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Constraints on the variations of fundamental couplings by stellar models

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Abstract. The effect of variations of the fundamental constants on the thermonuclear rate of the triple alpha reaction, ^{4}He$(\alpha\alpha,\gamma)^{12}$C, that bridges the gap between ^{4}He and ^{12}C is investigated. We follow the evolution of 15 and 60 $M_\odot$ zero metallicity stellar models, up to the end of core helium burning. They are assumed to be representative of the first, Population III stars which differ from later Pop I and II stars in their evolution due to the absence of initial CNO elements (zero metallicity). The calculated oxygen and carbon abundances resulting from helium burning can then be used to constrain the variations of the fundamental constants.

1. Introduction

The equivalence principle is a cornerstone of metric theories of gravitation and in particular of General Relativity. This principle, including the universality of free fall and the local position and Lorentz invariances, postulates that the local laws of physics, and in particular the values of the dimensionless constants such as the fine structure constant, must remain fixed, and thus be the same at any time and in any place. It follows that by testing the constancy of fundamental constants one actually performs a test of General Relativity, that can be extended on astrophysical and cosmological scales (for a review, see Uzan 2003).

The synthesis of complex elements in stars (mainly the possibility of the 3α-reaction as the origin of the production of ^{12}C) sets constraints on the values of the fine structure and strong coupling constants. This effect was investigated for red giant stars (1.3, 5 and 20 $M_\odot$ with solar metallicity) up to thermally pulsing asymptotic giant branch stars (TP-AGB) (Oberhummer et al. 2001) and in low, intermediate and high mass stars (1.3, 5, 15 and 25 $M_\odot$ with solar metallicity) up to TP-AGB (Schlattl et al. 2004). It was estimated that outside a window of 0.5% and 4% for the values of the strong and electromagnetic forces respectively, the stellar production of carbon or oxygen will be reduced by a factor 30 to 1000. We consider here (see Ekström et al. 2000), instead, the very first generation of stars which are thought to have been formed a few $\times 10^8$ years after the big bang, at a redshift of $z \sim 10 - 15$, and with *zero initial metallicity*. For the time being, there are no direct observations of those Population III stars but one may expect that their chemical imprints could be observed in the most metal-poor halo stars.

2. The triple–alpha reaction

Figure 1 displays the low energy level schemes of the nuclei participating in the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ reaction: ^{4}He, ^{8}Be and ^{12}C. First, two alpha particles fuse to produce a ^{8}Be nucleus whose lifetime is only $\sim 10^{-16}$ s, but sufficient to allow for a second alpha capture to the second excited level of ^{12}C, 7.65 MeV above the ground state. This corresponds to a $\ell = 0$ resonance as postulated by Hoyle in order to increase the cross section during the helium burning phase (Hoyle 1953). This level normally decays towards the first excited ^{12}C level at 4.44 MeV through a radiative transition. This reaction is very sensitive to the position of the Hoyle state as the corresponding resonance width is very small (a few $\sim$eV) as compared with the competing reaction $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$, dominated by broad ($\sim$100 keV) resonances and subthreshold levels. Hence, in this work, we will only consider variations of the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ reaction rate.

Assuming *i*) thermal equilibrium between the ^{4}He and ^{8}Be nuclei, so that their abundances are related by the Saha equation and *ii*) the sharp resonance approximation for the alpha capture on ^{8}Be, the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ rate, can be expressed (Iliadis 2007) as:

$$N_\mathrm{A}^2\langle\sigma v\rangle^{\alpha\alpha\alpha} = 3^{3/2} 6 N_\mathrm{A}^2 \left(\frac{2\pi}{M_\alpha k_\mathrm{B} T}\right)^3 \hbar^5 \omega\gamma \exp\left(\frac{-Q_{\alpha\alpha\alpha}}{k_\mathrm{B} T}\right) \quad (1)$$

with $\omega\gamma \approx \Gamma_\gamma(^{12}\mathrm{C})$, the radiative partial width of the Hoyle level and $Q_{\alpha\alpha\alpha}$ =380 keV its energy relative to the triple alpha threshold. The variation of $Q_{\alpha\alpha\alpha}$ with the nucleon nucleon interaction dominates the variation of the reaction rate in Eq. (1). However, a more accurate calculation (Nomoto & Thielemann 1985) requires a numerical integration, as done by Angulo et al. (1999), to take into account *i*) the two step process, two alpha particle fusion into the ^{8}Be ground state at a resonance energy of $E_R(^8\mathrm{Be})$ followed by another alpha capture to the Hoyle state at a resonance energy of $E_R(^{12}\mathrm{C})$ [$Q_{\alpha\alpha\alpha} \equiv E_R(^8\mathrm{Be}) + E_R(^{12}\mathrm{C})$] and *ii*) the energy dependent finite widths of those two resonances.

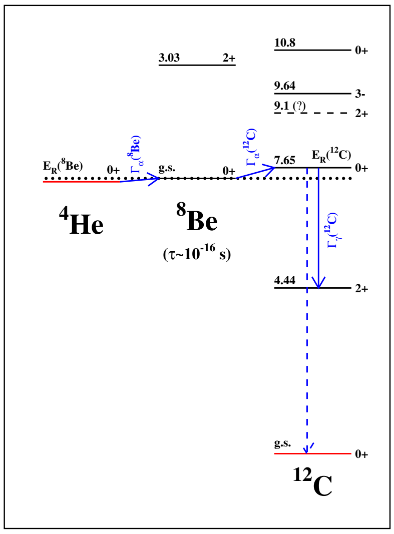

Fig. 1. Level scheme of nuclei participating to the $^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$ reaction.

In order to analyze their variation with the nuclear interaction, we have used a microscopic cluster model (see Korennov & Descouvemont 2004, and references therein). The Hamiltonian of the system is given by

$$H = \sum_i^A T(\mathbf{r_i}) + \sum_{j<i=1}^A V(\mathbf{r_{ij}}), \quad (2)$$

where A is the nucleon number, and $T(\mathbf{r_i})$ the kinetic energy of nucleon *i*. The nucleon-nucleon interaction $V(\mathbf{r_{ij}})$ depends on the relative coordinate $\mathbf{r_{ij}} = \mathbf{r_i} - \mathbf{r_j}$, and is written as

$$V(\mathbf{r}_{ij}) = V_C(\mathbf{r}_{ij}) + (1 + \delta_{NN})V_N(\mathbf{r}_{ij}). \quad (3)$$

where $V_C(\mathbf{r})$ is the Coulomb force, $V_N(\mathbf{r})$ the nuclear interaction (Thompson et al. 1977), and δ_{NN} the parameter that characterizes the change in the nucleon-nucleon interaction. (The variation of the Coulomb interaction is assumed to be negligible compared to the nuclear interaction.) When $A > 4$, no exact solution can be found and approximate solutions have to be constructed. For those cases, we use a cluster approximation in which the wave functions of the ^{8}Be and ^{12}C nuclei are approximated by a cluster of respectively two and three α particle wave functions. This approach has been shown to be well adapted to cluster states, and in particular to the ^{8}Be and ^{12}C levels of interest (Suzuki et al. 2007).

We obtained:

$$\Delta B_D / B_D = 5.7701 \times \delta_{NN} \quad (4)$$

for the binding energy of deuterium,

$$E_R(^8\mathrm{Be}) = (0.09208 - 12.208 \times \delta_{NN})\ \mathrm{MeV} \quad (5)$$

and

$$E_R(^{12}\mathrm{C}) = (0.2877 - 20.412 \times \delta_{NN})\ \mathrm{MeV} \quad (6)$$

for the resonance energies.

The strong energy dependence of the particle widths $\Gamma_\alpha(E)$ follows the Coulomb penetrability factor (Iliadis 2007) while the radiative width $\Gamma_\gamma(E)$ scales as $E^{2\lambda+1}$ where λ is the multipolarity (here 2 for $E2$) of the electromagnetic transition. Taking into account the variation of $E_R(^8\mathrm{Be})$ and $E_R(^{12}\mathrm{C})$ and of the partial widths, as a function of δ_{NN}, after numerical integration, we obtain the variation of the rate as depicted realtive to the NACRE (Angulo et al. 1999) rate in Figure 2. Indeed, a simple analytical estimate of the sensitivity of the rate to δ_{NN}, based on Eq. (1), is in agreement with our numerical integration.

3. Stellar evolution

We focus on Population III stars with typical masses, 15 and 60 $M_\odot$ and zero metallicity.

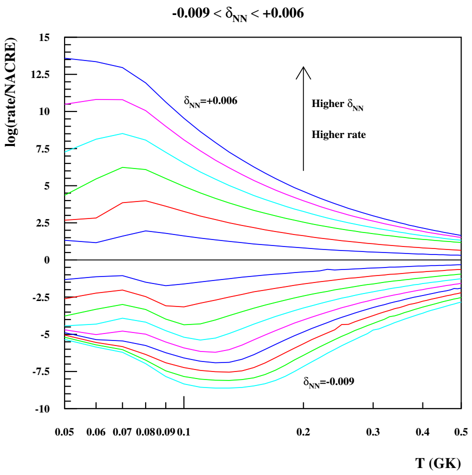

Fig. 2. Ratio of the rates for different values of δ_{NN}, relative to the δ_{NN}=0 rate.

We use the Geneva code, assuming no rotation (see Ekström et al. 2008, and references therein). The computations are stopped at the end of the core helium burning (CHeB). Note that the ^{4}He$(\alpha\alpha,\gamma)^{12}$C reaction is also important during the hydrogen burning phase at zero metallicity. In the absence of initial CNO, core hydrogen burning proceeds trough the slower pp-chain until sufficient ^{12}C has been produced by the triple alpha reaction. This explains the different main sequence tracks in the HR diagram of Figure 3.

Figure 4 shows the central abundances at the end of core He burning for the 60 $M_\odot$ model, and the 15 $M_\odot$ model is qualitatively similar. From these characteristics, we can distinguish four different cases:

I During He burning, ^{12}C is produced, until the central temperature is high enough for the ^{12}C$(\alpha,\gamma)^{16}$O reaction to become efficient: in the last part of the CHeB, the ^{12}C is processed into ^{16}O. The star ends the CHeB phase with a core composed of a mixture of ^{12}C and ^{16}O.

II If the 3α rate is weaker, the ^{12}C is produced at a slower pace, and T_c is high from the beginning of the CHeB phase, so the ^{12}C$(\alpha,\gamma)^{16}$O reaction becomes efficient

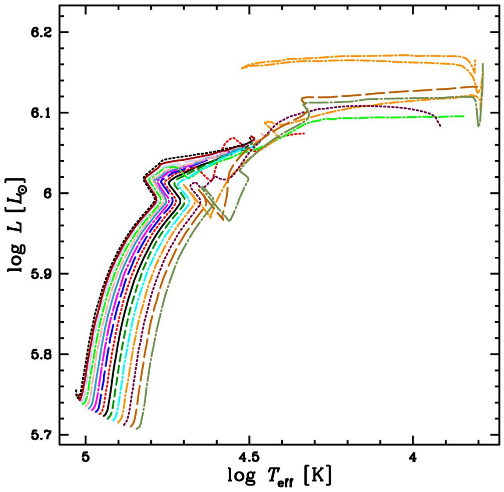

Fig. 3. HR diagram for a 60 $M_\odot$ star for different values of δ_{NN}.

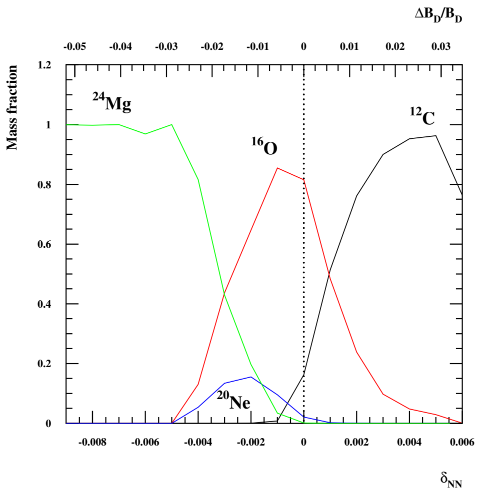

Fig. 4. Central abundances at the end of CHe burning as a function of δ_{NN} for a 60 $M_\odot$ star.

very early: as soon as some ^{12}C is produced, it is immediately transformed into ^{16}O. The star ends the CHeB phase with a core composed mainly of ^{16}O, with very little ^{12}C and an increasing fraction of ^{24}Mg for decreasing δ_{NN}.

III For still weaker 3α rates, the central temperature during CHeB is such that the ^{16}O$(\alpha,\gamma)^{20}$Ne$(\alpha,\gamma)^{24}$Mg chain becomes efficient, reducing the final ^{16}O abundance. The star ends the CHeB phase with a core purely composed of ^{24}Mg.

IV If the 3α is strong ($\Delta B_D/B_D > 0$), the ^{12}C is very rapidly produced, but the T_c is so low that the ^{12}C$(\alpha,\gamma)^{16}$O reaction can hardly enter into play: the ^{12}C is not transformed into ^{16}O. The star ends the CHeB phase with a core almost purely composed of ^{12}C.

4. Limits on the variations of the fundamental constants

The results of the 15 and 60 $M_\odot$ models shows that the variation of the N–N interaction should be in the range $-0.0005 < \delta_{NN} < 0.0015$ to insure that the C/O ratio be of the order of unity. The previous nuclear model introduces the parameter δ_{NN} which is not directly related to a set of fundamental constants such as the gauge and Yukawa couplings. In order to make such a connection, we use our computation of the deuterium binding energy B_D. Using Eq. (4) the limits become $-0.003 < \Delta B_D/B_D < +0.009$.

Using a potential model, the dependence of B_D on the the nucleon, σ-meson and ω-meson mass has been estimated (Dmitiev et al. 2007) so that it can be related to the u, d and s quark masses and thus to the corresponding Yukawa couplings, the Higgs vacuum expectation value (vev) v and the QCD scale Λ. For instance it was concluded (Coc et al. 2007) that

$$\frac{\Delta B_D}{B_D} = 18\frac{\Delta\Lambda}{\Lambda} - 17\left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s}\right) \tag{7}$$

which can then link any constraint on ΔB_D and δ_{NN} to the three fundamental constants (h_s, v, Λ). As explained in (Coc et al. 2007) this set can be replaced to (h, v, α) if grand unification is taken into account and if one assumes all the Yukawa couplings vary identically. It can then be reduced to e.g. (h, α) in models in which the weak scale is determined by dimensional transmutation and even to α $[\Delta h/h = 1/2\Delta\alpha/\alpha]$, in certain string models where the couplings are related by a dilaton (Campbell & Olive 1995). In this context, the limits given by BBN (i.e. at a redshift of $z \sim 10^8$) on the variation of the fine structure constant are given by $-3.2 \times 10^{-5} < \Delta\alpha/\alpha < 4.2 \times 10^{-5}$. Within the same model and parameter values, the constraints obtained above would lead to $-3. \times 10^{-6} < \Delta\alpha/\alpha < 10^{-5}$ at $z = 10$–15.

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