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The electrostatic screening of nuclear reaction - present status

Giora Shaviv

Dept. of Physics, Israel Institute of Technology, Haifa 32000, Israel

Abstract. We review the present state of the weak electrostatic screening theory of nuclear reactions in dense astrophysical plasma of Main Sequence stars.

Key words. Stars: Screening – Plasma: Debye potential – Nuclear reactions – Mean field

1. Introduction

In the 2001 Vulcano conference, we reported (Shaviv & Shaviv, 2001b, S&S) on the ongoing controversy about the proper treatment of the screening of nuclear reactions in stellar plasma in general, and in the Sun in particular. The discussion about the screening continued for a while. However, recently, Mussack *et al.*, (2007) and Mao *et al.*, (2009) checked the assumptions of Shaviv & Shaviv, (2001a) (hereafter SS01) and confirmed their validity, as well as the basic physical results. Effectively, this puts an end to the controversy and opens a new era in the calculations of the screening in stars. In view of this development, we summarize here the involved physics and report numerical results for the screening of nuclear reactions.

The effect of the screening of nuclear reactions is based on the classical Debye-Hückel (DH) theory. Each positive ion in the plasma is surrounded with a cloud of negative and positive ions, creating an effective neutralizing potential of the form $\exp(-r/R_D)/r$, where R_D is the Debye radius. Thus, each ion in the plasma is immersed in a neutralizing cloud.

In liquid electrolytes, for which the DH theory was developed, the number of particles within the Debye sphere is very large and a mean field description is valid. However, the conditions inside Main Sequence stars imply that the number of particles (ions or electrons) is only a few, and the necessary condition for a mean field description is not satisfied.

The relevant particles for the nuclear reactions are in the Gamow peak, which as a rule is at an energy greater than kT (e.g., $5kT$ for the pp-chain, and $15-20kT$ for higher Z reactions). Thus, the colliding particles are generally much faster than the speed of the thermal particles comprising the Debye cloud.

Salpeter, (1954) assumed the so called weak screening limit, which corresponds to the DH mean field theory, and stated the approximations involved. In fact, Salpeter, (1954) has already stressed the points raised here. The following questions arise:

- If no nuclear reaction takes place, then the extra energy gained must be returned to the plasma. How can this happen when the soft Debye cloud contains just a few particles?
- The negative electrons are even faster than the fast ions in the Gamow peak, while the

positive ions are slower. What is the effect of the net polarization during the collision?
- How could a Debye neutralizing sphere that is composed of thermal particles screen a particle which is faster than all of them?

In summary, the collision between fast particles takes a shorter time than the classical relaxation time for the Debye cloud. Is mean field description valid on this short time scale?

S&S raised exactly these questions and claimed that indeed, the mean field description is not appropriate under these conditions, and consequently, the Salpeter theory for screening requires modifications. S&S proposed to address the problem using Molecular Dynamics, an "ab initio" method which bypasses the question of basic assumptions.

The problem of the weak electrostatic screening of nuclear reactions was shrouded in uncertainty. The results of S&S were challenged by Brueggen & Gough, (1997), Gruzinov & Bahcall, (1998) and Bahcall *et al.*, (1998), who published a series of papers claiming the correctness of the Salpeter weak screening limit under solar conditions.

The second question was raised by Carraro *et al.*, (1988) who pointed out that the potential energy of a particle moving with the Gamow energy is not identical to the potential energy of a thermal particle and hence the screening should be reduced. It can be shown that the above authors essentially assumed a particle with an infinite mass and hence ignored the back reaction from the plasma on the particle. In other words, the thermal particles in the cloud cannot relax sufficiently fast during a collision of faster than thermal particles.

Recently, Bahcall *et al.*, (2002, BBGS) discussed the Salpeter formula for calculating the electrostatic screening of nuclear reactions in plasma and argued to have shown that it should apply to the Sun and that all claims concerning dynamic effects are wrong, including the recent results of SS01. The discussion of BBGS is based on thermodynamics and statistical mechanics in which the mean field electrostatic potential is assumed. Further, the authors assume that two interacting ions *always* gain energy from the plasma, as the particles approach each other. This additional energy, the screening energy, is taken to be just the mean field potential of the ions in the plasma, namely, the *long time average* (the thermodynamic time - much longer than a typical collision) of the potential felt by an ion in the plasma. As the particles move apart the tacit assumption is that this extra energy is *exactly* returned to the plasma.

The source of the problem to our mind, is the confusion between the static and long time averaged behavior and the dynamic behavior of dense plasma. BBGS are in the idea that the screening problem is a static one while we claim that under the conditions in the Sun, it is a dynamic one, and should be treated as such. In Sahrling (1994) words, the classical Salpeter theory assumes "the approximation that the ions in the screening cloud are always fully relaxed" (See also Salpeter, 1954).

SS01 have shown how to obtain the screening energy from first principles using ab initio Molecular Dynamics (hereafter MD). SS01 carried out extensive MD simulations that show that on the *average*, particles with low relative energy gain energy from the plasma when they scatter off each other, while particles with high relative energy lose on the average energy to the plasma, as they scatter. The sum of the exchange over all particles vanishes in equilibrium. SS01 showed the connection between the screening and the dynamic friction in the system.

The enhancement of nuclear reactions in plasma involves the fundamental physics of particles collisions in plasma. Frequently the effect of the surrounding particles is summed into a mean field.

- Can we treat this field as a rigid field attached to the particles, such that the particles effectively scatter off each other with a revised central potential?
- How far can the collision between two particles be considered as a binary collision, while ignoring non central effects which cause a change in the motion of the center of mass?

– To what extent does the condition of particles within a plasma in equilibrium affects the collisions?

The above, as well as other similar questions, are connected to the relaxation process in the plasma, and the rate enhancement cannot be separated from these questions.

To demonstrate the physics, we start with a simple case of a single specie but a modified Coulomb potential which takes into account the screening by the electrons. The simple model allows the understanding of the collision mechanism in a gas in thermal equilibrium but with few particles in the screening cloud. Once we demonstrate the effect in a simplified case, we turn to the actual plasma and (1) Infer the critical properties of the collisions between plasma particles, (2) Define again the screening energy, (3) Discuss typical results and (4) Compare with the classical Salpeter result.

2. The conditions in the Sun

The necessary condition for the Debye mean field theory to be valid is that $N_D = (4\pi/3)n_e R_D^3 \gg 1$ where n_e in the number density of electrons and R_D is the Debye radius. In the case of the Sun we find that $N_D \sim 2-3$, namely the number of particles in a Debye sphere is small. Landau & Lifshitz, (1958) p 231 expressed it as follows: *The Debye-Hückel radius must be large compared with the distance between ions*. They also point out that this condition is equivalent to assuming that the Coulomb potential energy is small.

3. A simple demonstration of the physics

In view of the complexity of the problem and the mixing of internal relaxation processes in the gas with the screening phenomenon, we run a set of simple minded cases in which we control the interaction length or radius of neutralization R_n. By varying R_n we can see how the relaxation process and the screening change.

3.1. The neutralization radius idea

Wolf *et al.*, (1999) developed the idea of a neutralization radius to overcome the problem of truncated Coulomb potential and in an attempt to avoid the lengthy Ewald sum calculations. Here we borrow the same idea to see how the screening comes into play in a system of interacting particles. To this goal, we assume the effective binary potential to be given by:

$$V(r) = \begin{cases} 1/r + r/R_n^2 - 2/R_n^2 & \text{for} \quad r < R_n \\ 0 \quad \text{for} & \geq R_n \end{cases} \tag{1}$$

and the force is given by:

$$f(r) = \begin{cases} 1/r^2 - 1/R_n^2 & \text{for} \quad r < R_n \\ 0 \quad \text{for} & \geq R_n \end{cases} \tag{2}$$

where R_n is the neutralization radius and is taken here as a free parameter. Clearly, when $R_n \to 0$ we get a short range force and in principle, no screening is expected.

The fundamental processes taking place in the plasma are seen in fig. 1. $\langle \Delta E(E) \rangle$ is the mean energy exchange between a particle with energy E and the plasma. The curves $'$down$'$ and $'$up$'$ give the probability for the particle to lose or gain energy, respectively. $n(E)$ is the number of particles in the energy bin. We see that energetic particles have a higher probability to lose energy in a binary collision and vice versa.

We turn now to the mean potential energy of particles with a given kinetic energy. In fig. 2 and 3 we show the results for $R_n = 0.4\,\langle r \rangle$ and for $R_n = 0.8\,\langle r \rangle$, where $\langle r \rangle$ is the mean interparticle distance. The figures show that the potential energy at the classical turning point is a function of the relative kinetic energy and exhibits a fast rise for very slow encounters.

3.2. The distribution of the potential energy of particles

The number of particles in a Debye sphere is given by:

$$N_D = \frac{4\pi}{3} n_e R_D^3; \quad R_D \approx 6.9\sqrt{\frac{kT}{n_e}}. \tag{3}$$

This expression assumes that both the electrons and the ions contribute to the Debye potential.

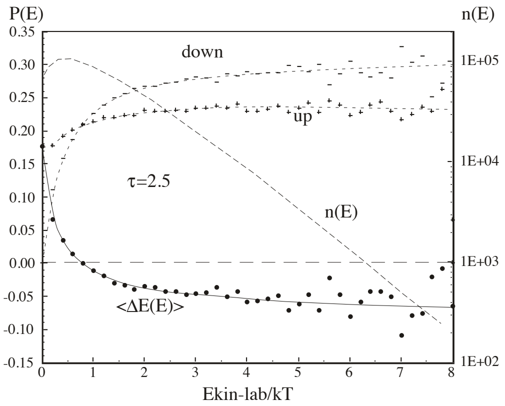

Fig. 1. The dynamic friction in the plasma. Energetic particles mainly lose energy in collisions and low energy particles gain energy. $\langle \Delta E(E) \rangle$ is the average energy change in the E energy bin.

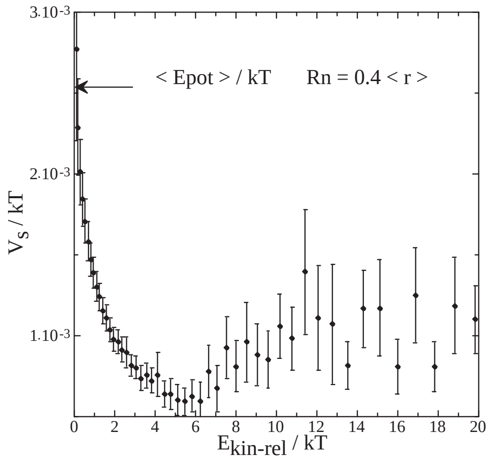

Fig. 2. The screening and ensemble mean potential felt by particles with given relative kinetic energy at the classical turning point. The arrow marks the ensemble mean potential felt by a particle in the system. Here $R_n = 0.4 \langle r \rangle$.

and the plasma contains only trace amounts of heavy elements. Assuming pure Hydrogen plasma, $n_e = 10^{26}$ and $T = 1.5 \times 10^7 K$ we find that in the core of the Sun $R_D = 0.877 \langle r \rangle$ where $\langle r \rangle = n_{ion}^{-1/3}$ and $N_D = 2.83$. Actually, equating N_D to unity yields a condition on the number density of electron below which the mean field theory of the Debye cloud is certainly not valid. The conditions at the centers of

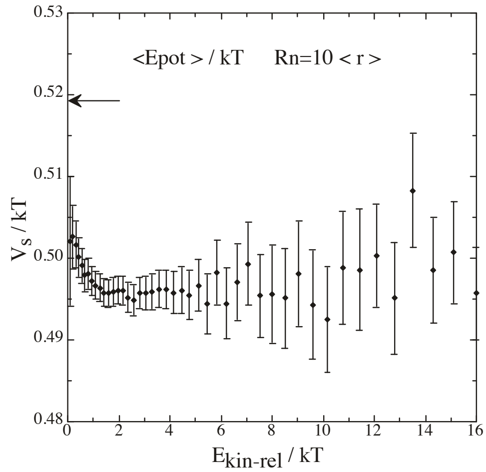

Fig. 3. The screening and ensemble mean potential felt by particles with given relative kinetic energy at the classical turning point. The arrow marks the ensemble mean potential felt by a particle in the system. Here $R_n = 10 \langle r \rangle$.

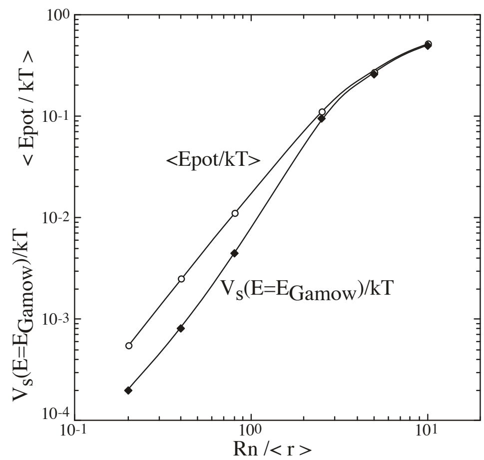

Fig. 4. The ensemble mean potential and the mean screening energy at the Gamow peak for the pp reaction as a function of the interaction rang. Here $C_f = 1$.

Main sequence stars fall very close to this condition. Alternatively, one can implement the condition given by Landau & Lifshitz, (1958, p. 230), namely,

$$\ll \left(\frac{kT}{z^2 e^2} \right)^3 , \qquad (4)$$

where z is the mean atomic change of the ions.

In fig. 4 we see how the mean field limit, where $N_D \to \infty$ is obtained as R_n increases. The open circles are the results obtained applying the mean field (even with a too small number of particle, while the full circles show the results obtained for relatively small values of R_n, for which the mean field is not expected to be valid. The difference in the potential energy is in this case about a factor of four.

As the distribution function of the particles is $F_N \propto \exp(-\Sigma_{ij}\phi_{ij}/kT)$, where ϕ_{ij} is the Coulomb interaction between two particles, one expects that the potential energy of the single particle will have also a distribution (in contrast to a constant value). Namely, not all particles have the same potential energy. The Salpeter's approximation assumes a constant E_{pot}. Indeed, in fig. 5 we show the potential energy distribution in equilibrium (after more than 50 dynamic time scales) found in a snap shot. The width of the distribution is proportional to $1/\sqrt{N_D}$ which in the present case is slightly less than unity. According to Ginzburg, (1960), the fact that the distributions have a significant width (due to the small number of particles giving rise to the potential) means that fluctuations are large the mean field is not a good approximation in this case. (cf. Landau & Lifshitz, 1958)

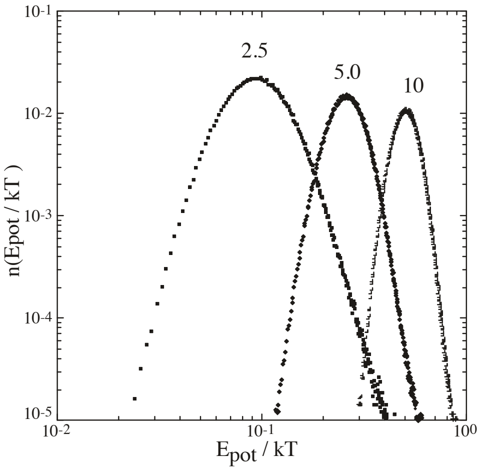

Fig. 5. The distribution of the potential energy among the particles for $R_n > 1$. (Obtained in a snap shot calculated over all particles irrespective of their particular dynamic state).

4. A simple definition of the screening

The Molecular Dynamic method allowed us to define the screening energy in a simple and clear way.

Let $E_i^{tot-L,f}$ be the total energy of particle i when it is moving in the plasma and $E_i^{tot-L,c}$ the total energy when it reaches the distance of closest approach during the scattering off particle j. The screening energy is then given by:

$$E_{i,scr} = E_i^{tot-L,c} - E_i^{tot-L,f}, \quad (5)$$

where the total energy of the particle in given by

$$E_i^{tot,L} = E_i^{kin} + \sum_{k\neq i} \phi(i,k) + \frac{1}{2}\phi(i,j), \quad (6)$$

namely, the kinetic energy plus all the potential interactions between all pairs plus the specific interaction between the scattering pair. Here all energies are evaluated in the laboratory. The screening energy is the energy gained/lost by the scattered proton as it moves from far away to the distance of closest approach. This definition is very close to the original definition given by Salpeter, (1954) to the screening energy. (cf. Salpeter, (1954) eq. 2, where the expression for the energy is given essentially in the center of mass system). It is equally simple to define the screening energy in the center of mass system of the two scattering particles. However, as in the next section we solve the equations in the laboratory system, we use here a definition in the laboratory system.

Other definitions exist as well. For example, Ichimaru, (1994) defines the screening potential as follows: Let $W(r)$ be the potential between a pair of particles in vacuum and let $g(r)$ be the pair distribution function, then the screening potential (or interaction potential) is given by:

$$H_{int}(r) = W(r) + \frac{1}{\beta}\ln g(r), \quad (7)$$

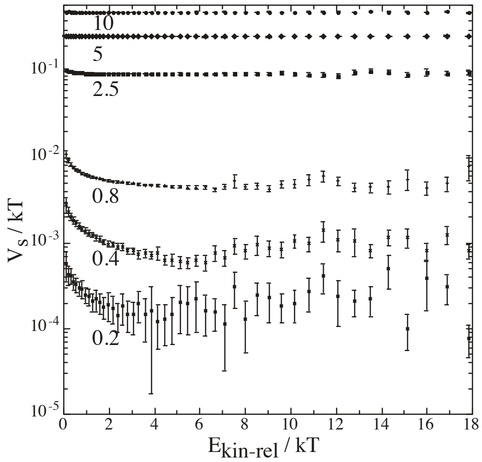

Fig. 6. The potential energy V_s as a function of the kinetic energy for various values of R_n.

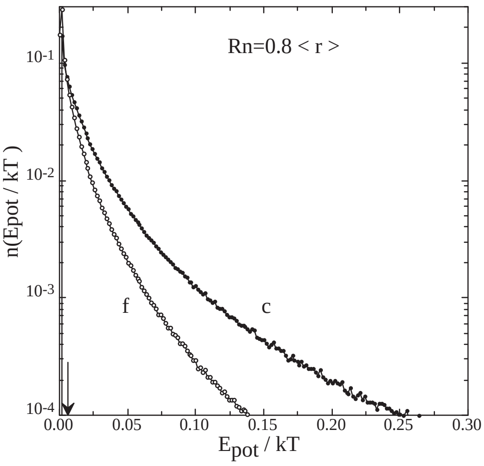

Fig. 7. The potential energy distribution for $R_n = 0.8\langle r\rangle$. The letters f and c imply far and close (at the classical turning point). The arrow marks the ensemble mean.

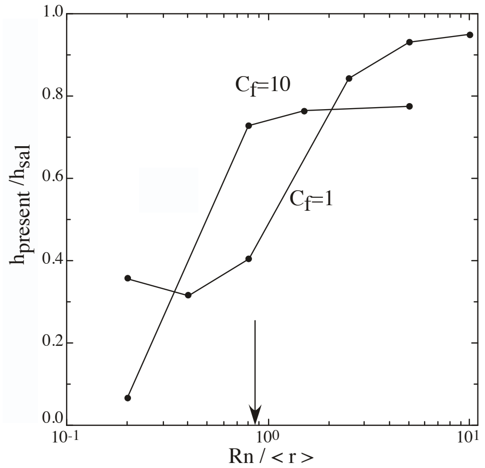

Fig. 8. The ratio of the new screening enhancement factor as a function of the range of the potential for two values of the potential strength.

where $\beta = 1/kT$. Again, this definition can by applied provided the mean field approximation is valid.

The MD code was adapted to calculate the screening using the above definition. The scattering particles are first identified and then followed to the distance of closest approach, where all the dynamic quantities are registered. Then the moving away pair is followed till a point which is few Debye lengths away. Checks about the actual point where to declare the pair as "far away— were carried out and we found that 3-4 Debye lengths are sufficient. One can follow the pair to a longer distance at the cost of a larger scatter in the results. In all cases we require that the distance of closest approach be smaller than $1/2\langle r\rangle$. Hence, very low energy particles are ignored.

The polarization of the screening shielding cloud is nicely seen in fig. 7, where the relative potential energy between the colliding particles is shown when they are far away (f) and when they are at the classical turning point (c). The arrow marks the ensemble average value.

In fig. 6 we show the results for the screening energy as a function of the relative kinetic energy at maximum separation for several values of R_n. It is obvious that the effect (the variation with relative kinetic energy) is largest for small R_n and diminishes as R_n increases. The non linearity of the effect is shown in fig. 8, where $E_{scr,present}/E_{scr,Salpeter}$ is depicted as a function of R_n for two values of the strength of the interaction.

It is interesting to consider what happens to the energy of the center of mass during the scattering process, namely $E_{CM} = 2m_p v_{CM}^2$, where m_p is the proton mass and v_{CM} is the velocity of the center of mass. If the scattering would have been via a central force, then

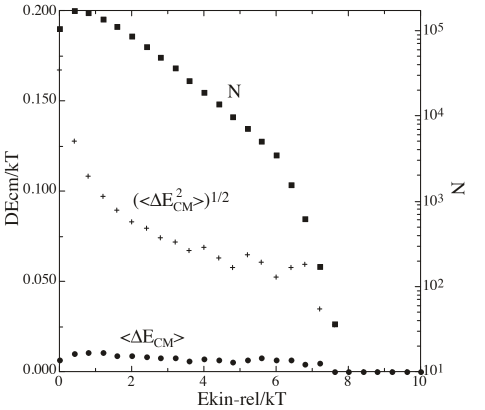

Fig. 9. The change in the center of mass energy $\Delta E^2_{cm}(E)$ as a function of the relative kinetic energy of the scattering particles. The long time average of $\Delta E_{cm}(E)$ tends zero.

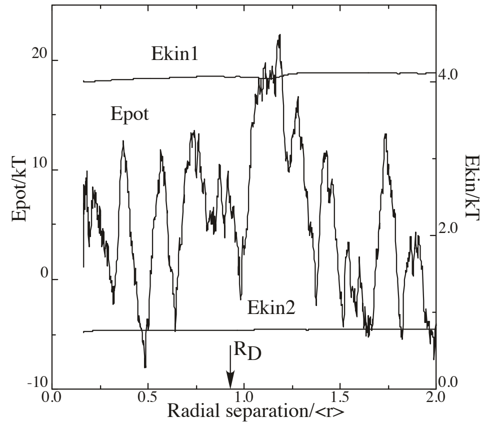

Fig. 10. The evolution of the potential and kinetic energy of two colliding protons in a typical scattering. The kinetic energies are the absolute kinetic energies and the potential energy is the potential energy between the two colliding particles.

the energy of the center of mass should be conserved. The average change in E_{CM} squared is shown in fig. 9 as a function of the relative kinetic energy.

The change in E_{CM} tends to zero. The conclusion is that three (or few) body interaction gives rise to a change in the energy of the center of mass of the scattering pair of particles. Alternatively, the fluctuations in the plasma are not spherically symmetric about the radius vector joining the two particles (they should not be so). In fig. 10 we show the evolution of the energies during a particular scattering of two protons starting from the classical turning point at about $0.35\,\langle r\rangle$. The evolution is presented as the function of the separation distance. Given are the potential energy and the kinetic energies (in the laboratory) of the two particles. The Debye radius is marked with an arrow. We see that the potential energy fluctuates during the scattering and the field is far from being smooth. Note that the amplitude of the fluctuations is large but because of the short time duration, the total kinetic energy change of the particles is small.

In fig. 11 we show the Fourier transform of the potential felt by a colliding particle as a function of time. Marked are the plasma frequency and the tunneling frequency indicating

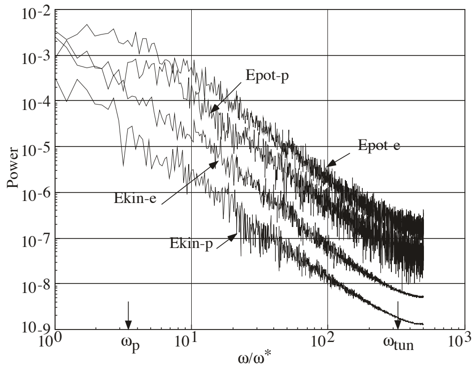

Fig. 11. The potential energy of the particles is a fluctuating function of time. ω_p is the plasma frequency and ω_{tun} is the tunneling frequency (derived from the time for complete tunneling). ω^* is reference frequency.

that the potential changes during the tunneling through the potential.

5. The screening in the real solar plasma

The actual results for several nuclear reactions in the solar plasma are given in tables 1 for several compositions. The results for the pp

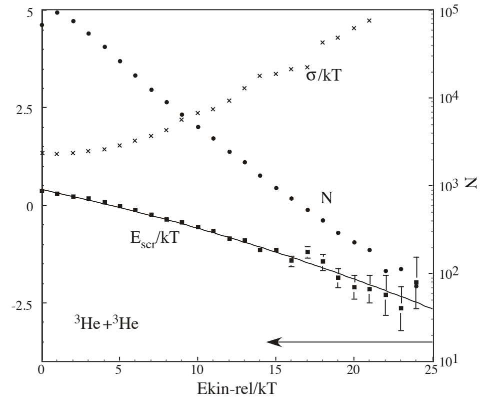

Fig. 12. The screening of He^3 + He^3. Note how the screening is a function of the relative energy between the particles. In the statistical limit, the screening does not depend on the relative energy between the particles. The horizontal arrow marks the range of the Gamow peak.

screening can be aproximated by:

$$F_{sc}(X = 0) = F_{\mathrm{Sal}}\times(0.59R_D+0.183)\times(1\pm0.05).$$

A graphical comparison between the present result and Salpeter's is shown in fig. 13. In table 2 we give the screening factor for unrealistic enhanced concentrations of ^{7}Be so that the screening factor for the ^{7}Be + p can be calculated. Finally, in table 5 we give the results for an enhanced amount of ^{12}C. The presence of ^{12}C and a strong effect on the result. For example the reactions ^{4}He +^{3} He and ^{12}C +^{12} C are suppressed rather then enhanced. The difference in the last reaction amounts to a factor of 84!

6. Conclusions

The particular conditions in Main Sequence stars are such that the number of particles in the Debye sphere is not sufficiently large and consequently, the fundamental assumption on which the theory of the mean field rests, is not valid. We showed how the mean field limit is obtained as $N_D \to \infty$.

The cloud surrounding each ion is composed mainly of thermal particles while the relevant particles for the nuclear reactions are the fast particles in the Gamow peak.

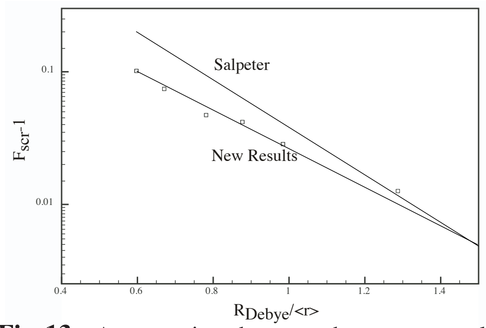

Fig. 13. A comparison between the present result for the pp reaction and the Salpeter's result. Plotted is the screening factor minus unity.

Consequently, polarization effects become important.

We showed how the method of molecular dynamics provides a way to handle the problem.

7. DISCUSSION

Antonino del Popolo: Is the plasma in your calculation homogenous? It is well known that there is clustering.

Giora Shaviv: Yes, you are right. The plasma is homogeneous on a large scale but non-homogenous on the scale of few mean inter-particle distances. We start the calculation with a homogenous distribution and run it for at least 5 relaxation times before we start our screening calculation.

Gennadi Bisnovatyi-Kogan: How do your results influence the interpretation of the solar neutrino experimets?

Giora Shaviv: We have to calculate the new screening for different densities, temperatures and compositions. Our goal is both to re-calculate the solar neutrino and solar models, as well as to extend the calculations to denser plasmas relevant to thermo-nuclear-runaways on white dwarfs. At the moment we are busy in accelerating the calculation which takes few months each.

Table 1. Comparison of screening factors between the Salpeter classical screening factor (S) and the new results (N) for $T_7 = 1.5$

n_{26}	X	^{4}He	^{4}He	calc.	pp	^{4}He + p	^{3}He + p
1.0	0.902	0.056	0.042	N	1.024	1.111	1.092
				S	1.064	1.133	1.133
0.5	0.902	0.056	0.042	N	1.022	1.088	1.048
				S	1.050	1.103	1.103
0.5	0.700	0.173	0.130	N	1.027	1.060	1.065
				S	1.047	1.096	1.096
0.5	0.767	0.098	0.098	N	1.022	1.072	1.070
				S	1.050	1.103	1.103
1.0	0.902	0.056	0.042	N	1.034	1.086	1.052
				S	1.063	1.129	1.129

Table 2. Comparison of screening factors between the Salpeter classical screening factor (S) and the new results (N) for $T_7 = 1.5$ and enhanced ^{7}Be

n_{26}	X	Y	^{7}Be	calc.	pp	^{4}He + p	^{7}Be + p
0.5	0.428	0.305	0.267	N	1.027	1.091	1.297
				S	1.059	1.122	1.402
1.0	0.384	0.384	0.287	N	1.038	1.083	1.338
				S	1.088	1.184	1.402

Table 3. Screening factors for several reactions with composition: $X = 0.573, Y = 0.214, C^{12} = 0.214$, $T_7 = 1.5$ and $n_{26} = 1$.

model	^{4}He + p	^{12}C + p	^{4}He +^{4} He	^{12}C +^{12} C	^{12}C +^{4} He
N	0.952	1.027	0.812	0.191	0.836
S	1.167	1.588	1.361	16.03	2.521

John Beckman: Can the screening affect element synthesis in the Big-Bang and in particular the synthesis of Lithium?

Giora Shaviv: The density in the Big-Bang at the time the temperature is fine for the synthesis of Lithium, is much lower than the density in stars and hence, practically no screening exists in the Big-Bang nucleosynthesis.

James Beall: A comment and a question. Comment: your screening potential is identical to the one calculated by Rutherford for scattering of α particles onto gold foil. Question:

How does the calculation change for heavier elements?

Giora Shaviv: Heavier elements show a greater effect. But it all depends on the composition. If you have a composition rich in hydrogen and few heavy elements, in a collision between the heavy particles and say the proton, the Debye cloud is composed mainly of light protons. The situation is different when say, most of the ions are heavy, because then the masses of the particles are equal. Note from the results I show in the table that for heavier elements the effect is larger and may even be negative. Namely, a slow down rather then enhancement.

Bozena Czerni: Helioseismology in principle gives strong constraints for the temperature profile in the sun. Do you plan to confront your model with such data?

Giora Shaviv: When we finish the calculation of the screening and the resulting equation of state we are going to do just that.

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