



# Doubling the CO<sub>2</sub> cools or heats?

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**Abstract.** We calculate the effect that CO<sub>2</sub> enhancement has on the radiative transfer in the Earth atmosphere. To this goal, we apply a recent algorithm developed by Wehrse and Shaviv. The calculation differs from previous calculations in that it includes several million molecular lines and in that it iterates for the temperature profile, so as to satisfy radiative equilibrium.

The main result we find is that the effective optical depth of the CO<sub>2</sub> band at 14,000-16,000Å is of the order of unity, and that by increasing the amount of CO<sub>2</sub>, the optical depth increases primarily at high altitudes. With it, the absorption of solar energy increases at high altitudes but it decreases at low altitudes above the planetary boundary layer. As a consequence, the temperature rises at altitudes of about 20km, it decreases at low altitudes, but the surface actually warms.

Note that the calculation does not include yet the effects of convection, nor does it include indirect effects on the  $T$  such as through water vapor or cloud feedbacks.

**Key words.** Planets, CO<sub>2</sub>, planetary atmosphere, radiative transfer, molecular absorption

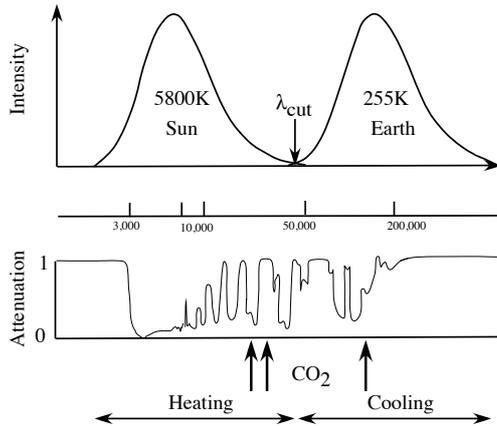
## 1. Introduction

CO<sub>2</sub> is considered to be one of the most potent greenhouse gases. It is also commonly believed that by raising the amount of any greenhouse gas in the atmosphere, and in particular that of CO<sub>2</sub>, the heat absorption by the atmosphere will increase and therefore lead to a global warming of the atmosphere.

The traditional methodology used to evaluate the radiative forcing of CO<sub>2</sub> is by calculating the transmission of both the solar radiation and the thermal radiation through every line, and therefore called the line-by-line method (see for example Bernstein et al. 2007 and Anderson et al. 2006 which describe the standard MODTRAN code).

With this method, one compares the transmission through a standard atmosphere with the transmission through an atmosphere with an increased concentration of CO<sub>2</sub>. However, this is carried out without iterating for the temperature as a function of altitude, and hence does not yield an atmosphere in radiative equilibrium. We will show that this procedure cannot provide a reliable answer to the fundamental question of what is the radiative forcing of CO<sub>2</sub>, instead, one has to alleviate at least some of the assumptions often made when solving the radiative transfer problem.

Thus, with the goal of assessing the accurate effect of CO<sub>2</sub> on the radiative balance and temperature of the atmosphere, we developed a very detailed radiative transfer model.



**Fig. 1.** The upper panel shows the relative intensities of the solar irradiation and the Earth's emission. The definition of  $\lambda_{cut}$  as the wavelength separation between the two radiations is indicated. In the lower panel the total attenuation of the radiation, as is frequently depicted as a function of wavelength. The arrows mark the location of CO<sub>2</sub> absorption. The rest of the absorption is due to O<sub>3</sub> and O<sub>2</sub> below 3,000Å, and mostly H<sub>2</sub>O and small contributions from CH<sub>4</sub>, NO<sub>2</sub> and other gases in small quantities. The horizontal arrows mark the region of heating of the atmosphere by absorption and cooling respectively.

## 2. Background

The radiation field in the Earth atmosphere can be divided into two main spectral regions. For wavelength shorter than 50,000Å, the “short-wave” solar radiation dominates, while for wavelengths longer than 50,000Å, it is the terrestrial “longwave” infrared radiation which dominates. This can be seen in fig.1. We define  $\lambda_{cut}=50,000\text{Å}$  as the separation wavelength between the solar dominated and Earth dominated spectral regions.

The absorption by each molecule takes place via electronic (with energies of the order of  $few \times eV$ ), vibrational (with energies of the order of  $10^{-3}eV$ ), and rotational states (with energies of the order of  $10^{-5} - 10^{-6}eV$ ). The electronic states absorb mainly in the UV and the visible range, the vibrational states absorb in the Near Infrared (NIR) region of the solar spectrum, while the rotational bands absorb

in the far infrared (FIR) where the terrestrial thermal radiation dominates. In particular, the bands for CO<sub>2</sub> are marked in fig.1. It is important to realize that the absorption in the NIR range is always accompanied by absorption in the FIR range.

With the above in mind, we can proceed to examine the heat balance equation which governs the temperature profile in the atmosphere. It is given by

$$\int_0^{\infty} \kappa(z, \lambda)(J(z, \lambda) - B(T(z, \lambda)))d\lambda + \nabla \cdot F_{conv} = 0, \quad (1)$$

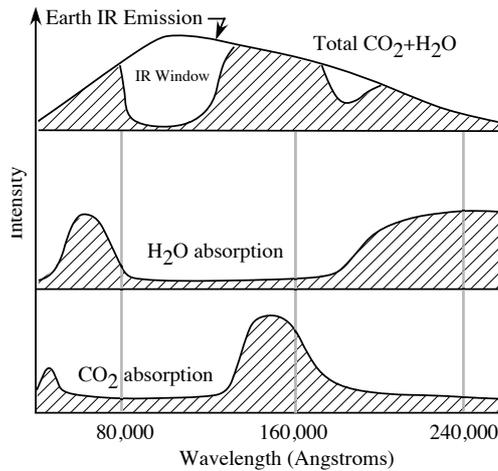
where  $J$  is the mean intensity. The latter is given by  $J = (1/4\pi) \int_{\Omega} d\Omega$ , namely, the mean intensity integrated over the entire sphere.  $\kappa(z, \lambda)$  is the absorption coefficient as a function of wavelength and height. The condition must be satisfied at all heights  $z$ .  $F_{conv}$  is the convective flux. Here we restrict the discussion to an atmosphere in pure radiative equilibrium, without any convection.

In the region  $\lambda < \lambda_{cut}$ , we have that  $B \ll J$  and we can therefore split the integral into two parts. We thus write

$$\int_0^{\lambda_{cut}} \kappa(z, \lambda)J(z, \lambda)d\lambda + \int_{\lambda_{cut}}^{\infty} \kappa(z, \lambda)[J(z, \lambda) - B(z, \lambda)]d\lambda = 0. \quad (2)$$

The first term is positive definite, implying that the second term must be negative. The first term represents radiation absorbed by the atmosphere, namely heating. The second term should therefore represent cooling of the atmosphere, as the atmosphere is in a thermodynamic balance. According to Kirchhoff's law, the higher is the absorption, the higher is the cooling. Hence, the enhancement of the concentration of any molecule increases the heating by increasing the absorption for  $\lambda < \lambda_{cut}$ , but it also increases the cooling by increasing the absorption for  $\lambda > \lambda_{cut}$ .

In addition, the surface could cool from the reduced transmissivity to the solar shortwave radiation, but it can also increase due to increased thermal radiation emitted from the atmosphere back to the surface.



**Fig. 2.** The absorption of water vapor and CO<sub>2</sub> in the Far IR range. The absorption is plotted as a continuous line, however, recall that each band contains about  $10^3$  lines.

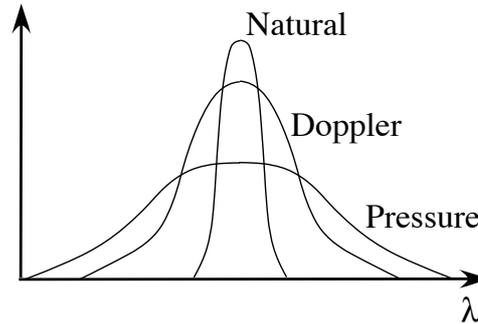
Thus, a priori, it is not clear without solving the resulting radiative transfer problem what the new balance will be. In particular, it is not obvious to ascertain the effect that changing  $\kappa$  has on the temperature profile. Loosely, we can say that heating takes place via the rotational levels and cooling via the vibrational levels.

It is obvious that  $J$  cannot be equal to  $B$  in the FIR range, as long as  $\kappa(\lambda < \lambda_{cut}) \neq 0$ . Moreover, if  $\kappa(\lambda < \lambda_{cut})$  does vanish,  $J - B$  can change sign in the FIR to produce radiative equilibrium.

Fig. 2 depicts some of the FIR absorption details of water vapor and CO<sub>2</sub>. Clearly, enhancing the CO<sub>2</sub> will increase the atmospheric cooling because of the increased emissivity from the CO<sub>2</sub> dominated bands in the FIR.

### 3. Importance of line broadening

Fig. 3 describes a typical spectral line, and the effects of Doppler and pressure broadening. Doppler broadening is Gaussian while pressure broadening is Lorentzian. As a consequence, they affect the absorption differently. The turn over height in the Earth's atmosphere, from pressure broadening at low altitudes to Doppler broadening at high altitudes, is given in table 1.



**Fig. 3.** The shape of the line depends on the temperature via Doppler broadening and on the pressure via pressure broadening.

In the calculations, we used the Voigt profile, which was calculated for every pressure and temperature during the iteration for the temperature.

**Table 1.** The transition height in the Earth atmosphere from pressure to Doppler broadening

Molecule	Band center ( $\mu\text{m}$ )	km
H <sub>2</sub> O	2.7	17
	6.3	23
	20	32
	40	37
C <sub>2</sub> O	4.3	26
	15	34
CH <sub>4</sub>	3.3	22
	7.7	27

### 4. Line absorption versus gray absorption

Molecular absorption takes place in a multitude of lines. The total number of lines is several millions, consequently, the approach generally taken is to average the absorption over wavelength and at best use several bands over which the absorption is averaged. The obtained averaged absorption coefficient is the gray absorption opacity, and it assumes that the absorption is constant over the given frequency

band. However, the averaging process eliminates some of the most important features of the radiative transfer, namely, the non-linearity of the radiative transfer equations as well as the temperature feedback.

This can be illuminated by the following example. Let us look at the following two cases. (a) Two molecules which absorb at a given frequency, physically placed one behind the other, and (b) two molecules which absorb at all wavelengths, placed side by side. It is obvious that even if the frequency integrated absorption coefficient is the same, the total absorption in the first case is smaller, as the system can reach saturation. In radiative transfer terms, a large optical depth over a short wavelength range, is not strictly equivalent to a small optical depth over a large wavelength range. These fine details are lost when averaging over a frequency band with many lines.

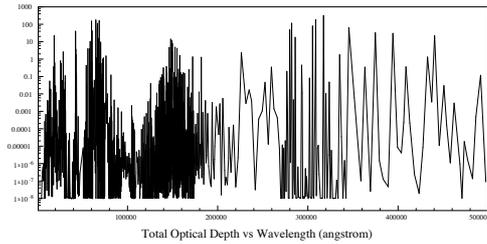
To see this in more detail, consider now the following comparison between line absorption and gray absorption. Assume that

$$\kappa_{gray} = \frac{\int_{\Delta\lambda} \kappa(\lambda, z) d\lambda}{\Delta\lambda}, \quad (3)$$

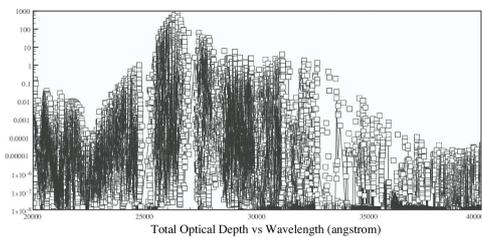
where  $\Delta\lambda$  is the wavelength range over which the averaging takes place. It appears that integration over the entire frequency band apparently accounts for all absorption. But this is not the case. For a small number of bands, one finds that

$$\kappa_{gray} \ll \kappa(\lambda \approx \lambda_0 \pm 1/2\omega), \quad (4)$$

where  $\lambda_0$  and  $\omega$  are the wavelength and width of the line respectively. In the particular case of the CO<sub>2</sub> band at  $\lambda = 14,000\text{--}16,000\text{\AA}$ , the total optical depth in the band is about 17, while  $\kappa_{gray} \ll 1$ . Thus, if the line absorption is assumed, the line is fully saturated and only an exponentially small amount of radiation in this band will reach the terrestrial surface. On the other hand, if the equivalent gray absorption is assumed, then some radiation reaches the Earth's surface everywhere in the frequency band. An enhancement of the CO<sub>2</sub> will, in the line absorption case, enhance the heating at an altitude above the point where the optical depth is about unity (in this case about 20km high).



**Fig. 4.** The total optical depth in the molecular lines in the standard atmosphere as a function of wavelength. Note that the heaviest bands have a total optical depth of several tens. When the transmission is plotted as a function of wavelength, all lines with optical depth larger than a few appear with zero transmission, hence drawing the transmission is misleading.

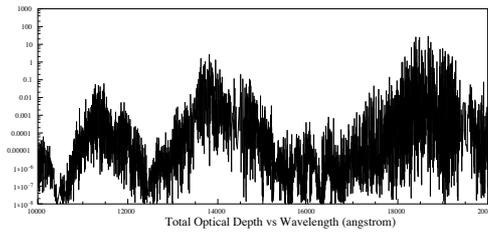


**Fig. 5.** Each small square describes a wavelength in the calculation. Here we show a small segment from the entire wavelength range.

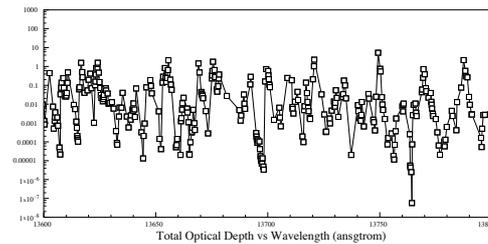
On the other hand, in the gray approximation, the radiation continues to reach the Earth's surface and heat it.

## 5. The basic data

We used the HITRAN molecular line list compiled by the Rothman et al. (2003). The compilation contains a few million of the most important molecular lines of the most important gases in the Earth and planetary atmosphere. Unfortunately, however, the data set does not include whether the lines are absorption or scattering. Even statistical information on the fraction of scattering is not available. For this reason, we assume the lines to be purely absorptive. This gives an upper limit to the amount of atmospheric heating and cooling.



**Fig. 6.** The molecular absorption at the CO<sub>2</sub> band. Shown is the total optical depth as a function of wavelength.



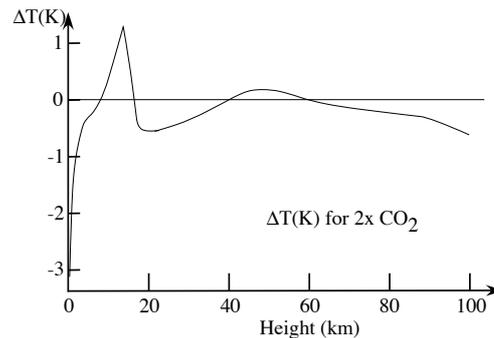
**Fig. 7.** The description of part of the CO<sub>2</sub> band. Shown is the total optical depth as a function of wavelength.

## 6. The calculation

We implemented the method of Shaviv & Wehrse (1987) for a fast solution of the radiative transfer in the two stream approximation. In view of the above, we prepared a special computer program which analyzes a planetary atmosphere with a given composition and selects all the lines with absorption strengths above a prescribed strength (usually greater than  $10^{-6}$  of the strongest line). In this way, about a million lines are selected. In the next step, the program discretizes wavelength space and distributes wavelength points for the calculation of the radiative transfer, such that all lines are covered. Interference between lines is identified and taken care of. In this way, a total of about  $10^7$  wavelengths are used in the calculation. A typical example is shown in figs. ?? and ??, where we depict how the wavelength points are distributed with respect to the spectral lines.

About 50 fixed height layers are assumed up to an altitude of 100km. The width of each layer is inversely proportional to the density.

The top boundary condition is fixed to be the mean solar irradiation, averaged over the globe. The surface boundary condition is a convective-radiative equilibrium between the temperature of the surface and that of the atmosphere. Because the surface interacts with all the atmosphere, there is a discontinuity between the surface temperature and the lower atmospheric layer. This implies that the plan-



**Fig. 8.** The temperature change following a doubling in the atmospheric concentration of CO<sub>2</sub>. The temperature change of the surface is +0.1K. Thus, the gradient along the planetary boundary layer is expected to increase, even though we do not solve it explicitly.

etary boundary layer is neglected<sup>1</sup>. Note that the surface temperature is not given but iterated for.

The iteration for the temperature is carried out using the method of steepest descent, which releases us from finding the numerous partial derivatives needed for a faster method. The cost, however, is a slow convergence and some 500 to 1000 iterations are required. The temperature is solved to a very high accuracy (to a relative error of  $10^{-7}$ ). This is necessary because the minima in the multidimensional space are very shallow and the high accuracy is mandatory to secure the correct solution.

<sup>1</sup> Because the planetary boundary layer is optically thin to all but the most extremely thick lines, this is not a problem.

Two calculations were carried out. In Case A, we assume the present day CO<sub>2</sub> concentration. In Case B, we double the present day CO<sub>2</sub> level, while keeping all other parameters (solar heat flux, albedo and composition) unchanged. We assume also the terrestrial atmosphere, with N<sub>2</sub>, O<sub>2</sub>, H<sub>2</sub>O and CO<sub>2</sub>. We do not include clouds (except in the albedo we take), nor do we consider O<sub>3</sub>, lateral mass motions winds, convection, the flux of sensible and latent heat, or the air-surface convective energy exchange.

In essence, we calculate the *partial derivative* of the temperature with respect to the change in the concentration of CO<sub>2</sub>, since we do not consider secondary feedback effects, such as those which arise from changes in the water vapor content or in the cloud cover.

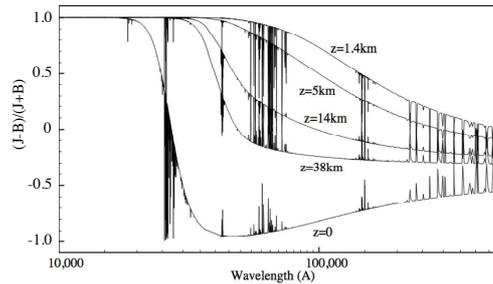
## 7. The results

In fig. 8, we show the results for the temperature difference between an atmosphere with a doubled CO<sub>2</sub> concentration and with the present concentration. We find that doubling the CO<sub>2</sub> reduces the atmospheric temperature at low altitudes, but it increases the temperature at an altitude of about 20km, whereupon doubling the CO<sub>2</sub>, more energy is absorbed.

It is of interest to see the behavior of  $J$  and  $B$ . For this reason, we plot a normalized  $J - B$  in fig.9 for different altitudes. In the solar dominant range, where  $J \gg B$ , the value is close to unity. However, it behaves quite differently in FIR range and it changes with altitude. It is clear that the assumption frequently implemented, that  $J = B$  in the FIR, does not hold.  $J - B$  changes sign both as a function of wavelength and altitude.

## 8. Conclusions

1. Due to the particular form of the molecular absorption, the gray approximation obtained by integrating the absorption over a finite range of wavelengths, leads to very inaccurate results. Such a calculation misses the peculiarities inherent to line absorption, namely Doppler and pressure



**Fig. 9.** The variation of  $(J - B)/(J + B)$  as a function of wavelength, for several altitudes.

broadening as well as the most important effect of saturation.

2. The method of line-by-line transfer to obtain the radiative forcing of a given molecule does not provide reliable answers. In particular, it fails to account for the feedback between line absorption and line broadening, as well as the temperature structure of the atmosphere. It also incorrectly describes the behavior in the presence of both optically thick and optically thin lines.
3. The standard description of the effect of CO<sub>2</sub> through an effective radiative forcing change, that is, by providing a single number such as  $4\text{W/m}^2$  for the entire atmosphere, does not represent faithfully the variation of the effect with height.
4. The radiative forcing we obtain at the bottom of the atmosphere is  $0.5\text{W/m}^2$ , which is much smaller than the canonical  $3.8\text{W/m}^2$ . As a consequence, the surface should warm by  $0.1\text{K}$  to compensate.
5. In essence, we calculated the *partial derivative*:  $\partial T/\partial[\text{CO}_2]$  and found that it depends on height. It is negative near the surface and positive at high altitude. We did not calculate the *total derivative*, namely  $DT/D[\text{CO}_2]$ , because we did not include, yet, the feedback effects of clouds, the change in the energy balance due to water vapor, the effects of atmospheric convection, or even the dynamic balance between atmospheric CO<sub>2</sub> and CO<sub>2</sub> in the oceans. This is work in progress.

It should be stressed that the results presented here are preliminary, in particular, in view of the limitations in point 5 above. It does show us however, that the present estimates in the literature for the effects of CO<sub>2</sub> should be taken with a grain of salt.

## 9. DISCUSSION

**Dmitry Bisikalo:** Could you explain the role of chemical reactions in your model?

**Giora Shaviv:** At the moment we do not include chemical reactions in the model. We assume a static atmosphere with a given composition. We verify the hydrostatic equilibrium and calculate the temperature run with altitude. The temperature run is such that it satisfies both the hydrostatic and radiative equilibria. Both the temperature and pressure are iterated for.

The chemical reactions in the atmosphere (about 650) and in the ocean, are now being calculated and will be introduced into the radiation code. We hope to report the results in the next Vulcano meeting.

**Bozena Czerny:** Can you comment on the role of clouds?

**Giora Shaviv:** Clouds have many effects, but the dominant one is reflecting the solar light, thereby changing the planetary albedo. More clouds imply less solar radiation reaching the Earth's surface and hence a direct cooling. But water vapor absorbs the radiation so clouds with water droplets also affect the IR radiation.

During the night, however, the role of clouds changes. A cloudy night is warmer than a clear bright night. The present calculation is of a mean atmosphere irradiated by a mean solar irradiation. The day/night effect is not yet included.

**Antonino Del Popolo:** In your model you use the steepest descent method. Nowadays there are more efficient methods.

**Giora Shaviv:** You are right in principle. The complications with the method is that the minima are very shallow. We use a software developed in the mathematical institute of the Heidelberg university (IWR). We work now on a new method. Our first goal was to assess the importance of the line absorption and hence were ready to use a safe and slow method rather than spend time on a faster method.

**John Beckman:** Please explain the principle of your calculation of the runaway temperature rise in Venus.

**Giora Shaviv:** First the conditions on Venus are very different. The atmospheric pressure is 90bar and the atmosphere contains more than 90% CO<sub>2</sub>. Consequently, pressure broadening has a much greater effect than on the Earth, and the molecular lines overlap. Second, the atmosphere contains several species for which HITRAN has no data. Third, given the fact that the detailed line list is not known and pressure broadening is cardinal, we used a semi-gray opacity model and found that irrespective of the magnitude of the opacity, the maximum temperature is 720-750K, in good agreement with observations. Actually, to the best of our knowledge, this is the first calculation which shows this saturation effect on Venus.

**James Beall:** Comment: You might find the Vostok data set of interest. Question: A paper I did using both the paleoclimate data (Vostok) shows an 8°K change in temperature for a CO<sub>2</sub> change from 180pp, to 280ppm. It shows a remarkably different slope when compared to the industrial period temperature vs CO<sub>2</sub>. These data show a (3/4)°K rise in temperature going from 280ppm to 380ppm.

**Giora Shaviv:** Excellent! Indeed there are more indications and evidence that the rise of the CO<sub>2</sub> lags behind the rise of the temperature and never the other way round (if it were a measurement error you expect both results to occur). Thus, it is the change in temperature which induces a change in CO<sub>2</sub> and not the other way round.

As for the different slopes, we hope to tackle this problem in the future. We have no answer at the moment.

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