*.
- Thermal energy is not the only source of WD luminosity. There are additional energy sources and sinks: residual contraction of the outer layers, nuclear burning, neutrino losses, energy released by possible chemical redistribution in the core, and the release of latent heat on crystallization.
- The core of a WD is never strictly isothermal, particularly in hot WDs.
- Changes in chemical composition during WD evolution, due to convective mixing, diffusion processes, nuclear reactions and accretion are expected to influence the cooling of WDs.

A proper treatment of these issues requires a detailed knowledge of the previous history of the WD. This is particularly true regarding the computation of the early phases of WD evolution at high luminosities, where contraction is not negligible. In addition, the mass and chemical composition of the core and outer layers — which, as we saw, have a marked influence on the WD evolution — are specified by the evolutionary history of the progenitor star. Thus, an accurate treatment of WD evolution requires the complete methods of the stellar evolution theory and calculating the evolutionary history of the progenitor stars all the way from the Main Sequence, to the mass loss and planetary nebulae phases. In what follows, I comment on the main aspects to be considered in a detailed treatment of WD evolution.

4.1. Chemical abundance distribution

The chemical composition of a WD is determined by the nuclear history of the WD progenitor. The chemical composition expected in a typical WD is displayed in Fig. 2 in terms of the outer mass fraction. The use of this coordinate strongly emphasizes the outer layers of the model. Three different regions are distinguished: the carbon-oxygen core (95% of the mass), which is the result of convective He core burning and the following steady He shell burning in prior evolutionary stages. Possible extra-mixing episodes during core He burning and existing uncertainties in the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate turn the carbon/oxygen composition of the core into one of the main sources of uncertainty weighing upon the determination of WD cooling times.

Overlying the carbon/oxygen core is the He– and C– rich intershell, built up during the thermally pulsing phase on the AGB. This intershell reflects the mixing episode during the last thermal pulse. Here, the carbon abundance depends on the occurrence of overshooting and the number of previous thermal pulses. Above on the intershell, there is the He–rich buffer which results from prior H burning. During the AGB, the mass of this buffer goes from almost zero — at the pulse peak when the He-flash convection zone is driven close to the base of the H layer — to $0.01 M_\odot$. The chemical composition expected in a WD depends also on the initial stellar mass of progenitor star: massive WDs are expected to have O/Ne cores and very low mass WDs, He cores.

The theory of stellar evolution provides upper limits for the mass of the various intershells. For a typical WD of 0.6 $M_\odot$, the maximum H envelope mass which survives hot pre-WD stages is about $10^{-4} M_\odot$ and the total mass of the He buffer and the He-intershell amounts to $\sim 0.02 M_\odot$. Asteroseismology inferences on individual pulsating WDs provide, in some cases, constraints to the thickness of the H and He envelopes and the core composition of WDs. In particular, there is evidence for thin H envelopes in some WDs. We conclude that the initial chemical stratification of WDs is not known in sufficient detail from the stellar evolution theory or from observations. This in turn leads to uncertainties in the evaluation of cooling times.

4.2. Changes in the chemical abundance distribution

During the WD stage, there are numerous physical processes which alter the chemical abundance distribution with which a WD is formed. The effects of these changes on the evolutionary properties of WDs may be more or less important, depending on the luminosity they occur. The most important of these processes is element diffusion. In particular, gravitational settling and chemical diffusion strongly influence the abundances produced by prior evolution. Gravitational settling is responsible for the purity of the outer layers of WDs. In fact, because of the extremely large surface gravity of WDs, and in the absence of competing processes (such as weak mass–loss episodes), gravitational settling rapidly leads to the formation of a pure H envelope. At the chemical interfaces, characterized by large chemical gradients, *chemical diffusion* strongly smoothes out the chemical profile. Here, diffusion time scale is comparable to WD evolutionary time scale, so equations describing diffusion have to be solved simultaneously with the equations describing WD evolution.

Under certain circunstances, diffusion processes may trigger the occurrence of thermonuclear flashes in WDs. Indeed, note from Fig. 2 that inside the He buffer, chemical diffusion has led to a tail of H from the top and a tail of C from the bottom. If the WD is formed with a thin enough He-rich buffer, a diffusion-induced H shell flash may be initiated, thus leading to the formation of a self-induced nova, during which the WD increases its luminosity by many orders of magnitude in a very short time (Iben & MacDonald 1986). During this process, the mass of the H envelope is believed to be strongly reduced.

Another physical process which may alter the chemical composition of a WD is convection. In particular, for cool WDs with thin enough ($< 10^{-6} M_\odot$) H envelopes, convective mixing will lead to dilution of H with the underlying He layer, thus leading to the formation of He–rich outer layers.

4.3. Energy sources of white dwarfs

4.3.1. Gravitational energy

Although WDs evolve at almost constant radius, the role of contraction is by no means negligible. The radius at the beginning of the WD phase can be up to twice the zero-temperature fully degenerate radius, and thus the contribution of compression of the outer, non-degenerate layers to the energy output of the star can be important in very hot WDs. In addition, changes in the internal density distribution, owing to the increase in the core mass from CNO H burning lead to an important release of gravitational potential energy from the core of pre-WDs (very young WDs). Gravitational contraction is also relevant in the *final* phases when Debye cooling has already depleted the thermal energy content. Here, the residual contraction of the thin subatmospheric layers may provide up to 30% of the star luminosity. But, as we mentioned in 3.2, for most of WD evolution compression barely contributes to the surface luminosity, because the compression energy released in the core is almost completely absorbed by degenerate electrons to increase their Fermi energy.

4.3.2. Nuclear energy contribution

Stable CNO H shell burning and He shell burning are the main source of luminosity during the evolutionary stages immediately prior to WD formation. As a result of CNO burning, the H–rich envelope is consumed, so that below $\sim 100 L_\odot$ nuclear burning is a minor contribution to surface luminosity — the density and temperature at the base of the H–rich envelope become too low once $M_{\rm H} \sim 10^{-4} M_\odot$. Thus, in a typical WD, the role of nuclear burning as a main energy source ceases as soon as the hot WD cooling branch is reached. But it never stops completely, and depending on the stellar mass and the exact amount of H left by prior evolution, it may become a non-negligible energy source for DA WDs. Detailed evolutionary calculations show that for WDs having *low-metallicity progenitors*, stable proton-proton H burning may contribute by more than 50% to the surface luminosity by the time cooling has proceeded down to $L \sim 10^{-2} - 10^{-3} L_\odot$. It must be noted that a small reduction in the H envelope by a factor of 2 with respect to the upper theoretical limit is sufficient to strongly inhibit H burning, a fact that explains the different role of H burning obtained by different authors. *Hence, a correct evaluation of nuclear burning during the WD stage requires the computation of the pre-WD evolution.*

Finally, stable H burning may be dominant in low-mass WDs, thus delaying their cooling for significant periods of time (Driebe et al. 1998; Althaus et al. 2001). These stars are the result of binary evolution and are characterized by thick H envelopes. In these low-mass WDs, H burning can also become unstable, thus giving rise to thermonuclear flashes at early stages of WD evolution. This bears important consequences for the interpretation of low-mass WD companions to millisecond pulsars.

4.3.3. Additional energy sources

The first-order phase transition that occurs during crystallization results in a new energy source, due to the release of latent heat — of about KT per ion, Lamb & Van Horn (1975). The impact of this energy release on the WD cooling depends on the stellar mass of the WD. Indeed, because crystallization occurs at higher luminosities in more massive WDs, the impact of latent heat in these WDs is less relevant. An additional source of energy that is expected to delay the cooling is the release of potential gravitational energy resulting from changes in the chemical profile upon cyrstallization (Salaris et el. 1997). In fact, a partial separation between carbon and oxygen, as compared with the initial distribution of these elements, is expected to occur after crystallization. The resulting release of potential energy depends on the initial unknown fractions of carbon and oxygen and on their distribution in the interior. The diffusion of minor species in the core may also make a contribution to the WD luminosity. In particular, the slow ^{22}Ne diffusion in the WD core releases enough potential gravitational energy as to impact the cooling times of massive WDs (Deloye & Bildsten 2002 and García–Berro et al. 2008). In fact, ^{22}Ne, which is the byproduct of nuclear burning during prior evolution, has a neutron excess that results in an imbalance between the gravitational and electric fields. This leads to a slow gravitational settling towards the center. The settling rate is quite uncertain because of the poorly known microphysics involved in the calculation of the interdiffusion coefficient of elements at very high densities.

4.3.4. Energy loss by neutrinos

Energy is lost from WDs not only in the form of photons, but also through the emission of neutrinos. Neutrinos are created in the very deep interior of WDs and provide a main energy sink for hot WDs. Because of their extremely small cross–section for interaction with matter — at a central density of $10^6 gr/cm^3$, typical of WDs, the mean free path of neutrinos is $l_\mu \approx 3000 R_\odot$ — once created, neutrinos leave the WD core without interactions, carrying away their energy. In young WDs, neutrinos increase the cooling rate and yield a temperature inversion. Neutrinos result from pure leptonic processes, as a consequence of the electro–weak interaction. Under the conditions prevailing in hot WDs, the plasma-neutrino process is usually dominant. Here, a so-called plasmon, resulting from a collective excitation of the electron gas, decays to a neutrino-antineutrino pair.

4.4. Results from detailed calculations

The time dependence of the luminosity contribution due to H burning ($L_{\rm nuc}$) via proton proton and CNO nuclear reactions, neutrino losses (L_ν), the gravothermal energy ($L_{\rm gravo}$) — the release of thermal plus gravitational potential energy — and photon emission (surface luminosity, $L_{\rm sur}$) for a typical WD is shown in Fig. 3. Evolution is depicted from the planetary nebulae stage to the domain of the very cool WDs. Luminosities are in solar units and the age is counted in years from the moment the progenitor departs from the AGB. At early times, i.e. in young WDs, nuclear burning via CNO mostly contributes to the surface luminosity of the star. Here, there is a near balance between $L_{\rm nuc}$ and $L_{\rm sur}$ and between $L_{\rm gravo}$ and L_ν (like in AGB star). During this stage, $L_{\rm gravo}$ results from the release of potential gravitational energy, owing to the change in the internal density caused by the increase in the core mass from CNO H burning. After $\sim 10^4$ yr of evolution, nuclear reactions abruptly cease and the star begins to descend to the WD domain and the surface luminosity of the star begins to decline steeply. Thereafter, evolution is dictated by neutrino losses and the release of gravothermal energy — now essentially the release of internal energy from the core. Nuclear burning (via CNO) is a minor contribution. Note that during this stage, neutrino losses are the dominant energy sink ($L_{\rm gravo} \approx L_\nu$). In fact, $L_\nu \approx 10 L_{\rm sur}$ and as a result the WD cooling is strongly accelerated. To this stage belong the pulsating DOV and DBV WDs, so an eventual measurement of the rate of period change in some of these stars would allow to constrain the plasmon neutrino productions rates (Winget et al. 2004). The time-scale for CN burning is larger than the evolutionary time-scale. Burning occurs in the tail of H distribution and, as a result, the H envelope is not reduced by burning during this stage.

As soon as the structure approaches a cooling age of 10^7 yr, the WD is cold enough that neutrino and CN burning are no longer relevant. $L_{\rm gravo} \approx L_{\rm sur}$ and the WD resembles the Mestel idea. This is the best understood stage in the WD life: no convection, neutrinos, nuclear burning, crystallization, and ion thermal energy are the main source of energy. At more advanced stages, proton-proton burning shell makes a significant contribution to surface luminosity. As a result, the cooling rate of the WD is reduced. Eventually, H burning becomes virtually extinct at the lowest luminosities, and surface luminosity is given by the release of internal energy from the core. During these stages, the contraction of the very outer layers may contribute to surface luminosity. Finally, we note that the mass of the H envelope decreases with increasing metallicity Z of progenitor star. Thus, the role of proton-proton burning at advanced stages will be much less relevant in WDs with solar metallicity progenitors. For the case of low metallicity progenitors we have analysed, residual proton-proton burning provides an appreciable contribution to luminosity, unless the H envelope is reduced below the maximum value predicted by stellar

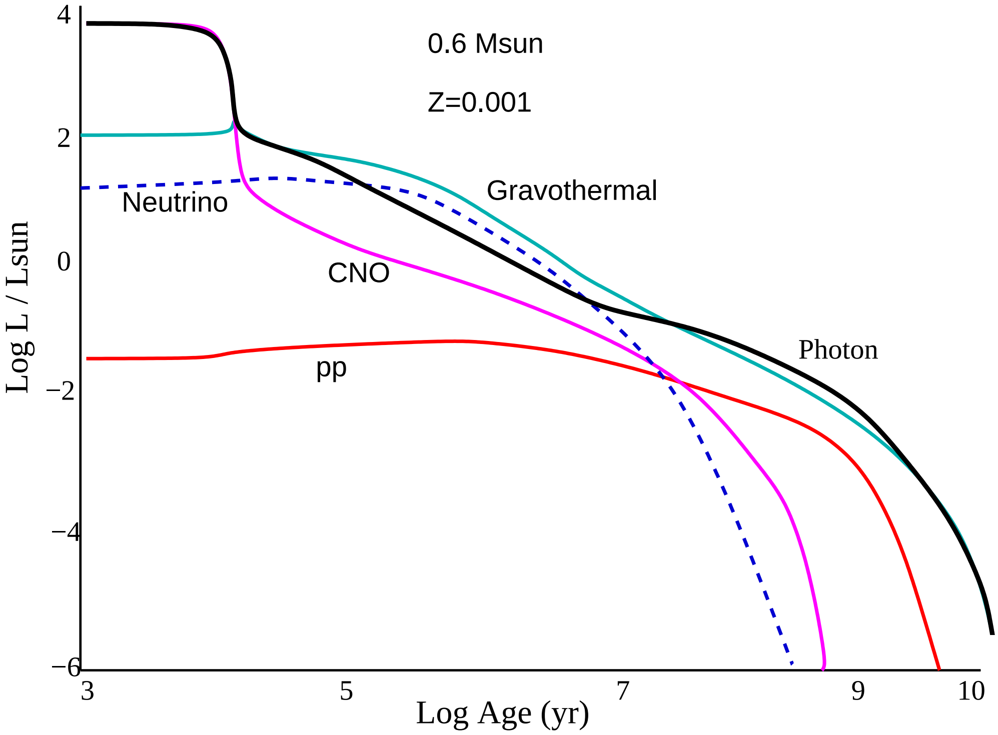

Fig. 3. Temporal evolution of different luminosity contribution for a typical WD from the hot pre–WD to the very advanced stages. The metallicity of the progenitor star is $Z = 0.001$

evolution. In this context, we note that the mass of the H envelope may be orders of magnitude smaller if the WD progenitor experiences a late thermal pulse on the WD cooling track (Althaus et al. 2005).

5. Conclusions

There are several reasons that turn WDs into cosmic clocks and laboratories to study different kind of problems. Indeed, they constitute the final evolutionary stage for vast majority of stars. They are a homogeneous class of stars with a narrow mass distribution and well-known structure, with slow rotation rates and low magnetic fields. And most importantly, their evolution, as we have seen, is relatively simple and can be described in terms of a cooling problem. However, there are still some open questions regarding both the evolutionary history of WD progenitors and the physics of WD cooling, which should be addressed in order to use WDs as reliable tools.

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