



GAME

A small mission concept for high-precision astrometric test of General Relativity

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Abstract. GAME (Gamma Astrometric Measurement Experiment) is a concept for a small mission whose main goal is to measure from space the γ parameter of the Parameterized Post-Newtonian formalism. A satellite, looking as close as possible to the Solar limb, measures the gravitational bending of light in a way similar to that followed by past experiments from the ground during solar eclipses. Preliminary simulations have shown that the expected final accuracy can reach the 10^{-7} level, or better if the mission profile can be extended to fit a larger budget. The estimation of the γ parameter at this level of accuracy, according to several theoretical claims, is decisive for the understanding of gravity physics, cosmology and the Universe evolution at a fundamental level. Moreover, thanks to its flexible observation strategy, GAME is also able to target other interesting scientific goals in the realm of General Relativity, as well as in those involving observations of selected extrasolar systems in the brown dwarf and planetary regime. We report on new estimation for the mission performances based on the latest updates on the mission configuration and simulation results, focusing on the main scientific goal of GAME, i.e. the measurement of the γ parameter.

Key words. General Relativity – fundamental physics – astrometry – telescopes – methods: numerical – instrumentation: miscellaneous

1. Introduction

GAME (Gamma Astrometric Measurement Experiment) is a concept for a small mission whose main goal is to put General Relativity and other alternative theories of gravity to test by estimating the γ parameter of the Parameterized Post-Newtonian (PPN) formalism (Will 2001). This result is reached by re-

peated measurements from space of the deflection of the light coming from stars close to the solar limb. In short, the mission is conceived as a novel implementation from space of the experiment conducted during the solar eclipse of 1919, when Dyson, Eddington and collaborators measured for the first time the gravitational bending of light (Dyson et al. 1920). This makes GAME a decisive experiment for the understanding of gravity physics, cosmol-

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ogy and the Universe evolution at a fundamental level.

It has long been known, in fact, that General Relativity (GR) can act as a cosmological attractor for scalar-tensor theories with expected deviations in the $10^{-5} - 10^{-7}$ range for the γ parameter (Damour & Nordtvedt 1993). Also, during the last decade, a strong experimental evidence of an acceleration of the expansion of the Universe at the present time has been deduced from several observational data. This has been interpreted as the effect of a long range perturbation of the gravity field of the visible matter generated by the so-called Dark Energy. These data add to those available since long time at different scale length, which are explained with the existence of non-barionic Dark Matter (e.g. galaxy rotation curves) or with some kind of modification of the General Relativity theory (e.g. Pioneer anomalies (Bertolami et al. 2008; Turyshev 2008; Leibovitz 2008; Toth & Turyshev 2008)). However, there are claims that these data can be explained with a modified version of General Relativity, in which the curvature invariant R is no longer constant in the Einstein equations ($f(R)$ gravity theories). Again, a 10^{-7} -level measure of γ seems to be the boundary of discrimination between GR and $f(R)$ theories (Capozziello & Troisi 2005).

Present experimental limits of this parameter span within the range $10^{-3} \div 10^{-5}$, depending on the effect measured and on the adopted technique (Reasenberg et al. 1979; Frøeschlé et al. 1997). The lower bound has been reached with the Cassini data, and exploiting the derivative of the Shapiro effect (Bertotti et al. 2003). This means that we have just started to explore the upper boundaries of the scientific range of interest.

In the forthcoming future, and limiting ourselves to astrometric measurements, the most promising effort presently under development is represented by the Gaia mission (Perryman et al. 2001) which, stemming from the same principles of Hipparcos, might achieve a level of accuracy of $10^{-6} - 10^{-7}$ during the second half of the next decade (Vecchiato et al. 2003).

Pushing even longer in the future, the proposed medium-class mission LATOR (Turyshev et al. 2004) claims to be able to reach the 10^{-8} level of accuracy for γ using a technique which involves also astrometric measurements. The same accuracy is the target of the ASTROD concept (Ni 2008) which, however, builds (and depends) on the laser ranging measurement technique planned for LISA.

Despite the encouraging possibilities that future space missions are opening on the measure of the γ parameter, there are many reasons supporting a modern rendition from space of the Dyson-Eddington experiment. The main consideration is that looking from space close to the solar limb, and observing with an acceptable S/N ratio, it is possible to retain the best from the original idea and at the same time to avoid or mitigate the historical disadvantages. These reasons have been summarized in a recent paper (Vecchiato et al. 2009), which also showed that, within the context of a small mission and less than one year of observations, it is possible to reach the 10^{-6} level or better, i.e. well within the scientific range of interest.

The same paper reports also on additional scientific goals which can be addressed by this mission profile. Basically, the observation time needed to achieve the main goal is about two months per year, leaving the remaining time available for other purposes. Preliminary studies have identified some of them again in the realm of General Relativity, but also in the observation of selected targets to study the planetary to brown-dwarf regime and the characteristics of known exo-planetary systems with the transit method. The interested reader can refer to the cited paper for a more detailed review of these goals.

The rest of the paper will focus on the main scientific target, with a review of the measurement principles, of the basic instrument design, and finally reporting on the latest results of the simulations which aim at estimating the mission performances on the γ parameter determination.

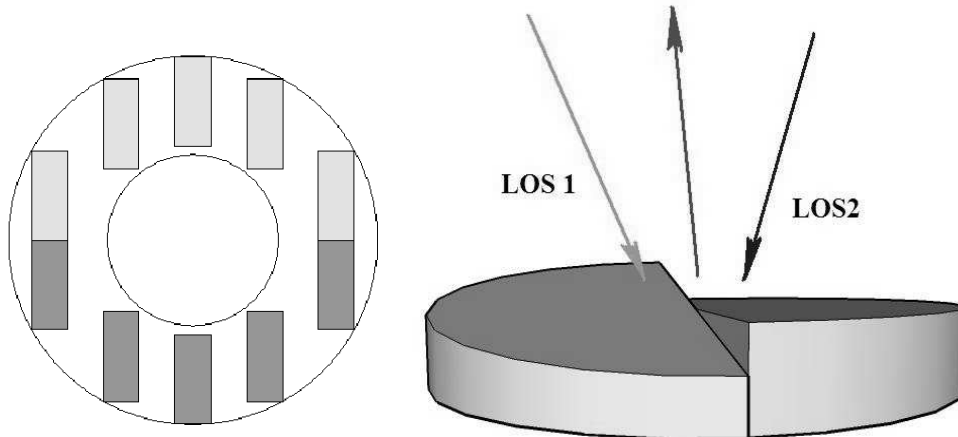


Fig. 1. Fizeau interferometer geometry. The two sets of elementary apertures (left panel) and the beam combiner (right panel). Each set of apertures is shown with different shades of grey, each referring to one direction on the sky, folded by the beam combiner.

2. Measurement principle and instrument design

The basic mechanism of GAME is to measure the arcs between the stars in two fields of view (FOVs) pointing symmetrically w.r.t. the ecliptic. The light deflection is estimated directly by measuring the same arcs in two epochs: (a) with the Sun in between and (b) when, after some months, our parent star is away from the observed region.

The GAME experiment is implemented as a small mission, from a satellite in a polar orbit at an altitude of 1500 km, with the payload of an optimized telescope observing simultaneously, in the visible, two fields of view with few degree separation. As mentioned above, the measurement sequence requires repeated observation in two epochs: (a) with the Sun between the two fields (maximum deflection observed close to the Solar limb), and (b) with a significant displacement from the Sun (minimum deflection). The planned mission lifetime is two years, thus allowing a repetition of the basic experiment for validation and improvement of the statistical precision. The observing time is maximized by the choice of a polar, Sun-synchronous orbit with proper inclination.

The optical design is described in more detail in (Gai et al. 2009) and consists of a multi-

ple aperture Fizeau interferometer, with line of sight split and folded over two sky regions by a *beam combiner* made by two flat mirrors set at a fixed angle. (Fig. 1) The proposed diluted optics solution alleviates the baffling requirements on each individual aperture of the overall Fizeau interferometer. The solar radiation baffling takes place in the section of the optical configuration devoted to folding of the input collimated beam, while the telescope proper is located in a section providing compression and focusing of the beams. The preliminary optical configuration is of the Ritchey-Chrétien type, and the long optical path through the additional folding mirror is used for optimization of the baffling. The offset between observing directions, set by the beam combiner, is called base angle. The design trade-off for the optimal base angle value depends on the competing requirements of higher astrometric signature of the deflection at smaller angles to the Sun, and lower solar photon background at larger distance. The selected value is 4° , setting the nominal observation in epoch (a) at $\pm 2^\circ$ from the Sun centre, where the deflection angle is 233 milli-arcsec (hereafter, mas) for each field, corresponding to a differential displacement of 466.7 mas of the stars with respect to their position in epoch (b). The fields are nominally set

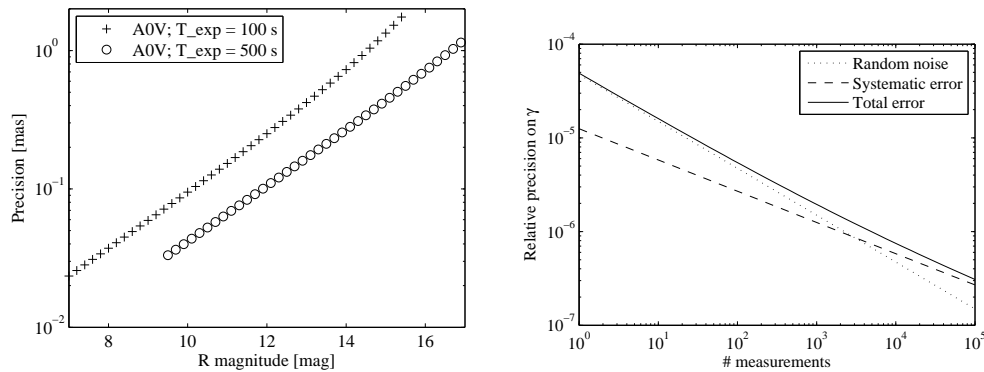


Fig. 2. Astrometric performance: elementary exposure error and final mission precision on γ . The elementary exposure error is shown for an A0V star for 100 s (asterisks) and 500 s (circles) integration, with background per square arcsec, respectively, 15.5 mag and 19.75 mag. The final mission precision is derived for an average stellar population close to the Galactic centre and up to four months observations.

in the North-South direction with respect to the current Sun centre position. At such distance, the corona is fainter than 22 mag per square arcsec. The GAME performance estimate is based on a background level 16.5 mag per square arcsec, i.e. more than two orders of magnitude brighter than the corona, and dominated by the residual Sun disc diffraction.

3. New simulation results

Simulations conducted in the case of a uniform stellar distribution, matching the average star counts of the GSC-II catalog (Lasker et al. 2008) in the latitudes of interest, had showed that the $\sigma_\gamma \approx 2.5 \cdot 10^{-6}$ level of accuracy is a target reachable with a 20 + 20 days measurement duration (i.e. 20 days of observations per epoch) with an $F = 17$ magnitude limit. (Vecchiato et al. 2009)

In that paper we justified our opinion that the result obtained so far was pessimistic. Among the others, we suggested that the unrealistic hypothesis of uniform stellar distribution was unfavorable and that the result on σ_γ could improve considering the real sky. Another improvement factor could also have been obtained with an optimization of the choice of the observed sky regions.

The present version of our simulation environment started to consider these two points.

We used the stellar distribution of the real sky from the GSC-II catalog, quitting the limitation of uniform star distribution adopted in our previous simulations. We also used an accelerated simulator to select the regions with the highest signal to noise ratio. The accelerated simulator, given a starting longitude, reckons the number of observed arcs between stars of given magnitudes, and then estimates the final SNR of the mission using the elementary exposure errors of Fig. 2. This estimation is iterated in the whole range of longitudes with a step of 1 degree, so the result of this code, since the present observation strategy consists in observing two continuous sky strips parallel to and at equal distance to the ecliptic, is just the starting ecliptic longitude that gives the best SNR. As expected (Fig. 3) the most convenient region is close to the Galactic center, with a starting longitude of about 270°.

This result is taken as an input by the new version of the simulator which, as the accelerated one, uses the error budget of Fig. 2, and considers the real stellar distribution as taken from the GSC-II catalog. We then repeated with this simulator the same kind of Monte Carlo tests performed with the previous version. These tests simply estimate the overall accuracy for σ_γ for a given magnitude limit, and were conducted varying this parameter from $F_{\max} = 14.5$ to $F_{\max} = 19$ (therefore

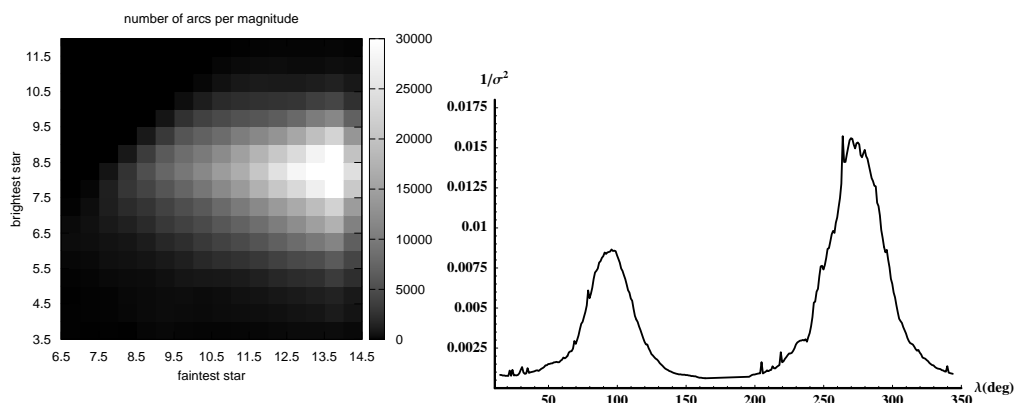


Fig. 3. Example of the results of the accelerated simulator used to choose the optimal initial conditions for the observations. In the left panel are shown, as a color map diagram, the number of arcs observed during the entire mission lifetime. The two axes are the magnitudes of the stars forming the arc. The right panel shows the SNR as function of the starting longitude.

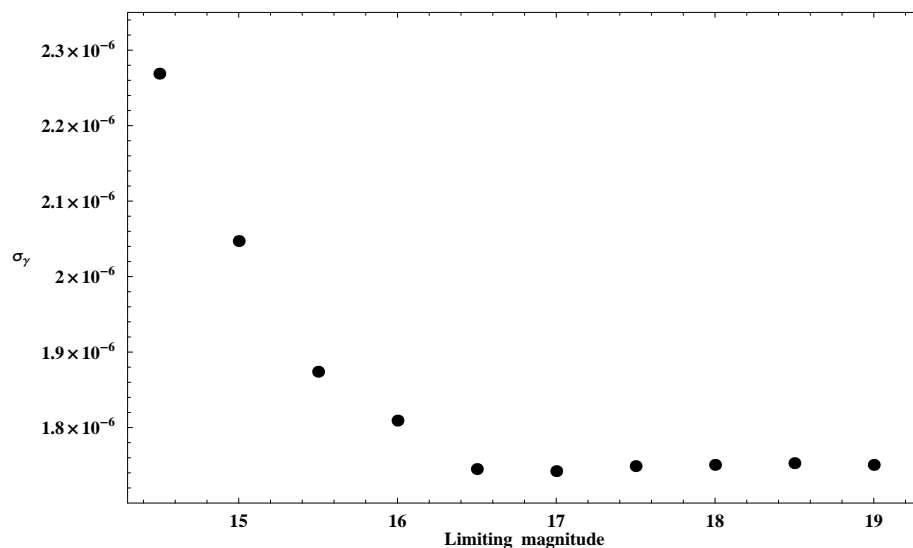


Fig. 4. Results of the Monte Carlo tests conducted with the new version of the simulator. Each point represents a Monte Carlo of 100 runs.

extending the magnitude range w.r.t. the past simulations, which stopped at $F = 17$) with a step of 0.5 mag. Present results of this simulations are reported in Fig. 4 and show that:

1. $\sigma_\gamma \simeq 2.6 \cdot 10^{-6}$ for a limiting magnitude of $F_{\max} = 14.5$. This corresponds to the same accuracy reached with the previous
2. a further improvement to $\sigma_\gamma \lesssim 1.7 \cdot 10^{-6}$ can be reached with $F_{\max} = 17$;

version of the simulator (i.e. considering a uniform stellar distribution) with limiting magnitude of $F_{\max} = 17$;

3. there is no advantage for the final result on σ_γ at pushing the magnitude of the observed stars to a fainter limit.

4. Conclusions and future developments

Latest developments on the GAME mission concept have confirmed that a number of improvements on the final accuracy for the main scientific goal are possible. Using the stellar counts of the real sky it has been shown that, depending on the value of the faintest magnitude, a factor approximately between 2.5 and 1.5 can be gained on the results obtained from the simulations on the uniform sky. This improvement is almost entirely related to the increased number of observations, which is obtained by selecting a convenient starting longitude close to a more crowded region of the sky. It has also been confirmed that, with the error budget of the present configuration, the lowest useful limit to the magnitude of the observed star is about $F = 17$.

Future investigations are needed in order to establish if and how the mission performances could be further improved. The options which are being investigated in the nearest future are dealing with the data reduction strategy and with the selection of the observed objects.

In the present data reduction strategy the measured arcs are formed by connecting the brightest star in one FOV with all the other ones of the opposite FOV. Preliminary calculations (Gai et al. 2009) shows that using a single arc connecting the two photocenters of each FOV will exploit all the information from the two fields and could bring another factor up to 1.4 to the final accuracy on the estimate of γ . Moreover, since the most important measurements are those involving the brightest stars, another option which is being investigated is that of exploring the potentiality on the final result of a modified observation strategy which favors these targets by relaxing the two conditions of continuity and parallelism of the present option.

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