Mem. S.A.It. Vol. 80, 770

Memorie della

Energy conservation and constants variation

L. Kraiselburd[1,2], M.M. Miller Bertolami[1,2], P. Sisterna[3], and H. Vucetich[1]

[1] Facultd de Cs. Astronómicas y Geofísicas – Universidad Nacional de La Plata – Argentina
[2] Consejo Nacional de Investigaciones Científicas y Técnicas – Argentina
[3] Facultad de Cs. Exactas – Universidad Nacional de Mar del Plata – Argentina

Abstract. If fundamental constants vary, the internal energy of macroscopic bodies should change. This should produce observable effects. It is shown that those effects can produce upper bounds on the variation of much lower than those coming from Eötvös experiments.

1. Introduction

The Standard Model of Fundamental Interactions (SM) together with General Relativity (GR) describe all low-energy (i.e. $E < E_P$) and short-scale (i.e. $L < 100$ kpc) physics, in good agreement with experiment and observation (Particle Data Group et al. 2008). These theories depend of a set of parameters called *fundamental constants*, that are assumed independent of space and time. The Equivlence Principle implies such invariance and so, the discovery of changes in "Fundamental Constants" imply the existence of "new physics". Indeed, many theories attempting the unification of SM and GR, such as Kaluza-Klein or Superstring theories, predict such variations. Variation of "fundamental constants" should produce a multitude of observable phenomena that may be used to test theories against observation. Several of these have been used already (Sisterna & Vucetich 1990); among the latest let us mention the Oklo phenomenon (Fujii 2004), spectra in absorption systems (Murphy et al. 2004), comparison of atomic clocks (Fortier et al. 2007), early Universe data (Landau et al. 2008) and Eötvös experiments (Chamoun & Vucetich 2002; Kraiselburd & Vucetich 2009).

We examine in this comunication some possibilities of testing these theories based on consequences of conservation laws, mainly energy-momentum conservation. This is because if a known electromagnetic system interacts with some other unknown one, the balance of the conserved quantity will be altered with respect to the known conservation law and some large unexpected phenomena should manifest.

2. Variation of α in Bekenstein's formalism

Bekenstein's model (Beckenstein 1982; Bekenstein 2002) is a well-defined theory for the variation of the "fine-structure constant". It may be considered a low energy limit of some string theory, since it satisfy the most important conditions that such a limiting theory should have. The theory is lagrangian-based and so it has well defined conservation laws for energy-momentum and charge. Let us briefly review its content. The theory is based on several postulates, embodying physical assumptions:

1. It must reduce to Maxwell's theory when α is constant.

2. α variationn is dynamic, generated by a field $\epsilon = \exp\psi$, the latter field being a mock dilaton.
3. All dynamics should be derived from a variational principle.
4. The theory must be causal and gauge and time reversal invariant.
5. Planck's scale ℓ_P is the smallest one in the theory.

All these assumptions, except the last one, have to be satisfied for any low energy limit of a string or Kaluza-Klein theory.

From these equations, the following action can be derived

$$\begin{aligned} S &= -\frac{1}{16\pi}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d^4x \\ &-\frac{\hbar c}{2\ell_B^2}\int\frac{\epsilon^{,\mu}\epsilon_{,\mu}}{\epsilon^2}\sqrt{-g}d^4x \\ &+S_{\rm RG}+S_{\rm m} \end{aligned} \tag{1}$$

where ℓ_B is a new parameter with dimension of length, that we shall call *Bekenstein length*. The main difference between Maxwell and Bekenstein theories is due to the connection between the vector potential A_μ and the fields:

$$\begin{aligned} \epsilon A'_\mu &= \epsilon A_\mu + f_{,\mu} \\ \nabla_\mu &= \partial_\mu - e_0\epsilon A_\mu \\ F_{\mu\nu} &= \frac{1}{\epsilon}\left[(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}\right] \end{aligned} \tag{2}$$

while the charge and the ϵ field are connected in the form

$$\begin{aligned} e(\boldsymbol{r},t) &= e_0\epsilon(\boldsymbol{r},t) \\ \epsilon(\boldsymbol{r},t) &= \left(\frac{\alpha}{\alpha_0}\right)^{\frac{1}{2}} \end{aligned} \tag{3}$$

In the above equations e_0 (α_0) are reference values corresponding to a certain event in space-time, where $\epsilon = 1$. From the above action the following equation of motion can be found

$$\left(\frac{1}{\epsilon}F^{\mu\nu}\right)_{,\nu} = 4\pi j^\mu \tag{4}$$

and

$$\Box\ln\epsilon = \frac{\ell_B^2}{\hbar c}\left(\epsilon\frac{\partial\sigma}{\partial\epsilon} - \frac{F^{\mu\nu}F_{\mu\nu}}{8\pi}\right) \tag{5}$$

where σ is the electromagnetic energy density of matter (Beckenstein 1982). From the above equations one can find a solution for the cosmological variation of ϵ

$$\frac{\dot\epsilon}{\epsilon} = -\frac{3\zeta_c}{8\pi}\left(\frac{\ell_B}{\ell_P}\right)^2 H_0^2\Omega_B\left[\frac{a_0}{a(t)}\right]^3(t-t_c) \tag{6}$$

$$\left.\frac{\dot\epsilon}{\epsilon}\right|_0 \simeq 1.3\times10^{-5}\left(\frac{\ell_B}{\ell_P}\right)^2 \tag{7}$$

As equation (7) shows, the predicted variation is small and large accuracy will be necessary to detect it.

3. Energy transfer in Bekenstein's formalism

From the action (1) the following energy-momentum tensor can be derived in the usual way

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{4\pi}\left(F_\mu^\lambda F_{\lambda\nu} - \frac{\eta_{\mu\nu}}{4}F_{\lambda\sigma}F^{\lambda\sigma}\right) \\ &+ \frac{\hbar c}{\ell_B^2}\left(\frac{\epsilon^{,\mu}\epsilon^{,\nu}}{\epsilon^2} - \frac{1}{2}\eta^{\mu\nu}\frac{\epsilon^\alpha\epsilon_\beta}{\epsilon^2}\right) \\ &+ T_{\rm m}^{\mu\nu} \end{aligned} \tag{8}$$

where we have assumed a locally flat space.

The energy-momentum conservation law takes the form

$$\begin{aligned} T^{\mu\nu}{}_{,\nu} &= -\epsilon j_\alpha F^{\mu\alpha} \\ &+ \phi_{,\nu}\left(\eta^{\mu\nu}\epsilon\frac{\partial\sigma}{\partial\epsilon} + T_{\rm em}^{\mu\nu} - \frac{\eta^{\mu\nu}}{16\pi}F_{\lambda\sigma}F^{\lambda\sigma}\right) \\ &+ T_{\rm m}^{\mu\nu}{}_{,\nu} = 0 \end{aligned} \tag{9}$$

The first line represents the Joule effect, the second line is the energy-momentm balance between the electromagnetic field, the ϵ field and matter. The second term is responsible for any unaccounted energy liberation.

The T_{00} component represents the energy transfer between the fields and matter. Using the equations of motion and expressing in terms of the field components $\boldsymbol{E}, \boldsymbol{B}$ we find

$$\frac{\partial e_m}{\partial t} + {\rm div}\boldsymbol{q} = \boldsymbol{j}\cdot\boldsymbol{E} + \frac{\dot\alpha}{\alpha}\epsilon\frac{\partial\sigma}{\partial\epsilon} \tag{10}$$

$$- \frac{\dot\alpha}{\alpha}\frac{B^2}{16\pi} - \frac{1}{2}\frac{\boldsymbol{\nabla}\alpha}{\alpha}\cdot\boldsymbol{S} \tag{11}$$

where e_m is the matter energy density, $\boldsymbol{q}$ the energy current (heat flow) and $\boldsymbol{S}$ is Poynting's vector. It is easy to show that

$$\frac{\partial e_m}{\partial t} = \frac{\dot{\alpha}}{\alpha}\epsilon\frac{\partial\sigma}{\partial\epsilon} \tag{12}$$

since $e_m \propto \epsilon$. It is important to note that electric energy density does not contribute to the energy transfer to matter and that space variation of α contributes to the transfer to/from a radiant system. The most important conclusion from equation (10) is that the cosmological time variation of α contributes to the liberation of energy only through the magnetic energy density of matter.

4. Magnetic energy of matter

To compute the magnetic energy of matter we must take into account that the main contribution arises from quantum fluctuations of currents in nuclear atomic systems. From the Biot-Savart law

$$\boldsymbol{A} = \frac{1}{c}\int d\boldsymbol{x}'\frac{\boldsymbol{j}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|} \qquad \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A} \tag{13}$$

where $\boldsymbol{j}, \boldsymbol{A}, \boldsymbol{B}$ are quantum operators. Besides, in nonrelativistic limit quantum mechanics

$$\boldsymbol{j} = e\frac{\boldsymbol{p}}{2m} + g\frac{e}{mc}\boldsymbol{s}\times\hat{\boldsymbol{p}} \tag{14}$$

with e the charge, $\boldsymbol{p}$ the momentum, $\boldsymbol{s}$ the spin and g the gyromagnetic ratio. Since $\boldsymbol{j}$ is a vector, its matrix elements will be nonzero only between states of opposite parity. Magnetic field fluctuations will then satisfy

$$\langle 0|\boldsymbol{B}|0\rangle = 0 \tag{15}$$

but the corresponding magnetic energy density is

$$e_m = \frac{\langle 0|B^2|0\rangle}{8\pi} > 0 \tag{16}$$

The total magnetic energy of a nucleus has been computed in a quasiclassical approximation and with the neglect of the spin contribution by Haugan and Will Haugan & Will (1977). An approximate interpolation formula can be found relating the matrix elements of the current operator to the nuclear dipole matrix elements and using sum rules to connect them to the nuclear giant dipole resonance (Haugan & Will 1977).

$$E_m \simeq \frac{3}{20\pi}\frac{E_{\mathrm{ma}}}{\hbar c R}\int\sigma_{\mathrm{rg}}dE \tag{17}$$

$$\sim 3\cdot 10^{-5}M_n c^2\left(\frac{A}{27}\right)^{\frac{2}{3}} \tag{18}$$

With this result, the effective energy transfer per unit mass from the ϵ field to matter results

$$\varepsilon_a = \frac{\dot{\alpha}}{\alpha}\frac{E_m}{Mc^2}c^2 = \frac{\dot{\alpha}}{\alpha}c^2\zeta_a \tag{19}$$

where

$$\zeta_a \simeq 3\times 10^{-6}A^{-\frac{1}{3}}. \tag{20}$$

This is a rather large energy transfer; with $H_0 \simeq 2.4\times 10^{-18}$ s^{-1} we find

$$\varepsilon \sim 2200\frac{\dot{\alpha}}{H_0\alpha}\zeta_a \ {}^{\mathrm{erg}}/_{\mathrm{g\,s}}; \tag{21}$$

of the order of magnitude of energy liberation in the sun center ($\varepsilon_\odot \sim 2$ ${}^{\mathrm{erg}}/_{\mathrm{g\,s}}$) and much greater than in the Earth or a white dwarf. This fact open the possibility of testing Bekenstein model through the energy liberation in "cold" bodies: planets or dead stars.

5. Heat flow in the Earth

Heat flow in the Earth crust has been measured since the XIX century. A good summary of the results up to 1990 is in the book by Jessop Jessop (1990). From the results there summarized it is found that the mean heat flow is

$$j_\oplus^{\mathrm{obs}} \simeq (60\pm 40)\ {}^{\mathrm{mW}}/_{\mathrm{m}^2} \tag{22}$$

where the standard deviation comes from the geological scatter of data. The wheighted mean is

$$\bar{j}_\oplus^{\mathrm{obs}} = 69.6\pm 3.3\ \mathrm{mW/m^2} \tag{23}$$

where the last number is the standard error of the mean. From the histograms shown in (Jessop 1990) one sees that the measured heat flow is positive. This heat flow can be explained by decay of long lived radionucleides

Table 1. High accuracy bounds on time variation of α

Experiment	Datum/H_0	Reference
Al/Hg ion clock	$(-1.8 \pm 2.6) \times 10^{-8}$	Rosenband et al. (2008)
Oklo Phenomenon	$(0 \pm 3) \times 10^{-8}$	Petrov et al. (2006)
Present Work	$(0 \pm 4) \times 10^{-6}$	—

as ^{238}U and ^{232}Th, in the crust and upper mantle. We can relate the measured heat flow to the heat liberation (19) in the form

$$j_{\oplus}^{\mathrm{an}} = -K \frac{dT}{dr}\bigg|_{\oplus} = \bar{\varepsilon} \frac{m_{\oplus}}{4\pi R_{\oplus}^2} \tag{24}$$

where $\bar{\varepsilon}$ is the averaged heat production. Substituting numerical values we obtain

$$j_{\oplus} \simeq 2.6\ ^{\mathrm{kW}}/_{\mathrm{m}^2} \frac{\dot{\alpha}}{H_0 \alpha} \tag{25}$$

This anomalous heat flow is much larger than the observed one by almost six orders of magnitude. From the data we obtain

$$\left|\frac{\dot{\alpha}}{H_0\alpha}\right|_0 < \frac{3\sigma}{2.6\ ^{\mathrm{kW}}/_{\mathrm{m}^2}} \simeq 5 \times 10^{-5} \tag{26}$$

from the geological dispersion of the data; and

$$\left|\frac{\dot{\alpha}}{H_0\alpha}\right|_0 < 4 \times 10^{-6} \tag{27}$$

from the mean value of the data. This should be compared to the strongest bound obtained from the Oklo phenomenon (Damour & Dyson 1996; Fujii 2004)

$$\left|\frac{\dot{\alpha}}{H_0\alpha}\right|_{\mathrm{Oklo}} < 1.4 \times 10^{-7} \tag{28}$$

This bound may be refined with a careful analysis of radiactive heat generation.

6. Conclusion

Our analysis shows that the analysis of energy conservation show several effects that can be used to test theories that predict time variation of fundamental constants. From these phenomena, strong bounds can be found for the time variation of α. The apparent contradiction between some indirect results and the direct measurment of α at cosmological distances shows that more research, both theoretical and observational, is needed in this field.

References

Beckenstein, J. D. 1982, Phys. Rev., D25, 1527

Bekenstein, J. D. 2002, Phys. Rev., D66, 123514

Chamoun, N. & Vucetich, H. 2002, Physics Letters B, 541, 291

Damour, T. & Dyson, F. 1996, Nuclear Physics B, 480, 37

Fortier, T. M., Ashby, N., Bergquist, J. C., et al. 2007, Physical Review Letters, 98, 070801

Fujii, Y. 2004, in Lecture Notes in Physics, Berlin Springer Verlag, Vol. 648, Astrophysics, Clocks and Fundamental Constants, ed. S. G. Karshenboim & E. Peik, 167–185

Haugan, M. P. & Will, C. M. 1977, Phys. Rev. D, 15, 2711

Jessop, A. 1990, Thermal geophysics, Develoopments in Solid Eearth geophysics (Elsevier)

Kraiselburd, L. & Vucetich, H. 2009, ArXiv e-prints

Landau, S. J., Mosquera, M. E., Scóccola, C. G., & Vucetich, H. 2008, Phys. Rev. D, 78, 083527

Murphy, M. T., Flambaum, V. V., Webb, J. K., & et al. 2004, in Lecture Notes in Physics, Berlin Springer Verlag, Vol. 648, Astrophysics, Clocks and Fundamental Constants, ed. S. G. Karshenboim & E. Peik, 131–150

Particle Data Group, Amsler, C., Doser, M., et al. 2008, Physics Letters B, 667, 1

Petrov, Y. V., Nazarov, A. I., Onegin, M. S., Petrov, V. Y., & Sakhnovsky, E. G. 2006, Phys. Rev. C, 74, 064610

Rosenband, T., Hume, D. B., Schmidt, P. O., et al. 2008, Science, 319, 1808

Sisterna, P. & Vucetich, H. 1990, Phys. Rev., D41, 1034