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Thermodynamics in variable speed of light theories

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Abstract. Planetary and stellar observational consequences of a variable speed of light (VSL) theory proposed by J. Magueijo is studied. The modified first law of thermodynamics together with a recipe to obtain equations of state are obtained. In the Newtonian limit is is obtained the non-relativistic hydrostatic equilibrium equation for the theory. We determine the time variation of the radius of Mercury induced by the variability of the speed of light (c), and the scalar contribution to the luminosity of white dwarfs. Using observational constraints a bound on the time variation of c is set.

1. Introduction

The coincidence of large dimensionless numbers constructed from different physical constants led Dirac to propose the Large Number Hypothesis and predict a time variation of them [Dirac , 1937], [Dirac , 1938]. Unification schemes such as superstring theories predict that some of the low-energy fundamental constants may vary in space or time.

Besides the theoretical motivations mentioned above, variable speed of light theories (VSL) are interesting because they could solve several cosmological puzzles [Moffat , 1993], [Albrecht and Magueijo , 1999], [Barrow , 1999]. In this work we study the thermodynamics and Newtonian limit of the varying speed of light theory developed by J. Magueijo [Magueijo , 2000].

2. Brief description of the VSL theory

In the covariant and locally Lorentz invariant VSL proposed by Magueijo c is a is an adimensional dynamical scalar field $c = c_0 e^{\psi}$, where c_0 is a constant. The General Relativity (GR) action is modified and becomes:

$$I = \int d^4x \ \sqrt{-g} \left(e^{a\psi}(R - 2\Lambda - \kappa \nabla_\mu \psi \nabla^\mu \psi) + \frac{16\pi G}{c_0^4} e^{b\psi} \mathcal{L}_m \right). \tag{1}$$

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$\mathcal{L}_m$ is the matter lagrangian and κ, a and b are three parameters of the theory. We will take $a-b = 4$ as in [Magueijo , 2000]. The matter lagrangian is required to have no explicit dependence on c (*minimal coupling condition*), fixing the scaling with c of all the lagrangian parameters up to the $\hbar(c)$ dependence, which is taken to be $\hbar \propto c^{q-1}$, where q is the fourth parameter of the theory.

We can derive the equations for the metric and ψ varying the action (1), then apply the Bianchi identities, and an equation for the divergence of the matter stress energy tensor $T^{\mu\nu}$ is obtained:

$$T^{\nu}{}_{\mu;\nu} = -\psi_{;\nu}T^{\nu}{}_{\mu}b + \psi_{;\mu}b\mathcal{L}_m - \Lambda_{m;\mu} \ . \tag{2}$$

where $\bar{\Lambda} = \Lambda + \frac{8\pi G}{c^4}\Lambda_m$. Note that matter energy is conserved only when $b = 0$, in all other cases there is exchange of energy between matter and the ψ field.

The presence of the ψ field can modify the law of conservation of the number of particles and the normalization condition for the four-velocity. Also the energy density and the total energy of a body in hydrostatic equilibrium will also vary if c isn't constant. In Racker et al. [2009] we parameterize these modifications with four parameters q_1,q_2,q_3,q_4.

3. The perfect fluid lagrangian and Magueijo's theory

The task of obtaining a lagrangian for the perfect fluid is not a trivial one due to the constraints imposed by the normalization of the velocity and the conservation of the number of particles. F. Schutz [Schutz , 1970] uses a formulation of relativistic hydrodynamics based on the utilization of 6 potentials to represent the velocity:

$$U_\nu = \mu^{-1}\left(\phi_{,\nu} + \xi\beta_{,\nu} + \theta s_{,\nu}\right), \tag{3}$$

where μ and s are the specific enthalpy (enthalpy per unit mass) and the specific entropy respectively.

The perfect fluid action is:

$$I = \int \left(R + \frac{16\pi G p}{c_0^4}\right)(-g)^{1/2}\, d^4x\,,$$

where p is the pressure of the fluid. Then the perfect fluid lagrangian is: $\mathcal{L}_m = p$. The steps that have to be taken to vary the action together with the Euler-Lagrange equations are detailed in [Racker et al. , 2009].

Varying $\mathcal{L}_m$ with respect to $g^{\mu\nu}$ leads to:

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + pg^{\mu\nu}\,, \tag{4}$$

where ρ, p, ρ_m, U^μ are defined from ϕ, ξ, β, θ and s exactly in the same way as in Schutz's theory. In the VSL theory they can depend on ψ, but they will coincide with the usual quantities when $\psi = 0$.

4. Thermodynamics: First law and Equations of state

When energy is conserved, the divergence of $T^{\mu\nu}$ (eq. (2)) is zero and the first law of thermodynamics is obtained projecting along U^μ. The results of this section will be applied to systems whose scales are much smaller than cosmological scales, so we take $\Lambda_m = 0$. Projecting $T^{\nu}_{\mu;\nu}$ along U^μ using Schutz's lagrangian and the second law of thermodynamics leads to the modified first law:

$$du + v\left[1 + \frac{c^{q_1}}{C}\right]dp + pdv - v(\rho + p)\left(\frac{q_1}{2} + q_2 - b\right)d\psi = Tds\,, \tag{5}$$

where s (identified with the specific entropy of the system) and T are two thermodynamic variables, and we have introduced the specific volume, energy and entropy $v = \dfrac{V}{N} = \dfrac{1}{n}$, $u = \rho v = \dfrac{U}{N}$, $s = \dfrac{S}{N}$. The first two terms are the only ones appearing in GR. The third term shows that ψ formally plays the role of a new thermodynamic variable.

We take as the time of reference the present epoch and choose s, v and ψ as the independent variables. From the equalities between the mixed partial derivatives we can obtain, up to first order in the scalar field, the following expression for the functional dependence of p on v, s and ψ(see [Racker et al. , 2009]):

$$p \simeq p_0(v, s) + b_1 v \frac{\partial p_0}{\partial v} \psi \,. \tag{6}$$

where $p_0(v, s)$ is the pressure as a function of v and s for $\psi = 0$ and therefore it is obtained from the usual theories in which c is constant.

5. Newtonian limit and transformation into a variable G theory

The Newtonian limit of this VSL theory can be obtained following the same steps as those used in GR. The hydrostatic equilibrium equation for a system with spherical symmetry is:

$$\frac{dp}{dr} = -\frac{\tilde{G}}{c^4} \frac{\rho(r) U(r)}{r^2} \,, \tag{7}$$

where $U(r)$ is the total energy inside the sphere of radius r, $\frac{d}{dr}$ and $\tilde{G} = G\left(1 + \frac{a^2}{2\kappa+3a^2}\right)$ is an effective gravitational constant.must be understood as a spatial derivative at constant time.

This equation plus an equation of state and boundary conditions determine the radius of a planet. The presence of ψ in these equations causes in general time variations of the radius. For Mercury its radius hasn't changed more than 1 kilometer in the last 3.9×10^9 years [McElhinny et al. , 1978]. This fact will allow to obtain a bound for the temporal variation of ψ.

In Racker et al. [2009] we showed that the hydrostatic equilibrium equation is equivalent to another equation in which the temporal dependence resides only in the gravitational constant. The result reads

$$\frac{dp_0}{dr^*} = -\bar{G}(t) \frac{M_0^{2/3}}{r_{b\,0}} \frac{\rho_{m0}(r^*) M_0^*(r^*)}{r^{*\,2}} \,, \tag{8}$$

with

$$\bar{G}(t) = \bar{G}\, e^{(b_1+q_3+q_4-4+2/3q+2/3)\psi(t)} \tag{9}$$

$r_{b\,0}$ the Bohr radius for $\psi = 0$, $\rho_{m0}(r^*)$ is the mass density for $\psi = 0$, $M_0(r^*)$ is the mass contained within radius r (also for $\psi = 0$), and

$$r^* = \frac{r}{r_b M^{1/3}} \,, \qquad M_0^*(r^*) = \frac{M_0(r)}{M_0} \,, \tag{10}$$

where M is the total mass of Mercury. Thus in this case the VSL theory is equivalent to a theory in which the only "constant" that varies is G.

Now we can use the results of [McElhinny et al. , 1978]. The variation in the radius of Mercury (R) produced by the variation in G can be parametrized as:

$$\frac{1}{R} \frac{dR}{dt} = -\frac{\delta}{\bar{G}(t)} \frac{d\bar{G}(t)}{dt} \,, \tag{11}$$

where δ is generally a function of $\bar{G}$ and M. Using models of Mercury, McElhinny et al. obtained the value $\delta = 0.02 \pm 0.005$ for that planet.

6. Bound for $\dot{c}/c$

We can express the parameters b_1, q_1, q_2, q_3 and q_4 that have been introduced in this work in terms of the parameters q and b of the VSL theory (see Racker et al. [2009]). The dependence for $\bar{G}$ is

$$\bar{G}(t) = \bar{G}\ \exp\left[\left(\frac{11}{3}q - b - \frac{10}{3}\right)\psi\right]. \tag{12}$$

so from eq. (11) one gets:

$$\left(\frac{11}{3}q - b - \frac{10}{3}\right)\dot{\psi}(t) = -\frac{1}{\delta}\frac{\dot{R}}{R} \simeq 0 \pm 5\times 10^{-12}\mathrm{y}^{-1}\,, \tag{13}$$

with $\mathrm{y}^{-1} = 1/\mathrm{year}$. We have taken $\delta = 0.02 \pm 0.005$ and $\frac{\Delta R}{R} = 0 \pm 0.0004$ [McElhinny et al. , 1978], where ΔR corresponds to a time interval approximately equal to 3.5×10^9 years.

This result can be combined with bounds for $\dot{\alpha}/\alpha$ that have been obtained using atomic clocks. We can use e.g. the one obtained in [Sortais et.al. , 2001]:

$$\frac{\dot{\alpha}}{\alpha} = (4.2 \pm 6.9)\times 10^{-15}\mathrm{y}^{-1}. \tag{14}$$

In this paper we will consider the $b = 0$ case, which gives an upper bound for all non negative values of b. Moreover, in the VSL theory $\frac{\dot{\alpha}}{\alpha} = q\dot{\psi}$, then eq. (13) can be written as $-\frac{10}{3}\dot{\psi} = 0 \pm 5\times 10^{-12}\mathrm{y}^{-1} - \frac{11}{3}\frac{\dot{\alpha}}{\alpha}$. Comparing this last equation with eq. (14) we see that $-\frac{10}{3}\dot{\psi} \approx 0 \pm 5\times 10^{-12}\mathrm{y}^{-1}$. The conclusive result is that

$$\frac{\dot{c}}{c} = \dot{\psi} = 0 \pm 2\times 10^{-12}\mathrm{y}^{-1}. \tag{15}$$

This can be rewritten as a bound for the adimensional quantity $\psi' = H_0^{-1}\dot{\psi}$ after multiplying by the Hubble time (H_0^{-1}) [1]:

$$\psi' = 0 \pm 3\times 10^{-2}. \tag{16}$$

7. White dwarfs luminosities and the scalar field

White dwarfs are excellent objects to test any energy injection from a scalar field [Stothers , 1976; Mansfield and Malin , 1980]. This is due both to their low luminosity, as well as their extremely high heat conductivity, making all energy microscopically released to enhance the total luminosity. Most of them are adequately described by Newtonian physics and a zero temperature approximation, the latter hypothesis providing a polytrope type equation of state (EOS).

The no-scalar-field polytrope equation is $p_0 = K_0\rho^\gamma$, so from eq. (6) we have

$$p = p_0(\rho) - b_1\gamma\rho\frac{p_0}{\rho}\psi, \tag{17}$$

so we recover a polytrope equation of state with a new constant $K_0 \to K = K_0(1 - b_1\psi\gamma)$. The Lane-Emden function with polytrope index $(\gamma - 1)^{-1}$ still applies, and consequently the expressions for the radius, mass and internal energy of the star will be the same in terms of the effective constant K (see for instance [Chandrasekhar , 1957]). We can show that $G(t) = \bar{G}\exp[(b_1 + q_3 + q_4 - 4)\psi] = \bar{G}\exp[(3q - b - 4)\psi]$ and that the dependence of the stellar internal energy is:

$$E \propto \exp\psi f(q, b, \gamma), \tag{18}$$

[1] We have taken $H_0^{-1} \approx 1.5\times 10^{10}$ years.

Table 1. Data and bounds for selected white dwarfs (data from [Provencal et al. , 2002]).

Object	$M/M_\odot$	$R/R_\odot$	$L/L_\odot$	E/E_0	$\lvert\dot\psi\rvert \le$
Procyon B	0.602	0.0123	5.8×10^{-4}	29.5	3.3×10^{-13}
40 Eri B	0.501	0.0136	0.014	18.5	1.2×10^{-11}
Stein 2015B	0.66	0.011	3.1×10^{-4}	39.6	1.4×10^{-13}

where $f(q,b,\gamma) = \frac{13}{3}q - \frac{13}{3}b - \frac{14}{3}$ for white dwarfs well described by the non relativistic value $\gamma = 5/3$.

To go on we assume that the star is in equilibrium in the sense that all the energy injected by the field ψ is radiated away. Therefore the equation for the luminosity induced by the ψ field (L_ψ) becomes

$$L_\psi = -\dot{E} = -f(\gamma, q, b)E\dot{\psi}. \tag{19}$$

Unfortunately, only a handful of white dwarfs have well measured masses, radii and luminosities: indeed, these four or five stars are used to test the theory of white dwarfs, since the mass-radius relation requires exactly the same parameters we need to carry our comparison of theory and experiment. These stars and their properties have been reviewed in [Provencal et al. , 2002]. The table shows the adopted values. We have excluded Sirius B from the sample, since relativistic effects are important in this case.

To obtain the bound for $\dot\psi$ we rewrite eq. (19) as

$$\dot{\psi} = \frac{L_\psi}{E}\frac{3}{13b+14} + \frac{13}{13b+14}\frac{\dot\alpha}{\alpha}\ . \tag{20}$$

Using again the upper bound (14) for the present time variation of α and bounding L_ψ by the observed luminosity of the white dwarfs (L) we obtain

$$\left|\dot\psi\right| \le \dot\psi_0 \left|\frac{3}{13b+14}\right| + \left|\frac{13}{13b+14}\right| 1.1 \times 10^{-14} y^{-1}, \tag{21}$$

where

$$\dot\psi_0 \equiv \frac{(L/L_\odot)}{(E/E_0)}\frac{1}{\tau_\odot}, \tag{22}$$

and

$$E_0 = -\frac{3}{7}\frac{GM_\odot^2}{R_\odot}, \tag{23}$$

is the would be internal energy of the sun were it described by a newtonian $\gamma = 5/3$ polytrope. $L_\odot$ is the solar luminosity and

$$\tau_\odot = \frac{E_0}{L_\odot} \simeq 1.32 \times 10^7 y, \tag{24}$$

is the Kelvin-Helmholtz solar contraction time scale.

The table shows upper bounds for $\dot\psi$ in y^{-1} again for $b = 0$.

Stein 2015B provides the strongest bound. Using a value for the Hubble time $H_0^{-1} \propto 1.5 \times 10^{10} y$ we obtain

$$\frac{1}{H_0}\frac{\dot c}{c} = \frac{1}{H_0}\dot\psi = 0 \pm 2.1 \times 10^{-3}. \tag{25}$$

Comparing the expressions (13) and (21) we see that white dwarf physics provides the strongest constraints on the VSL theory near the present epoch for almost all values of the b parameter, except for those near $b = -14/13$ which make eq. (21) uninformative. It's also clear that combining both bounds (13) and (21) we can obtain a bound for $|\dot\psi|$ independent from the value of b (although less strong than the bound given for $b \ge 0$): $|\dot\psi| \le 2.2 \times 10^{-12}\mathrm{y}^{-1}$.

8. Conclusions

We have obtained the equations that describe a perfect fluid in the non-relativistic limit and the first law of thermodynamics in the context of the covariant VSL theory proposed by J. Magueijo. We showed that the field ψ can formally be considered as a new thermodynamic variable and we also showed how to obtain the equations of state in the VSL theory when the corresponding equations for constant c are given.

The non-relativistic hydrostatic equilibrium equation has the usual form with the gravitational constant G replaced by an effective constant. The different variables (pressure, mass, mass density) depend on the ψ field and so the radius of a planet should vary in time. Using bounds for the variation of the radius of Mercury and the fine structure constant we have set limits on $\dot{c}/c$: $\dot{c}/c = \dot{\psi} = 0 \pm 2 \times 10^{-12}\mathrm{y}^{-1}$ (valid for positive values of the b parameter). The most interesting thing about this result is that it gives a bound for $\dot{\psi}$, whereas the known limits for the variation of α and e lead to bounds for the product $q\dot{\psi}$.

Under the same Newtonian approximation we obtained the dependence of the luminosity of a white dwarf on the time variation of the scalar field. The bound obtained is more stringent than the planetary radius bound by an order of magnitude. The $b = 0$ assumption suggests a null coupling of the scalar field with matter. However, the $q \neq 0$ assumption implies a quantum coupling between ψ and matter, not explicitly shown in the action (1). Of course this and other issues such as the microscopic origin of the energy exchange between the scalar field and ordinary matter as well as whether all the energy injected by ψ on a star is radiated away or not, deserve further work.

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