Mem. S.A.It. Vol. 81, 281

Memorie della

Mass of black holes:

The state of the art

B. Czerny[1] and M. Nikołajuk[2]

[1] Copernicus Astronomical Center – Bartycka 18, 00-716 Warsaw, Poland
e-mail: bcz@camk.edu.pl
[2] University of Bialystok, Faculty of Physics – Lipowa 41, 15-424 Białystok, Poland
e-mail: mrk@alpha.uwb.edu.pl

Abstract. In this small review we present the actual state the knowledge about weighting black holes. Black holes can be found in stellar binary systems in our Galaxy and in other nearby galaxies, in globular clusters, which we can see in our and nearby galaxies, and in centres of all well-developed galaxies. Range of values of their masses is wide and cover about ten orders of magnitude (not taking into account the hypothetic primordial black holes). Establishing the presence of black holes, and in particular the measurement of their mass is one on the key issues for many branches of astronomy, from stellar evolution to cosmology.

Key words. Stars: fundamental parameters – galaxies: fundamental parameters – galaxies: nuclei – X-rays: binaries – X-ray: galaxies

1. Introduction

Astrophysical black holes (BH) are customarily divided into three classes: stellar mass black holes, with masses of a few up to twenty-thirty solar masses, intermediate mass black holes (IMBH) with masses of a few hundreds to a few thousands of solar masses, and massive (or super-massive) black holes residing in galactic dynamical centres, with masses from 10^5 solar masses up.

Determination of values of black hole masses is important for the three main reasons: (i) to distinguish small mass black holes from neutron stars (ii) to analyse the luminosity states of accreting black holes through the luminosity to the Eddington luminosity ratio (ii) to study the growth of central black holes in galaxies. All three aspects are important, the last one is a key issue in cosmology, in studies of the galaxy evolution and AGN feedback.

The methods used for mass determination can be broadly divided into the following three classes:

– dynamical methods
– spectra fitting methods
– scaling methods

although such a division is not necessarily unique. For example, the reverberation method is rather an example of a dynamical method while the Broad Line Region (BLR) radius vs. luminosity relation combined with a line width fitting is somewhere in between the di-

Send offprint requests to: B. Czerny

rect dynamical method and a simple scaling law. In this review we will include it as a scaling method.

There are several recent reviews covering the topic of black hole mass determination (e.g. Casares 2007; Ziółkowski 2008; Wandel 2008; Vestergaard 2009). Here we will not go into the details of the various methods and its caveats but instead we will provide the collection of laws with short description of applicability. The possibilities are broad so black hole mass estimate is possible *nearly* for every object. Different methods use observational data from various energy bands from radio through NIR and optical to X-ray domain. Therefore, the black hole mass measurement is one of the nice examples of the multifrequency approach.

2. Black hole mass determination methods

2.1. Dynamical methods

Dynamical methods are mostly based on the Keplerian motion of a *test particle* around the black hole. The basis idea is simple: if for a test particle we can measure the velocity v at the circular orbit and the radius R of the orbit we obtain the value of the black hole mass:

$$M = \frac{v^2 R}{G}. \tag{1}$$

The application is complicated by several factors. First, we have to find a suitable *test particle*. Next, there is a problem of the inclination of the orbit. Even for a circular orbit the measurement of one of the velocity components does not allow for a unique determination of the orbital speed.

We have a choice of the following *test particles* and measurements:

- Companion star (in binary systems) - mass function
- Companion black hole (e.g. OJ 287) - orbital period
- Nearby star (e.g. S02 star in Sgr A*) - apparent motion, radial velocity
- Nearby stars - stellar rotational velocity, stellar velocity dispersion and/or stellar luminosity profile
- Broad Line Region clouds - line width and BLR radius from reverberation
- H2O masers - velocity and radius from mapping

Of those methods the last one - water maser method - is the most accurate since we observe water maser emission in an AGN only when observing almost in the orbital plane of the emission. Additionally, since the emission of several blobs located at various distances from the black hole is measured we have a test whether the motion is indeed Keplerian. The emission comes from the geometrically thin disk, from distances of order of less than a fraction of a parsec from the central black hole. Such measurement was done for NGC 4258 (Miyoshi et al. 1995; Herrnstein et al. 2005), and the most recent measurements give the black hole mass value of $(3.82 \pm 0.01) \times 10^7 M_\odot$ water maser (see Siopis et al. 2009, and the references therein). Similar measurements were done for a few other sources, but the water maser emission is rare (detected in about 10 percent of the nearby Seyfert 1.8 - Seyfert 2 galaxies, Kondratko et al. 2006), as it requires special inclination and relatively low absorption as for a Seyfert 2 galaxy (highly inclined AGN are extreme type 2 AGN), the data quality is usually not good enough to allow for mass measurement (emission is unresolved), and in some cases like in NGC 1068 the implied motion is clearly non-Keplerian so the mass measurement cannot be performed reliably. The method does not apply to Seyfert 1 galaxies or lower mass black holes, and the majority of Seyfert 2 galaxies also do not show maser action so the method has very limited applicability.

In case of stellar mass black holes, since they are generally in binary systems we have a secondary playing the role of the *test particle*. The companion star, however, is itself massive so the measurement of the companion semi-amplitude of the radial velocities, K_c, and the period, P, of the orbital motion results only in determination of the mass function

$$f(M) = \frac{(M \sin i_c)^3}{(M + M_c)^2} = \frac{K_c^3 P}{2\pi \mathrm{G}}, \tag{2}$$

which gives only the lower limit for the black hole mass M. The further determination of the black hole mass requires both an estimate of the inclination of the orbit, i_c, as well as the estimate of the mass of the companion star, M_c. Best constraints are achieved if multiwavelength measurements are used (e.g. to confirm the binary period) and evolutionary constraints applied. New determinations of the black holes were recently performed for example by Orosz et al. (2009) (LMC X-1, $M = 10.91 \pm 1.41 M_\odot$).

Overall, black hole masses in binary systems of the Milky Way were determined for 24 objects (confirmed BH). The accurate values (error less then 10%) were obtained for 7 objects (Casares 2007; Ziółkowski 2008).

Similar analysis of the binary motion was also used in the case of two massive black holes. In OJ 287 the observed periodic outbursts every 12 years are interpreted as a result of a smaller massive black hole ($\sim 1.3 \times 10^8 M_\odot$ crossing the accretion disk of a more massive ($\sim 1.8 \times 10^{10} M_\odot$) black hole (see Valtonen et al. 2009 and the references therein). However, this object is exceptional, with the optical data covering more than 100 years. The search for other massive binary black hole continues but the list of candidates is short (see e.g. Xu & Komossa 2009) and the available data do not allow for black hole mass measurement at the basis of binarity itself.

Single star as a *test particle* can also be used in the case of the massive black hole mass measurement for Sgr A*. In this case the star is indeed a test particle, but stellar orbits are elliptical and inclined. The peculiar motion of these young bright stars can be followed using the active optics in the IR band. One of the stars (S02) already completed the whole orbit and the most recent determination of the Sgr A* mass based on that result is $(4.1 \pm 0.6) \times 10^6 M_\odot$ (Ghez et al. 2008). Similar, and even more accurate value is obtained if all 28 monitored stars are used ($(4.31 \pm 0.06) \times 10^6 M_\odot$, Gillessen et al. 2009). The provided uncertainty of the measurement is additionally enhanced to $0.36 \times 10^6 M_\odot$ due to systematic error in the estimate of the distance to the Galactic centre.

The method based on a single star cannot be used now for other, even nearby galaxies or for centres of the globular clusters (due to the crowded fields).

Thus the next approach is to use the measurement of the rotational velocity or/and velocity dispersion of the stellar distribution to infer the presence of the central black hole. The method is not a simple one since it requires modelling of the stellar motion on the top of the measurement of the stellar velocity from the width of absorption lines. If the measured stars are not close enough so that the gravitational potential contains the contribution from the stars themselves the problem is particularly difficult.

The black hole mass measurement for nearby non-active black holes based on stellar dispersion was performed for several objects observed by HST or ground-based telescopes, preferably with active optics (Magorrian et al. 1998; Gebhardt et al. 2000; Ferrarese & Merritt 2000; Krajnović et al. 2009). As was shown for example by Gültekin et al. (2009a) for NGC 3607, the fit with a central black hole of mass of $(1.2 \pm 0.4) \times 10^8 M_\odot$ is better by $\chi^2 = 10.6$ than the fit without a central black hole. Application of the method to globular clusters in our Galaxy is even more difficult since the size of the region dominated by the central black hole has smaller angular size. Minor changes about the assumption of the stellar dynamics change the result drastically. For example, the fit to the globular cluster ω Centauri gives no black hole for core assumption for the stellar distribution but for the cusp profile assumption it gives the black hole mass of $(8.7 \pm 2.9) \times 10^3 M_\odot$ (van der Marel & Anderson 2009). Another example of the potential problems was shown by Fregeau et al. (2009): the inclusion of natal kicks of white dwarfs in globular cluster NGC 6397 removes the need for the IMBH at the cluster centre.

The determination of the black hole mass directly from the stellar dispersion velocity profile should not be confused with a global measurement of bulge velocity dispersion and black hole mass determination based on the scaling between the two since the motion of the stars in bulge is not determined dynamically by the black hole mass; this method will be described in Sect. 2.3.

Dynamical methods can be extended to distant objects for active galactic nuclei since in this case the gas of the Broad Line Region (BLR) can play the role of the *test particle*. The BLR is never resolved but the emission line profile provides the information on the velocity and the *orbital radius* can be measured from the time delays between the variations of the continuum and the response of the line, i.e. from reverberation (see e.g. Peterson & Horne 2004 for the general description).

The line profile is converted to the velocity either through measurement of the full-width-at-half maximum (FWHM), or through line dispersion (second moment of the line profile), v_σ. As argued by Denney et al. (2009), the first method is better for low signal to noise spectra while the second method is more accurate for higher signal to noise spectra since it is then less sensitive to details of the subtraction of the narrow component of the line profile.

The black hole mass is then determined from the relation:

$$M_{\rm BH} = f_1 \frac{R v_{\rm FWHM}^2}{\rm G} \quad (3)$$

or from the relation:

$$M_{\rm BH} = f_2 \frac{R v_\sigma^2}{\rm G}. \quad (4)$$

The problem of the method is the geometry of the BLR. If we assume the spherical geometry, as introduced by Netzer (1990), the coefficient $f = \frac{3}{4}$ in the formula above is determined uniquely. This approach was used by Wandel, Peterson, & Malkan (1999); Kaspi et al. (2000).

However, the BLR is likely to be flattened (see e.g. McLure & Dunlop 2001) so the coefficient was later determined at the basis of the comparison between two mass determination methods (reverberation and stellar dispersion, Onken et al. 2004) and this coefficient was adopted in Eqs. (3) and (4). It is by a factor ~ 1.8 higher than in spherically symmetric case. In addition, Collin et al. (2006) reported that the value of the coefficient f is dependent on type of AGN (for example f_1 in Eq. (3) for Seyfert 1 galaxies with broad emission lines is less than in case of Narrow Line Seyfert 1 (NLS1) galaxies).

Reverberation is time consuming but nevertheless it was performed for over 40 objects (see Peterson et al. 2004; Bentz et al. 2009a; Bentz et al. 2009c, and the references therein). It includes several Seyfert 1 galaxies, about 15 quasars and 7 NLS1 galaxies.

The problem in this method is the potential dependence of the scaling factor on the inclination angle. The issue was discussed in a number of papers (e.g. Collin et al. 2006; Nikołajuk et al. 2006), and the systematic error even by a factor of 3 can arise due to this effect in some cases (see discussion by Krolik (2001)).

Additionally, Marconi et al. (2009) suggested that the effect of radiation pressure should be included in determination of the black hole mass based on dynamical methods (i.e. virial theorem). However, Onken (2009) argued against the importance of this effect.

2.2. Spectra fitting methods

The spectra fitting methods are based on the assumption that we have a reliable model of the emission coming from the vicinity of a black hole. Such a model usually depends on the black hole mass, but also on some other parameters, mostly accretion rate, but also the spin of the black hole and the viewing angle of the nucleus. Therefore, using those methods is rather difficult and the results should be taken with much care.

In high accretion rate sources, like Galactic black holes in their soft states or Narrow Line Seyfert 1 galaxies and quasars, the emission is mostly dominated by the emission of an accretion disk seen in X-rays (GBH) or optical/UV (AGN). In exceptional AGN the peak emission is also seen in X-rays (RE J1034+396, Soria & Puchnarewicz 2002).

If we have a single measurement of the monochromatic flux at ν_0 (in the power-law region of the big blue bump) and the position of the maximum of the disk component spectrum on νF_ν diagram, $\nu_{\rm max}$, then assuming a non-rotating black hole, disk inclined at 60 deg and extending to the innermost stable circular orbit (ISCO) and a local black body without any correction connected with the colour temperature we can have a black hole mass estimate:

$$\log M_{\rm BH} = \frac{1}{2}\log L_{\nu_0} - \frac{1}{6}\log \nu_0 + -\frac{4}{3}\log \nu_{\rm max} + 16.515 \quad (5)$$

(Tripp et al. 1994). This can be applied to bright quasars if only broad band low quality spectrum is available. In Seyfert galaxies the spectral slopes are much softer than expected from the Shakura-Sunyaev disk emission (i.e. the effects of X-ray irradiation, extinction and outflow are important) so the formula is inadequate. For higher quality broad band spectra more advanced models can be fitted (see e.g. Davis, Woo, & Blaes 2007 based on Hubeny et al. 2000 models; irradiation effect in Loska, Czerny, & Szczerba 2004 and Cackett et al. 2007; Czerny & Janiuk 2007).

For galactic sources, more advanced disk spectra model are used (e.g. Sadowski et al. 2009) and with the knowledge of the black hole mass from dynamical method (binary motion) they are used instead to infer the black hole spin (e.g. McClintock et al. 2006).

However, in order to rely on such mass determination we must be sure that the measured emission does come from a fairly standard accretion disk. In some sources, like ULX, this is highly uncertain. For example, thermal disk + power law fits to X-ray spectra imply IMBH (e.g. Miller, Fabian, & Miller 2004) while other decomposition of the spectra implies the ultraluminous high states of 100 $M_\odot$ BH (e.g. Done & Kubota 2006; Roberts 2007). Recent study of the sample of 94 Chandra sources (Berghea et al. 2008) shows that the cool disk spectral component is unrelated to luminosity which implies that indeed we do not understand the spectra! Arguments against low masses in ULX sources (i.e. against strong beaming) are rather indirect (see the discussion of the sources ULX-1 ESO 243-49 by (Farell et al. 2009) for a recent example of such analysis). Therefore, conclusions on the presence or absence of IMBH based on spectral analysis cannot be done.

Accreting black holes also show the presence of the X-ray emission and the reflection component coming from X-ray reprocessing by an accretion disk. The iron line formed in this process can be also used for measurement of the black hole parameters. The line profile is mostly sensitive to the black hole spin but if X-ray time variability is included then iron line reverberation also should allow (in principle, when next generation large area X-ray telescope will operate) for black hole mass measurement (Dovčiak et al. 2004).

2.3. Scaling methods

Scaling methods are invaluable for black hole mass determinations in large AGN surveys as well as when a simple black hole mass estimate is needed for a specific object. There are numerous methods based on various observables. The accuracy of the methods is difficult to assess for a single object so two or more methods should be used whenever possible. These methods are based on scaling of measured properties with the mass found earlier by independent mass and the discuss quantity measurement.

Most broadly used nowadays is the method of the black hole mass measurement from a single spectrum in IR, optical or UV band. The method originally comes from reverberation approach but is based on empirical connection established between the BLR radius and the source monochromatic luminosity (see e.g. Wandel, Peterson, & Malkan 1999; Kaspi et al. 2000). If the starlight contamination is likely to be unimportant, or it is accounted for, the best relation to use is

$$\log \frac{R_{\rm BLR}}{1 \text{ lt day}} = -21.3 + +(0.519^{+0.063}_{-0.066})\log(\lambda L_\lambda(5100\text{Å})) \quad (6)$$

(based on HST spectra, Bentz et al. 2009a) and consequently the formula for the black hole mass determination based on Hβ line reads:

$$\log M_{\rm BH} = \log\left[\left(\frac{\rm FWHM(H\beta)}{1000\,{\rm km s^{-1}}}\right)^2 \times\right.$$

$$\times\left(\frac{\lambda L_\lambda(5100\text{Å})}{10^{44}\text{ergs}^{-1}}\right)^{0.519}\Bigg] + \log f_1 + 6.81\,, \qquad (7)$$

where f_1 is the factor from Eq. (3) and $M_{\rm BH}$ is in units of $M_\odot$. If the starlight contamination is likely, the older version of the phenomenological fit may be more accurate in practice. Kaspi et al. (2005) give the relation

$$\frac{R_{\rm BLR}}{10\text{ lt days}} = 2.39\left(\frac{\lambda L_\lambda(5100\text{Å})}{10^{44}\text{ergs}^{-1}}\right)^{0.67\pm0.05} \qquad (8)$$

and from this it follows that

$$\log M_{\rm BH} = \log\Bigg[\left(\frac{\text{FWHM(H}\beta)}{1000\,\text{kms}^{-1}}\right)^2 \times$$

$$\times\left(\frac{\lambda L_\lambda(5100\text{Å})}{10^{44}\text{ergs}^{-1}}\right)^{0.67}\Bigg] + \log f_1 + 6.66\,. \qquad (9)$$

In their AGN analysis Vestergaard & Peterson (2006) use two formulae:

$$\log M_{\rm BH} = \log\Bigg[\left(\frac{\text{FWHM(H}\beta)}{1000\,\text{kms}^{-1}}\right)^2 \times$$

$$\times\left(\frac{\lambda L_\lambda(5100\text{Å})}{10^{44}\text{ergs}^{-1}}\right)^{0.5}\Bigg] + (6.91 \pm 0.02)\,, \qquad (10)$$

or

$$\log M_{\rm BH} = \log\Bigg[\left(\frac{\text{FWHM(H}\beta)}{1000\,\text{kms}^{-1}}\right)^2 \times$$

$$\times\left(\frac{L(\text{H}\beta)}{10^{44}\text{ergs}^{-1}}\right)^{0.5}\Bigg] + (6.67 \pm 0.03)\,. \qquad (11)$$

If $H\beta$ is not available, other lines can be used. For example, the recent formula based on the Mg II line reads:

$$\log M_{\rm BH} = \log\Bigg[\left(\frac{\text{FWHM(Mg II)}}{1000\,\text{kms}^{-1}}\right)^2 \times$$

$$\times\left(\frac{\lambda L_\lambda}{10^{44}\text{ergs}^{-1}}\right)^{0.5}\Bigg] + zp(\lambda)\,, \qquad (12)$$

where $zp(\lambda)$ = 6.72, 6.79, 6.86 and 6.96 for λ1350Å, λ2100Å, λ3000Å, and λ5100Å, respectively (Vestergaard & Osmer 2009). There is also a version of this formula based on the line flux instead of the continuum monochromatic flux (see e.g. Kong et al. 2006).

If even Mg II is outside the measured spectrum, there is a possibility to use C IV line (Vestergaard & Peterson 2006):

$$\log M_{\rm BH} = \log\Bigg[\left(\frac{\text{FWHM(C IV)}}{1000\,\text{kms}^{-1}}\right)^2 \times$$

$$\times\left(\frac{\lambda L_\lambda(1350\text{Å})}{10^{44}\text{ergs}^{-1}}\right)^{0.53}\Bigg] + (6.66 \pm 0.01)\,, \qquad (13)$$

This line is perhaps less reliable as belonging to High Ionisation Lines (HIL) and being more likely influenced by the radiation pressure that Low Ionisation Lines (LIL) (like Hβ and Mg II).

Those methods work for type 1 AGN, i.e. Seyfert 1 galaxies and quasars, in Seyfert 2 galaxies we do not see the BLR region. However, in those sources we can use scaling based on the bulge properties which generally seem to apply for all AGNs well as non-active galaxies.

The best relation, with very low dispersion, was found between the stellar dispersion in the bulge and the black hole mass (Gebhardt et al. 2000; Ferrarese & Merritt 2000). Gültekin et al. (2009b) based on observations of over 49 observations of galaxies give formula:

$$\log M_{\rm BH} = 8.12 + \log\left(\frac{\sigma}{200\,\text{kms}^{-1}}\right)^{4.24\pm0.41} \qquad (14)$$

which implies the surprising relation between the black hole mass and the bulge mass (Kormendy & Richstone 1995; Magorrian et al. 1998). If the stellar dispersion in the bulge is not measured we can use first the relation between the bulge luminosity and the bulge mass

$$\log\left(\frac{M_{\rm bulge}}{M_\odot}\right) = -1.11 + 1.18\log\left(\frac{L_{\rm bulge}}{L_\odot}\right) \qquad (15)$$

Then we can combine it with the black hole mass vs. bulge mass relation in order to receive $M_{\rm BH}$ (Häring & Rix 2004):

$$M_{\rm BH} = 1.58\times10^8\left(\frac{M_{\rm bulge}}{10^{11}M_\odot}\right)^{1.12\pm0.06} M_\odot\,. \qquad (16)$$

If the bulge luminosity is measured in B band then we have to recalculate it to V band through empirical relation $B - V = 0.8$ (e.g. Bian & Zhao 2003) or directly use the formula proposed by Kormendy & Gebhardt (2001)

$$M_{\rm BH} = 0.78 \times 10^8 \left(\frac{L_{\rm B,bulge}}{10^{10} L_{\rm B,\odot}}\right)^{1.08} M_\odot\ . \tag{17}$$

For AGN the most recent scaling relation found by Bentz et al. (2009b) is:

$$\log M_{\rm BH} = 7.98 + \log\left(\frac{L_{\rm V,bulge}}{10^{10} L_\odot}\right)^{0.80\pm0.09} \tag{18}$$

For highly inclined radio galaxies the following relation between host galaxy absolute magnitude at R-band, $M_{\rm R}$ and $M_{\rm BH}$ can be applied

$$\log M_{\rm BH} = -0.5 M_{\rm R} - 2.74, \tag{19}$$

(McLure & Dunlop 2002) where $M_{\rm R}$ is the absolute R-Cousins bulge luminosity.

For BL Lac objects the similar relation, but with a different shift, was proposed by Bettoni et al. (2003):

$$\log M_{\rm BH} = -0.50 M_{\rm R} - 3.00\ . \tag{20}$$

More complex method of deriving the black hole mass from the Two-Micron All-Sky Survey K-band bulge luminosity was presented by Vasudevan et al. (2009).

Finally, 278 radio-loud AGNs (including 146 BL Lac sources) were observed by Zhou & Cao (2009). A significant correlation was found between the Lorentz factor of the jet, $\gamma_{\rm min}$, and black hole mass which opens a possibility to estimate the last quantity:

$$\log M_{\rm BH} = 3.26 \log \gamma_{\rm min} + 5.81\ . \tag{21}$$

In case of AGN we have also a possibility to use the profile of optical [O III] line (from NLR) as a proxy for the stellar dispersion velocity as advocated by several authors and use the Eq. (14). The accuracy is improved with the use of a relation

$$\sigma_* = {\rm FWHM}([{\rm O\,III}])/2.35 \tag{22}$$

(Gaskell 2009). This possibility is particularly interesting for radio galaxies where obscuration of the central region is considerable due to the host galaxy when the source is seen in the host galaxy plane, as it is in CSO sources. When mid-IR spectrum is available, other NLR lines can be used, like [Ne V], [O IV] (Dasyra et al. 2008).

Finally, also X-ray data provide us with the black hole mass measurement possibility. One of the advantages of those methods is the expected independence on the inclination effects since the X-ray emission is rather isotropic.

X-ray lightcurves of accreting black holes are dominated by aperiodic red noise and power spectral densities (PSDs) are well represented by several Lorentzians. Such analysis is mainly done using high quality X-ray data for X-ray Binaries in our Galaxy (see e.g. Nowak 2000; Pottschmidt et al. 2003, for Cyg X-1, GX 339-4). In case of low quality X-ray data, generally in AGNs and ULXs, PSD of those sources are represented by a power law with one or two breaks (Uttley, McHardy & Papadakis 2002; Heil, Vaughan & Roberts 2009). However, recently McHardy et al. (2007) reported finding 2 Lorentzian components in PSD of nearby NLS1 galaxy Ark 564.

The overall shape of the power spectrum depends on the luminosity state (the effect well studied in galactic black holes) and shifts linearly with the black hole mass. High frequency break was suggested to scale with mass in AGN (Papadakis 2004) as

$$M_{\rm BH} = 10^7 \frac{1.7 \times 10^{-6}}{f_{\rm hfb}} M_\odot, \tag{23}$$

where $f_{\rm hfb}$ is the high frequency break in Hz. McHardy et al. (2006) suggest a formula which includes also the luminosity term:

$$\log f_{\rm hfb}[{\rm day}] = -2.17 \log\left(\frac{M_{\rm BH}}{10^6 M_\odot}\right) + \\ +0.90 \log\left(\frac{L_{\rm bol}}{10^{44} {\rm erg s^{-1}}}\right) + 2.42\ . \tag{24}$$

However, the form and strength of the luminosity dependence is an open issue and likely depends of the AGN type (Seyfert 1 vs. NLS1 vs. LLAGN). In galactic sources the low frequency part of the X-ray power spectrum clearly shows a dependence on the hard or soft state of the source. No strong variability of the high frequency part of the X-ray power spectrum is seen for galactic sources in their hard states (Gierliński et al. 2008b). Similar disconnection with luminosity and dependence only on $M_{\rm BH}$ is seen in Sgr A* variability observed in Near IR (Meyer et al. 2009): the break in the power spectrum of the NIR lightcurve fits expectations from AGN despite the huge difference in the bolometric luminosity between AGN and Sgr A*.

An interesting version of this scaling emerged from the approach of Hayashida et al. (1998) and uses only the high frequency part of the power spectrum, or, in practice, the X-ray excess variance measured in the timescales shorter than the timescale corresponding to $1/f_{\rm hfb}$ in previous formulae. The relation

$$M_{\rm BH} = 1.92\frac{T - \delta t}{\sigma^2_{\rm rms}} M_\odot \qquad (25)$$

(Nikołajuk et al. 2006, 2009) is based on the scaling with Cyg X-1 and uses 20 $M_\odot$ for Cyg X-1 mass; study for a number of galactic sources gives somewhat lower coefficient of 1.24 (Gierliński et al. 2008b). Here T is the duration of the lightsurve and δt is the time bin, both in seconds, and the X-ray excess variance, $\sigma^2_{\rm rms}$ is dimensionless, normalized by the average flux.

In addition to broad band power spectra, quasi-periodic oscillations (QPO) are seen occasionally in X-ray Binaries and ULXs. Their use for black hole mass measurement is complicated by the fact that there are several types of QPO, and firm identification of the QPO type is needed to infer the black hole mass. QPO specific example of the application can be found in Shaposhnikov & Titarchuk (2007) who derived the black hole mass in Cyg X-1 of $8.7 \pm 0.8 M_\odot$ from the black hole mass in GRO J1655-40 at the basis of their QPO scaling with mass and X-ray spectral slope. There were several claims about the presence of QPO also in AGN, and recently Gierliński et al. (2008a) reported about discovery of an unmistakable QPO in NLS1 RE J1034+396. Middleton & Done (2009) have estimated the mass of a black hole in this object ($2 - 3 \times 10^6 M_\odot$).

However, caution in the case of X-ray methods is also needed since some weak inclination effect can be expected due to the general relativity causing in general slight enhancement in variability for highly inclined objects seen in theoretical studies (e.g. Abramowicz & Bao 1994; Czerny et al. 2004).

3. Consistency checks

Since all the methods may contain systematic errors the best test of the accuracy of the black hole mass determinations is to use two or more independent methods. This is particularly important for all the scaling approaches. In the case of AGN, also black hole mass determinations done at various epochs are interesting since they rely on the spectra which vary in time. Here we list a number of such comparisons.

A test of the reliability of a single epoch mass measurement based on the BLR radius-luminosity scaling (see Sect. 2.3) was performed using multi epoch measurements for two galaxies (NGC 5548 and PG 1229+204, Denney et al. 2009). The dispersion of the mass measurement due to the source variability was less than 0.1 dex for high signal to noise spectra (above 20).

The use of various optical/UV lines is nicely illustrated in Vestergaard (2009) (see her Fig. 3). There are visible discontinuities in the mass distribution with redshift due to the forced change from Hβ to Mg II and finally C IV, but less than a factor 2. Risaliti, Young & Elvis (2009) confirm previously suggestions (see e.g. Shen et al. 2008) that C IV line is not a reliable indicator of BH masses. Much better is to use Mg II. However, BH masses determined from this ion has systematic error so the Mg II mass can be corrected using the relation $\log[M_{\rm BH}({\rm H}\beta)] = 1.8\times \log[M_{\rm BH}({\rm Mg\,II})] - 6.8$.

The comparison of two methods was done for a water maser source NGC 4258, with the black hole mass of $(3.82 \pm 0.01) \times 10^7 M_\odot$ from water maser (Herrnstein et al. 2005) and

$(3.3 \pm 0.2) \times 10^7 M_\odot$ from stellar dispersion (Siopis et al. 2009). Stellar dispersion against reverberation was also tested for NGC 4151, stellar dispersion gives $(4 - 5) \times 10^7 M_\odot$ (Onken et al. 2007) and reverberation gives $4.57^{+0.57}_{-0.47} \times 10^7 M_\odot$ (Bentz et al. 2006).

Based on such tests, Vestergaard (2009) estimates that the systematic error in secondary estimates (i.e. scaling laws) is likely to be from 0.3 dex for stellar dispersion in the bulge to 0.7 dex when using bulge luminosity or NLR line width.

Interesting tests of the effects of inclination in AGN were performed by comparison of the reverberation method and the X-ray excess variance (Nikołajuk et al. 2006). The change of the inclination from ~ 15 deg to ~ 60 deg is likely connected with the change in the factor f in Eq. 3 by a factor 4 although errors in this analysis are large.

4. Summary of the results

4.1. Galactic black holes

The typical accuracy in the determination of the black hole masses is about a factor 2 or better. The best known black hole mass (microquasar GRO J1655-40, $M_{BH} = 6.3 \pm 0.5$, Greene, Bailyn & Orosz 2001) was achieved due to the dynamical method (accurate period determination, inclination from modelling of the lightcurve due to the companion ellipticity, mass ratio also from modelling the ellipticity). Overall, there are over 40 (24 confirmed BH) determinations of the black hole mass in such systems in our Galaxy and a few in the binaries in nearby galaxies like in LMC and M33 (e.g. eclipsing binary M33 X-7, with black hole mass of $15.65 \pm 1.45 M_\odot$; Orosz et al. 2007).

4.2. Intermediate mass black holes

This is the hottest subject in the Art of black hole mass measurement. First, the exact range is not specified, so it is not clear whether relatively small but otherwise typical AGN (i.e. located at the centres of their host galaxies) black holes from the range $\sim 10^5 - 10^6 M_\odot$ (see sample of Desroches, Greene & Ho 2009) belong to this class. Their existence does not cause much doubt (e.g. Ho 2008). The on-going stellar dispersion measurements in hundreds of near-by galaxies (see for example Ho et al. 2009) may bring more of them although measuring black holes with masses below $10^6 M_\odot$ is very difficult.

More questionable is the existence of the black holes with the masses of hundreds-thousands of the solar mass. Determination of $M_{\rm BH}$ in several ULX sources, which are hosted by galaxies taken from Messier and NGC catalogues, and the discussion of the stellar mass versus IMBH interpretation can be found in Zampieri & Roberts (2009) (see also references therein).

4.3. Non-Active galaxies or Weakly Active galaxies

The best example is of course the Milky Way galaxy, with the recent mass determination (see Sect. 2.1). The mass in other nearby galaxies is known less accurately (for example, triple nucleus of Andromeda hosts a black hole mass in P3 of $(1.1 - 2.3) \times 10^8 M_\odot$, Bender et al. 2005) but mass estimate (usually through stellar dispersion measurement) is available for hundreds of objects (Ho 2008).

4.4. Active Galactic Nuclei

The number of known AGN is of order of 10^5 (in SDSS DR7) and rising due to massive surveys. The best results are obtained from water maser discussed in Sect. 2.1. Generally, mass determination for tens/hundreds/thousands of objects were performed by several authors (e.g. Woo & Urry 2002; Vestergaard & Peterson 2006; Vestergaard et al. 2008; Shen et al. 2008; Fine et al. 2008; Kelly et al. 2009; Wu 2009).

Numerous large surveys supplemented with the black hole mass and the Eddington ratio determinations (e.g. Hickox et al. 2009) the available scaling laws from the list discussed in Sect. 2.3. The Eddington ratio is the frequently determine using another scaling to derive the bolometric luminosity, for example from the optical monochromatic flux. Richards et al. (2006) (see their Fig. 12) analysed spectra energy distributions of quasar type 1 and gave a formula:

$$L_{\rm bol} = {\rm BC}_\lambda \times \lambda L_\lambda \, , \tag{26}$$

where the bolometric corrections $\mathrm{BC}_\lambda = 3.81$, 5.15, 9.26 are for measurements of luminosity, λL_λ, made at λ= 1350, 3000 and 5100 Å, respectively.

It is also possible to use the near-IR monochromatic luminosity

$$L_{\rm bol} = 6.4 L({\rm NIR}) \tag{27}$$

(Cao 2005) or from the broad band X-ray luminosity (Hopkins, Richards & Hernquist 2007)

$$L_{\rm bol} = (10-20) L_{\rm X}(0.5-8{\rm keV}). \tag{28}$$

More complex, luminosity-dependent corrections can be found in (Hopkins, Richards & Hernquist 2007). Hickox et al. (2009) used the coefficient in Eq. (27) by a factor of two lower than derived by (Hopkins, Richards & Hernquist 2007) in order to have consistent results for AGN with both IR and X-ray measurements available.

In heavily obscured Seyfert 2 galaxies or radio galaxies the bolometric luminosity can be estimated from the [O III] line luminosity:

$$L_{\rm bol} = {\rm C} L([{\rm O\,III}]), \tag{29}$$

where C = 87, 142 and 454 for logarithmic luminosity ranges of [O III] line 38 - 40, 40 - 42 and 42 - 44, correspondingly (Lamstra et al. 2009).

5. Discussion

The large number of various methods allow to estimate the black hole mass in most astronomical objects quite reliably, i.e. within a factor of a few. Accurate determination of the error of a particular measurement is difficult since the errors are mostly systematic.

However, there are still classes of objects in high need for better mass determination. Those are first of all black holes in ULX and in globular clusters. The existence of IMBH is an interesting question both from the point of view of the accretion pattern (stellar mass black hole hypothesis for ULX sources requires very high Eddington ratios and strong beaming). Also some other types of active nuclei like weak line quasars (WLQ) and radio-loud objects like BL Lacs are still in a need for mass determinations although some estimates are possible even in those difficult cases (see e.g. Hryniewicz et al. 2009).

The mass determination together with the estimate of the bolometric luminosity opens a possibility of statistical studies in case of massive black holes since thousands of objects are available. Such a black hole spends most of the time in a relatively quiet state. Eddington ratios cover the range from 10^{-9} to ~ 1, with LINERS (and Seyfert 2 galaxies) grouped at $\sim 10^{-4}$ (Ho 2008), Seyfert 1 galaxies at $\sim 10^{-2}$, and quasars as well as Narrow Line Seyfert 1 galaxies close to 1. Future evolutionary studies should explain the observed ratios of these types of galaxies.

6. DISCUSSION

ANDRZEJ ZDZIARSKI: What is the current range of black hole masses in binaries and AGN?

BOZENA CZERNY: The smallest one currently measured is 3.8 solar masses in object J1650-500 and comes from the paper by Shaposhnikov & Titarchuk. As for the heaviest one, they are among the most distant quasars. For example, the black hole mass of $z = 6.41$ quasar SDSS J1148+5251 was estimated to be $3 \times 10^9 M_\odot$ (Willot et al. 2003), and some quasars in Verstergaard (2009) sample have black hole masses up to $3 \times 10^{10} M_\odot$. The black hole in OJ 287 mentioned in the context of binary black holes, with its mass of $\sim 1.8 \times 10^{10} M_\odot$ is also among the largest ones.

WOLFGANG KUNDT: In Vestergaard et al. SDSS plot of the quasars as a function of redshift z, their average masses evolve from a few times $10^9 M_\odot$, at $z <= 5$, down to a few $10^6 M_\odot$ at $z <= 0.2$. How can you downsize this inverted evolution to their predicted growth via accretion?

BOZENA CZERNY: The plot shows only very active galaxies. The apparent rise of the quasar black hole mass with the redshift in that plot, and in the fits by Labita et al. (2009) to the quasar mass as $\propto (1+z)^{1.64}$ only shows that more massive local over-densities evolve faster, which is known in cosmology as anti-hierarchical evolution. Those large mass over-densities becomes quasars at high z, and later on pass to a quiet non-active stage, not showing up in AGN surveys. Smaller over-densities evolve slowly, they become active much later and never grow up to give large black hole masses. For example, Sgr A* is likely to have occasional more active stages than seen at present but will never become a billion solar mass black hole.

Acknowledgements. We are grateful to Joasia Kuraszkiewicz and Amri Wandel for helpful discussions. Part of this work was supported by the grant NN 203 38 0136 and Polish Astroparticle Network 621/E-78/BWSN-0068/2008.

References

Abramowicz, M. & Bao, G., 1994, PASJ, 46, 523
Bender, R., Kormendy, J., Bower, G. et al. 2005, ApJ, 631, 280
Bentz, M. C., Denney, K. D., Cackett, E. M. et al. 2006, ApJ, 651, 775
Bentz, M. C., Peterson B. M., Netzer H., Pogge R.W. et al. 2009a, 697, 160
Bentz, M. C., Peterson B. M., Pogge R.W. et al. 2009b, 694, L166
Bentz, M. C., Walsh, J. L., Barth, A. J, et al. 2009c, ApJ, ArXiv Astrophysics e-print, arXiv:0908.0003
Berghea, C. T., Weaver, K. A., Colbert, E. J. M. et al. 2008, ApJ, 687, 471
Bettoni, D., Falamo, R., Fasano, G. et al. 2003, A&A, 339, 869
Bian, W.-H. & Zhao Y.-H. 2003, PASJ, 55, 143
Cackett, M., Horne K., & Winkler H., 2007, MNRAS, 380, 669
Cao, X., 2005, ApJ, 619, 86
Casares, J., 2007, IAUS, 238, 3
Collin, S., Kawaguchi, T., Peterson, B. M., et al. 2006, A&A, 456, 75
Czerny, B. & Janiuk, A. 2007, A&A, 464, 167
Czerny, B., Rozanska, A., Dovciak, M., Karas, V., Dumont, A.-M., 2004, A&A, 420, 1
Dasyra, K. M., Ho, L. C., Armus, L. et al. 2008, ApJ, 674, L9
Davis, S. W., Woo, J.-H., & Blaes, O. M. 2007, ApJ, 668, 682
Denney, K. D., Peterson, B. M., Dietrich, M. et al. ApJ, 692, 246
Desroches L-B., Greene J.E., Ho L.C., 2009, ApJ, 698, 1515
Done, C. & Kubota, A. 2006, MNRAS, 371, 1216
Dovčiak, M., Bianchi, S., Guainazzi, M., et al. 2004, MNRAS, 350, 745
Falomo, R., Kotilainen J.K., Treves, A., 2002, ApJ, 569, L35
Farell, S.A. et al., 2009, Nature, 460, 73
Ferrarese, L. & Merritt, D. 2000, ApJ, 539, L9
Fine, S., et al., 2008, MNRAS, 340, 1413
Fregeau, J. M., Richer, H. B., Rasio, F. A. et al. 2009, ApJ, 695, L20
Gaskell, C. M. 2009, ArXiv Astrophysics e-print, arXiv:0908.0328
Gebhardt, K., Bender, R., Dressler, A. et al. 2000, ApJ, 539, L13
Gierliński, M., Middleton, M., Ward, M. et al. 2008a, Nature, 455, 369
Gierliński, M., Nikołajuk, M., & Czerny, B., 2008b, MNRAS, 383, 741
Gillessen, S., Eisenhauer, F., Trippe, S. et al. 2009, ApJ, 692, 1075
Ghez, A. M., Salim, S., Weinberg, N. N. et al. 2008, ApJ, 689, 1044
Greene, J., Bailyn, Ch. D., & Orosz, J. A. 2001, ApJ, 554, 1290
Gültekin, K., Richstone, D. O., Gebhardt, K. et al. 2009a, ApJ, 695, 1577
Gültekin, K., Richstone, D. O., Gebhardt, K. et al. 2009b, ApJ, 698, 198
Häring, N. & Rix, H.-W. 2004, ApJ, 604, L89
Heil, L. M., Vaughan, S., & Roberts, T. P. 2009, MNRAS, 397, 1061
Hayashida, K., Miyamoto, S., Kitamoto, S. et al. 1998, ApJ, 500, 642

Herrnstein, J. R., Moran, J. M., Greenhill, L. J. et al. 2005, ApJ, 629, 719
Hickox R. C., Jones, Ch., Forman, W. R. et al., 2009, ApJ, 696, 891
Ho, L. C., 2008, ARA&A, 46, 475
Ho, L. C., Greene, J. E., Filippenko, A. V., et al. 2009, ApJS, 183, 1
Hopkins, P. F., Richards, G. T., & Hernquist, L. 2007, ApJ, 654, 731
Hryniewicz, K., Czerny, B., Nikolajuk, M. et al. ArXiv Astrophysics e-print, arXiv:0904.3517
Hubeny, I., Agol, E., Blaes, O. et al. 2000, ApJ, 533, 710
Kaspi, S., Smith, P. S., Netzer, H. et al. 2000, ApJ, 533, 631
Kaspi, S., Maoz, D., Netzer, H. et al. 2005, ApJ, 629, 61
Kelly B.C., Bechtold, J., Siemiginowska, A., 2009, ApJ, 698, 895
Kondratko, P. T., Greenhill, L. J., Moran, J. M., et al. 2006, ApJ, 638, 100
Kong, M.-Z., Wu, X.-B., Wang, R. et al. 2006, ChJAA, 6, 396
Kormendy, J. & Gebhardt, K. 2001, AIPC, 586, 363
Kormendy, J. & Richstone, D. 1995, ARA&A, 33, 581
Krajnović D., McDermid, R. M., Cappellari, M. et al. 2009, MNRAS, ArXiv Astrophysics e-print, arXiv:0907.3748
Krolik, J.H., 2001, ApJ, 551, 72
Labita, M., Decarli, R., Treves, A., et al. 2009, MNRAS, ArXiv Astrophysics e-print, arXiv:0907.2963
Lamastra A., Bianchi, S., Matt, G. 2009, A&A, 504, 73
Loska, Z., Czerny, B., & Szczerba, R. 2004, MNRAS, 355, 1080
Marconi, A., Axon, D. J., Maiolino, R. et al. 2009, ApJ, 698, L103
Magorrian, J., Tremaine, S., Richstone, D. et al. 1998, AJ, 115, 2285
McClintock, J. E., Shaffe, R., Narayan, R., Remillard, R. A. et al. 2006, ApJ, 652, 518
McHardy, I. M., Koerding, E., Knigge, C. et al. 2006, Nature, 444, 730
McHardy, I. M., Arévalo, P., Uttley, P. et al. 2007, MNRAS, 382, 985
McLure, R. J.; Dunlop, J. S. 2001, MNRAS, 327, 199
McLure, R.J., Dunlop, J.S., 2002, MNRAS, 331, 795
Meyer, L., Do, T., Ghez, A., Morris, M. R. et al. 2009, ApJ, 694, L87
Middleton, M. & Done, C., 2009, ArXiv Astrophysics e-print, arXiv:0908.0224
Miller, J. M., Fabian, A. C., & Miller, M. C. 2004, ApJ, 614, L117
Miyoshi, M., Moran, J., Herrnstein, J. et al. 1995, Nature, 373, 127
Netzer, H. 1990, in Active Galactic Nuclei, ed. R. D. Blandford et al. (Berlin: Springer), 57
Nikołajuk, M., Czerny, B., Ziółkowski, J. et al. 2006, MNRAS, 370, 1534
Nikołajuk, M., Czerny, B., & Gurynowicz, P. 2009, MNRAS, 394, 2141
Nowak, M. A. 2000, MNRAS,318, 361
Onken, Ch. A., Ferrarese, L., Merritt, D. et al. 2004, ApJ, 615, 645
Onken, Ch. A., Valluri, M., Peterson, B. M. et al. 2007, ApJ, 670,105
Onken, C. A., 2009, ArXiv Astrophysics e-print, arXiv:0907.4192
Orosz, J. A., McClintock, J. E., Narayan, R. et al. 2007, Nature, 449, 872
Orosz, J. A., Steeghs, D., McClintock, J. E. et al. 2009, ApJ, 697, 573
Papadakis, I. E. 2004, MNRAS, 348, 207
Peterson, B. M., Ferrarese, L., Gilbert, K. M. et al. 2004, ApJ, 613, 682
Peterson, B. M. & Horne, K. 2004, AN, 325, 248
Pottschmidt, K., Wilms, J., Nowak, M. A. et al. 2003, A&A, 407, 1039
Richards, G. T., Lacy, M., Storrie-Lombardi, L. J. et al. 2006, 2006, ApJS, 166, 470
Risaliti, G., Young, M., Elvis, M. 2009, ApJ, 700, L6
Roberts, T. P. 2007, Ap&SS, 311, 203
Sadowski, A., Abramowicz, M. A., Bursa, M. et al. 2009, A&A, 502, 7
Shaposhnikov, N. & Titarchuk, L. 2007, ApJ, 663, 445
Shakura, N. I. & Sunyaev, R. A. 1973, A&A, 24, 337
Shen, Y., Greene, J. E.; Strauss, M. et al. 2008, ApJ, 680, 169
Siopis, Ch. Gebhardt, K., Lauer, T. R. et al. 2009, ApJ, 693, 946

Soria, R. & Puchnarewicz, E. M. 2002, MNRAS, 329, 456
Tripp, T., Bechtold, J., Green, R.F. 1994, ApJ, 433, 533
Uttley, P., McHardy, I. M. & Papadakis, I. E. 2002, MNRAS, 332, 231
Valtonen M. J., Nilsson, K., Villforth, C. et al. 2009, ApJ, 698, 781
van der Marel, R. P. & Anderson, J., 2009, ArXiv Astrophysics e-print, arXiv:0905.0638
Vasudevan, R. V., Mushotzky, R. F., Winter, L. M. et al. 2009, MNRAS, ArXiv Astrophysics e-print, arXiv:0907.2272
Vestergaard 2009, ArXiv Astrophysics e-print, arXiv:0904.2615
Vestergaard, M, et al., 2008, ApJ, 674, L1
Vestergaard, M. & Peterson B. M. 2006, ApJ, 641, 689
Vestergaard M. & Osmer P. S. 2009, ApJ, 699, 800
Wandel A. 2008, AIP Conference Proceedings, 1053, 7
Wandel, A., Peterson, B. M. & Malkan, M. A. 1999, ApJ, 526, 579
Willot, C. J., McLure, R.J. & Jarvis, M. J. 2003, ApJ, 587, L15
Woo, J.-H., Urry, C.M., 2002, ApJ, 579, 530
Wu, Q., 2009, MNRAS, 398, 1905
Xu, D., & Komossa, S., 2009, ApJL, Astrophysics e-print, arXiv:0908.3140
Zampieri, L. & Roberts, T. P. ArXiv Astrophysics e-print, arXiv:0909.1017
Ziółkowski 2008, ChJAS, 8, 273
Zhou, M. & Cao, X.-W. 2009, RAA, 9, 293