Mem. S.A.It. Vol. 81, 144

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The gravitational lens as a radiospectrometer

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Abstract. Gravitational lensing is predicted by general relativity and is found in observations. Weak lensing and observational examples of lenses, as well as strong lensing with relativistic rings, are considered. When a gravitating body is surrounded by a plasma, the lensing angle depends on a frequency of the electromagnetic wave due to refraction properties, and the dispersion properties of the light propagation in plasma. We consider here the effects of weak and strong gravitational lensing, observations of gravitational lenses, and spectrometric properties of a gravitational lens in a plasma.

Key words. General relativity. Gravitational lensing. Waves in plasma.

1. Introduction

An ordinary theory of the gravitational lensing is well developed for the light propagation in the vacuum. Gravitational lensing in the vacuum is achromatic because the deflection angle for the photon does not depend on the frequency of the photon, see Schneider et al. (1992). In the limit of a weak lensing in vacuum by a body with a mass M, the deflection angle for the photon (Einstein angle) is

$$\hat{\alpha} = 4GM/c^2 b = 2R_S/b, \quad \text{at} \quad b \gg R_S, \qquad (1)$$

where b is impact parameter, R_S is the Schwarzschild radius, see Schneider et al. (1992).

Propagation of the light in the medium at presence of the gravity field was considered by many authors see Bliokhn and Minakov (1989) and references therein. In these papers concerning gravitational deflection, the inhomogeneous medium was considered, and the deflection due to the gravitation, and due to the inhomogenity of the medium, have been considered separately, without an account of the influence of the dispersion in plasma on the light propagation in the gravitational field. It was shown by Bisnovatyi-Kogan and Tsupko (2009) that even in the homogeneous medium the dispersion in the plasma leads to dependence of the light deflection angle on the wavelength, contrary to the vacuum case.

Plasma is a dispersive medium, where the refraction index depends on the frequency of the photon. Therefore in the plasma the photons with different frequencies move with different velocities, namely the photons with smaller frequency (or bigger wavelength) move with smaller group velocity of the light signal. This is the reason why in a homogeneous plasma, in the presence of gravity, the deflection angle of the photon depends on the frequency of the photon.

In the book of Synge (1960), the geometrical optics in the medium with gravity was considered in great details, and he had de-

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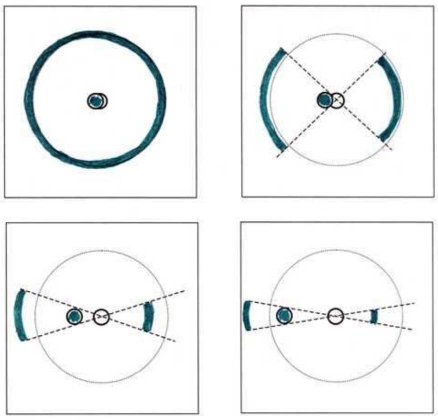

Fig. 1. Change of the lensing image, with increasing of the angle β between lines, connecting the observer with a source, and with a lens. Circle is formed when all three are on one straight line (upper left). Off line lensing produces arc images, β is increasing from the left to the right images. The figure is due to the courtesy of O.Yu. Tsupko.

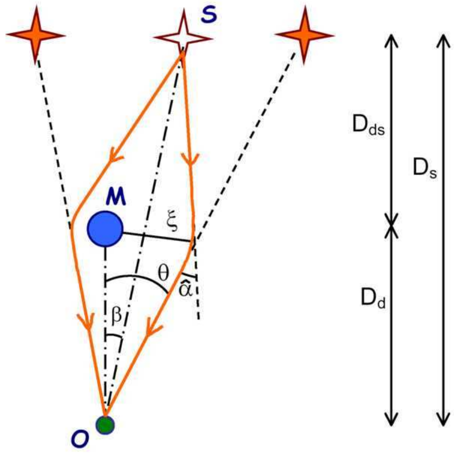

Fig. 2. The scheme of lensing, suggested by Chwolson (1924). The figure is due to the courtesy of O.Yu. Tsupko.

rived equations for the photon propagation in an arbitrary medium with gravity. Bisnovatyi-Kogan and Tsupko (2009) calculate, on the basis of the equations of Synge (1960), the deflection angle for the photon wave packet, moving in the gravitational field in presence of a uniform plasma. The Schwarzschild black hole metric was considered. The deflection angle increases with decreasing of the frequency. The effect is becoming large when the frequency is approaching the electron plasma frequency $\omega_p^2 = \frac{4\pi e^2 n_e}{m_e}$.

2. Weak gravitational lensing

Weak lensing corresponds to the case, when the impact parameter of the rays, coming to the observer, is much larger than the gravitational radius of the lens, and the deviation angle is small.

The flux from the source is determined by its surface brightness and by the solid angle it occupies in the sky. Since the gravitational deflection of the light does not involve emission or absorption, the intensity of the radiation remains constant along a beam. In addition, the gravitational deflection of the light by a local, essentially static lens does not introduce an additional shift in frequency. Thus, the surface brightness of an image of a lensed source is equal to the brightness of the image in the absence of the lens. Consequently, the ratio of the fluxes in the images of a lensed and a nonlensed source is determined by the ratio of the solid angles of these sources in the sky Schneider et al. (1992), i.e., by the ratio of the areas of the corresponding images on a photographic plate or CCD array (in the case of ideal imaging). The ratio

$$\mu = \frac{\Delta\Omega}{(\Delta\Omega)_0} \tag{2}$$

where $(\Delta\Omega)_0$ and $(\Delta\Omega)$, are the visual solid angles of the unlensed and lensed sources, respectively. The value μ is referred to as a magnification factor. The gravitational lens produces a circle on the sky, when the source, lens and the observer are situated on one straight line, see fig.1. When a lens is shifted from the line, connecting the source in the observer, the circle is transforming into two separate arcs, which angular size is decreasing with increasing of this shifting, see figs.1,2.

3. Observations of gravitational lenses

The first gravitational lens was discovered in 1979 year by Walsh at al. (1979), in the optical observation of the quasar QSO 0957+561. This lens has one of the largest separation between the two images, equal to 6.1 arcsec, image redshift = 1.41; lens redshift = 0.36; brightness ratio of two images is equal to 2/3, see radio image in Harvanek et al. (1997). The separation in most lenses is usually about 1 arcsec.

Similarity of spectra of two images in fig.3 is a very convincing argument in favor of the lens origin of these two images, belonging to the same quasar.

When the lens is not a point mass, but have a complicated structure, the image may be also complicated. and consisting of more than two images, like in the case of fig.4. The lensing image, close the the classical Einstein ring is given in fig.5.

The visible complicated structure may be identified with lensing image of the same source when several criteria are fulfilled.

1. Two or more point images are close together on the sky

2. Arc or ring (extended sources) is observed.

3. Line flux-ratios and shapes that are similar for all images.

4. Redshifts that are he same for all images

5. Flux ratios in different spectral bands that are the same for all images.

6. A possible lens is visible in the vicinity of the images, but at redshift smaller than that of the images.

7. Temporal variations in different images that are correlated.

Many lensing images, produced by the rich galactic cluster Abell 1689 are presented in fig.6.

4. Strong gravitational lensing by a Schwarzschild black hole

This case was considered in the paper of Bisnovatyi-Kogan and Tsupko (2008). The shape of the orbit of a photon coming from the infinity to a black hole is determined by its impact parameter b. A detailed analysis of the photon orbits for all values of the impact parameter given by Mizner et al. (1973) reduces to the following picture.

1. If $b < 3\sqrt{3}M$, then the photon falls to the gravitational radius $R_s = 2M$ and is absorbed by the black hole.

2. If $b > 3\sqrt{3}M$, then the photon is deflected by an angle $\hat{\alpha}$ and flies off to infinity. Here there are two possibilities:

(a) If $b \gg 3\sqrt{3}M$, then the orbit is almost a straight line with a small deflection by an angle $\hat{\alpha} = 4M/R$, where R is the distance of closest approach. This is the case customarily exam-

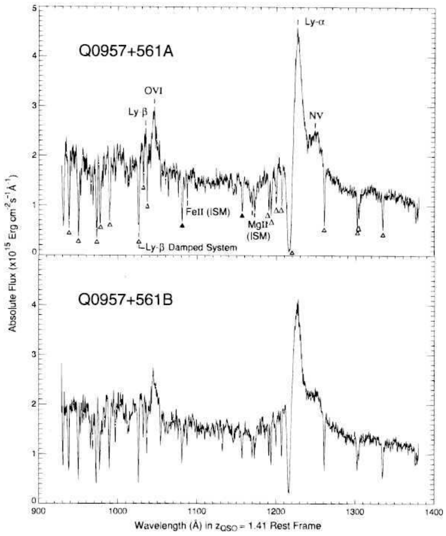

Fig. 3. Co-added spectra of lens components Q0957+561A (top) and B (bottom). $Ly\alpha$, O VI, and N V quasar emission lines are present. The wavelength scale corresponds to the quasar redshift $z_{QSO} = 1.41$. Absorption lines associated with the damped $Ly\alpha$ system at $z_{damped} = 1.3911$, and $Ly\alpha$ and O I absorption at $z_{Ly\alpha} = 1.1249$ are shown as open and filled , triangles, respectively. Interstellar absorption (ISM) from Mg and Fe II absorption is also indicated. Note the strong absorption associated with features formed in the damped $Ly\alpha$ system. QSO $Ly\beta$ emission is affected by the strong wings of O VI which degrades its flux. The absolute flux scale should be multiplied by a factor $(1 + z_{QSO})$to properly correct for the transformation into the quasar rest frame, from Michalitsianos (1997)

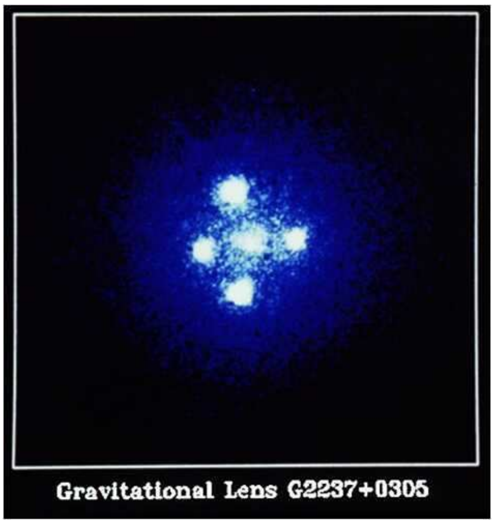

Fig. 4. Einstein's cross, HST FOS image, provided by NASA and ESA, accessed through `http://en.wikipedia.org/wiki/Gravitational_lens`.

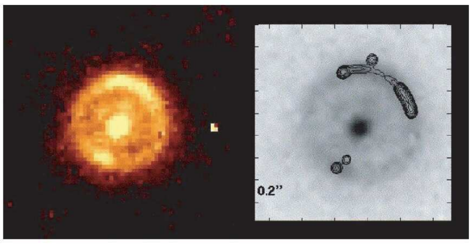

Fig. 5. The gravitational lens system B 1938+666. The left panel shows an image obtained by the experiment NICMOS on HST, of the system, clearly showing a complete Einstein ring into which the Active Galaxy is mapped, together with the lens galaxy situated near the center of the ring. The right panel shows the NICMOS image as gray-scales, with the radio observations superposed as contours. The radio source is indeed a double, with one component being imaged twice (the two images just outside and just inside the Einstein ring), whereas the other source component has four images along the Einstein ring, with two of them close together, from Schneider (2006).

ined in the theory of weak gravitational lensing, when the impact parameter is much greater than the Schwarzschild radius of the lens.

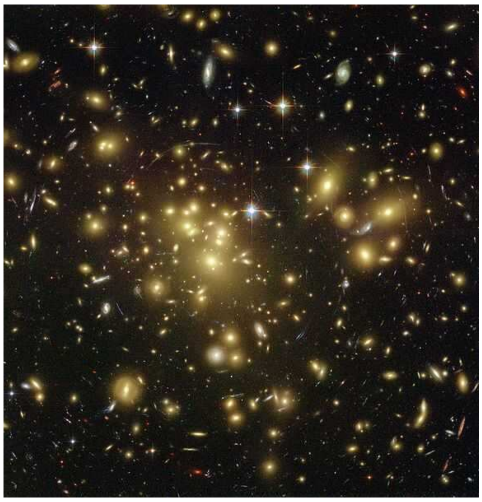

Fig. 6. Actual gravitational lensing effects as observed by the Hubble Space Telescope ACS in Abell 1689, image provided by NASA and ESA, accessed through `http://en.wikipedia.org/wiki/Gravitational_lens`.

(b) If $0 < b/M - 3\sqrt{3} \ll 1$, then the photon makes several turns around the black hole near a radius $r = 3M$ and flies off to infinity.

Let us consider the case when the source, lens (black hole) and observer lie along a straight line, fig.7. In this case inside the "main" Einstein ring, there are rings formed by photons which have been deflected by 2π, 4π, 6π etc. These rings are sometimes referred to as relativistic rings fig.8. Equating α to 2π, 4π, 6π, 8π, 10π we find that these relativistic rings are localized at the impact parameters

$$b/M - 3\sqrt{3} = 0.00653,\ 0.0000121,\ 0.0000000227,\ 0.423 \cdot 10^{-10},\ 0.791 \cdot 10^{-13}. \tag{3}$$

The corresponding distances of a closest approach are

$$R/M - 3 = 0.0902,\ 0.00375,\ 0.000162,\ 0.699 \cdot 10^{-5},\ 0.302 \cdot 10^{-6}. \tag{4}$$

The values of the impact parameters can also be obtained using the strong field approxi-

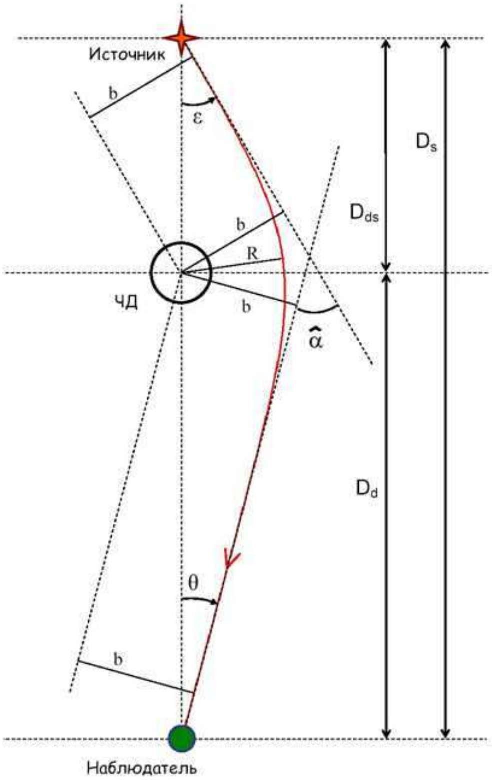

Fig. 7. The case when the source, lens (black hole) and observer lie along a straight line, $R_S = 2$, $R = 7$, $b \approx 8.3$, $D_d = 32$, $D_{ds} = 16$, from Bisnovatyi-Kogan and Tsupko (2008).

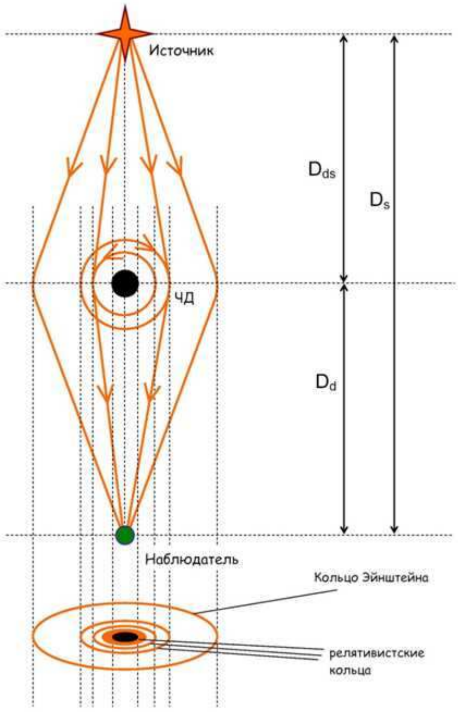

Fig. 8. The case when the source, lens (black hole) and observer lie along a straight line. The angular radius of the Einstein ring $\theta_0 = \sqrt{4M\frac{D_{ds}}{D_d D_s}}$. Relativistic rings are inside the Einstein ring, from Bisnovatyi-Kogan and Tsupko (2008).

mation, which is good (0.4% error)even for the first relativistic ring

$$\hat{\alpha}(b) = 2\pi n, \quad n = 1, 2, 3, ..., \quad (5)$$
$$\theta(n) = b_{cr}(1 + e^{C_1 - 2\pi n}), \quad C_1 = -0.40023.$$

The angular sizes of the relativistic rings are

$$b(n) = \frac{b_{cr}}{D_d}(1 + e^{C_1 - 2\pi n}), \; b_{cr} = 3\sqrt{3}M. \quad (6)$$

Regretfully, the magnification factors of the relativistic rings μ_n are incredibly small. It was found by Bisnovatyi-Kogan and Tsupko (2008)

$$\mu_n = 27\frac{R_s^2 D_s^2}{R_* D_{ds} D_d^2}(1 + e^{C_1 - 2\pi n})e^{C_1 - 2\pi n} \ll 1, \quad (7)$$

and $\mu_0 = 2\theta_0/\beta$. Here D_s, D_d, D_{ds} are indicated in figs.7,8, and R_* is a radius of the source (distant quasar), β is the observed angular size of the source. For the mass of the distant quasar $M_* = 10^9 M_\odot$, $R_* = 15R_{*s} = 30GM_*/c^2$, $D_{ds} = 10^3$ Mpc, D_d=3 Mpc, $D_s \approx D_d s$, mass of the lens $M = 10^7 M_\odot$ Bisnovatyi-Kogan and Tsupko (2008) have obtained

$$\mu_0 = 10^6, \; \mu_1 = 2 \cdot 10^{-15}, \; \mu_2 = 4 \cdot 10^{-18}. \quad (8)$$

5. Gravitational radiospectrometer

Using the formalism, developed by Synge (1960) the deviation angle $\hat{\alpha}$, depending on a frequency in a uniform plasma, was found for a weak lensing by Bisnovatyi-Kogan and Tsupko (2009) as

$$\hat{\alpha}_b = \int_0^\infty \frac{\partial}{\partial b}\left(h_{33} + \frac{1}{1-\omega_0^2/\omega^2}h_{00}\right)dz \qquad (9)$$
$$= -\frac{R_s}{b}\left(1 + \frac{1}{1-\omega_0^2/\omega^2}\right),$$

where b is the impact parameter, $\omega_0^2 = \frac{4\pi e^2 N_0}{m_e}$, ω is the photon frequency at infinity. Remind, that in the vacuum

$$\hat{\alpha} = \frac{2R_s}{b} = \frac{4GM}{bc^2}, \qquad (10)$$

for $b \gg R_s$. The angular separation between images α_0, according to Schneider et al. (1992), with definitions from figs.7,8, is written as

$$\alpha_0 = \sqrt{\frac{2R_s D_{ds}}{D_d D_s}}, \qquad (11)$$

with a typical angular separation between images about $1''$. In the uniform plasma we have from Bisnovatyi-Kogan and Tsupko (2009) and Bisnovatyi-Kogan and Tsupko (2009a)

$$\alpha_0 = \sqrt{\left(1 + \frac{1}{1-\omega_0^2/\omega^2}\right)\frac{R_s D_{ds}}{D_d D_s}}, \qquad (12)$$

It was shown by Kulsrud and Loeb (1992) that in plasma the photon with a frequency ω moves like a massive particle with a rest mass $m_{pl} = \omega_0$, energy $E_{pl} = \omega$, and velocity equal to the photon group velocity $v_{pl} = c\sqrt{1 - \frac{\omega_0^2}{\omega^2}}$. The dispersive properties of the lens in the homogeneous plasma can be derived, using this analogy, see Bisnovatyi-Kogan and Tsupko (2009a)

6. Perspectives for the radioastron

Radioastron is a Russian project of the space radio telescope for making VLBI observations with an angular resolution up to 10^{-5} arcssecond. It should have four frequency bands with the lowest frequency ω_{min} = 327 MHz. Angular difference in the same image between the optical image (vacuum case), and radio image at frequency ω_{min} = 327 MHz, will be equal to 0.00001 arcsec, when the rays propagate through plasma at $N_e \sim 50000$. This estimation is done for the lensing angle equal to 1 arcsec.

Different lensing images of the same source may have different spectra in the radio band, when the light of different images propagates through regions with different plasma density. This difference is connected with a dependence of the amplification factor on the lensing angle. For the Schwarzschild lens the amplification factor is proportional to the angular radius of the Einstein ring, which increases when the frequency approaches the plasma frequency.

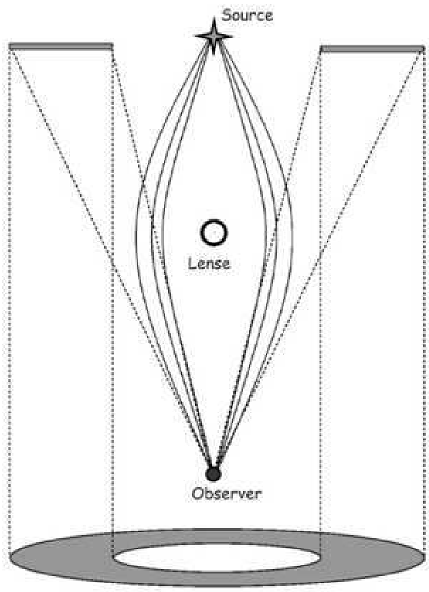

Fig. 9. Axis on lensing by the Schwarzschild point-mass lens. The case of the Einstein ring. Instead of a thin ring corresponding to the vacuum lensing (the inner circle of the ring) we have a thick ring, formed by the photons of different frequencies. from Bisnovatyi-Kogan and Tsupko (2009).

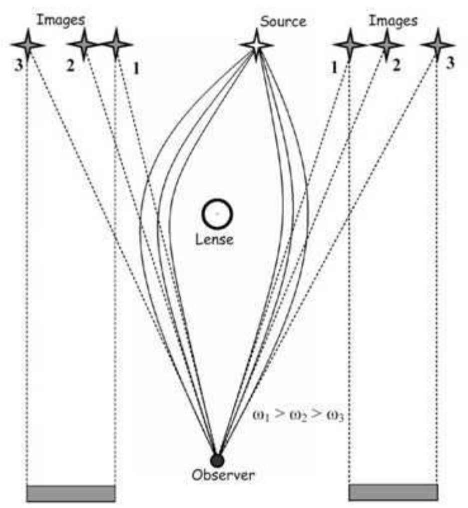

Fig. 10. Lensing of the point source by the Schwarzschild point-mass lens. Instead of two point images due to lensing in the vacuum we have two line images. The pairs of images, corresponding to the same photon frequency, are indicated by the same numbers. Two images with number 1 correspond to the vacuum lensing. from Bisnovatyi-Kogan and Tsupko (2009).

7. Conclusions

At lensing of point sources spectra of images may be different in the long wave side; extended image have different spectra along the image. Besides the refraction, the presence of plasma changes the time of the light propagation of different images. This property implyes the spectrum of fluctuations of the CMB to be slightly different for different wavelengths, in the long wave side. Plasma influence also timing effects in binary relativistic systems, similar to the double pulsar system J0737-3039A - J0737-3039B.

Regretfully, all plasma effects in gravitational lensing are very small and their observations are scarcely possible at the moment.

8. DISCUSSION

BOZENA CZERNY: I doubt whether the plasma effects on light propagation you discussed can be tested. the plasma become optically thick due to synchrotron self absorption at frequencies you requested. This is certainly for Sgr A*.

GENNADY BISNOVATYI-KOGAN: Yes, different kinds of absorption, as well as refraction properties of the nonuniform plasma contaminate this effect. Special conditions should exist for possibility to detect it observationally.

Acknowledgements. The author is grateful to Oleg Tsupko for help and cooperation; to F. Giovannelli, and other organizers of the workshop for support and hospitality. This work was partially supported by RFBR grants 08-02-00491, 08-02-90106, and President grant for a support of leading scientific schools 2977.2008.2.

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