



Probing the variation of fundamental constants using QSO absorption lines

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Abstract. Absorption lines seen in the spectra of distant QSOs allow us to probe the space and time evolution of various fundamental constants. Here, we summarize results on the variation of α obtained by our group and others using UVES/VLT. Most upper limits reside in the range $0.5-1.5 \times 10^{-5}$ at the 3σ level over a redshift range of approximately $0.5 \leq z \leq 2.5$. In addition, we also briefly report on preliminary results based on the analysis of 21-cm absorbers detected with Giant Meterwave Radio Telescope (GMRT) that lead to $\Delta x/x = (0.0 \pm 1.5) \times 10^{-6}$ at $z=1.3$. Discussions on future improvement are also presented.

Key words. Atomic processes – Methods: numerical – Methods: statistical – Quasars: absorption lines – Quasars:

1. Introduction

Most of the successful physical theories rely on the constancy of few fundamental quantities (such as the speed of light, c , the fine structure constant, α , the proton-to-electron mass ratio, μ , etc...). However, some of the modern theories of fundamental physics, that try to unify fundamental interactions naturally lead to the prediction of cosmological variation of some dimensionless fundamental constants (Uzan 2003 and reference therein). Therefore, constraining the possible time variations of these fundamental physical quantities is an important step toward a complete physical theory.

However, current laboratory constraints exclude any significant time variation of the dimensionless constants in the low-energy regime. It is not excluded, however, that they could have varied over cosmological timescales. Savedoff (1956) first pointed out the possibility of using redshifted atomic lines from distant objects to test the evolution of dimensionless physical constants. The basic idea is to compare the rest wavelengths of the same transition measured in the remote universe and in the laboratory. As we live in an expanding universe, one has to disentangle the contributions of the global redshift due to the expansion of the universe and the shift due to the variation in α . In principle we need at least two

transitions whose frequencies shift by different amplitude for a given change in α . This basic principle has been first applied to QSO absorption lines by Bahcall et al. (1967). Availability of 10m class telescopes fitted with high resolution echelle spectrographs and early claim of variation in α by Murphy et al. (2003) have generated considerable interest in this topic for the past 5 years.

A typical high resolution and high S/N spectrum of a distant QSO shows large number of absorption lines. These absorption lines arise when our line of sight to a distant QSO crosses a gaseous cloud by chance. Metal absorption lines that appear in the higher wavelength side of Lyman- α emission from the QSO are used for probing α variations. A small fraction of QSOs absorbers do show H₂ molecules in absorption (Petitjean et al. 2000; Ledoux et al. 2003; Noterdaeme et al. 2008). The Lyman and Werner band transitions of H₂ can be used for probing the variations in μ (Varshalovich & Levshakov, 1993). Like H₂ molecules, 21-cm absorption from the H I at high redshifts are very rare. However, the resonance frequency of this hyperfine transition is very sensitive to a small variations in either of α , μ and proton G-factor (G_p). Thus given the redshift of the absorbing gas it will be possible to probe the variation in the combination of these constants (Tubbs & Wolfe, 1980).

2. Variation of the fine-structure constant α

A possible time dependence of α will be registered as small shifts in the absorption line spectra seen toward high redshift QSOs as the energy of the atomic transitions depend on α . One has to disentangle the contributions of the global redshift due to the expansion of the universe and the shift due to the variation in α . To do so one needs at least two transitions with different sensitivity coefficients for the variations in α . As the redshift will be the same for all transitions the relative shift will therefore constrain $\Delta\alpha$. Initial attempts to measure the variation of α were based on alkali-doublets (e.g. Varshalovich et al. 1996) such as the well known Si IV doublet.

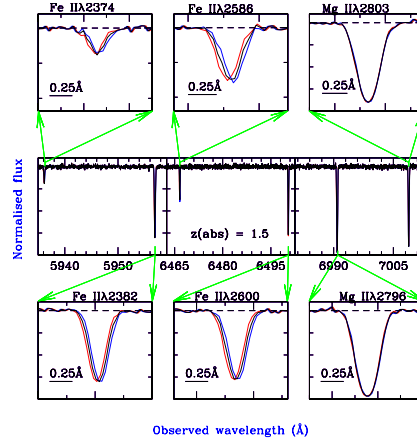


Fig. 1. Middle panel shows the simulated spectrum of Fe II multiplet and Mg II doublet produced by an absorbing gas at $z = 1.5$ for $\Delta\alpha/\alpha = 0.0$ (black), 5.0×10^{-5} (red) and -5.0×10^{-5} (blue). The zoomed in view of different absorption lines are shown in bottom and top panels. It can be seen that Mg II lines are insensitive to small values of $\Delta\alpha/\alpha$, while Fe II lines are very sensitive to these changes. Thus Mg II can be used for obtaining the absorption redshift and then Fe II lines will be used to probe the variation in α . This is the main idea behind the MM method. The difficulty in detecting the α variation can be appreciated from the fact that the values of $\Delta\alpha/\alpha$, used in this illustration are 10 times more than the detection claimed by Murphy et al (2003).

The generalization of this method, called many-multiplet (MM) method (Dzuba et al. 1999) gives an order of magnitude improvement in the measurement of $\Delta\alpha/\alpha$ compared to alkali-doublet method by using not only doublets from the same species but several multiplets from different species (e.g. Webb et al. 2001). The sensitivity to variations in α of different line transitions from different multiplets were computed using many-body calculations taking into account dominant relativistic effects (Dzuba et al. 2002). The wavenumber ω as a function of α is given by, $\omega = \omega_0 + qx$. Here, ω_0 is the vacuum wave number measured for the present day value of α (i.e. α_0), and $x = (\alpha/\alpha_0)^2 - 1$. The amplitude of the expected shift is more for heavier atoms and for a given atom the sign of the correction for dif-

ferent transitions will depend on the total electron angular momentum. Therefore, α induced shifts can have different amplitude and sign for different transitions.

In simple terms, MM method exploits the fact that the energy of different line transitions vary differently for a given change in α . For example rest wavelengths of Mg $\pi\lambda\lambda 2797, 2803$ and Mg $\lambda 2852$ transitions are fairly insensitive to small changes in α thereby providing good anchor for measuring the systemic redshift. Whereas the rest wavelengths of Fe π multiplets are quite sensitive to small variations in α . Thus measuring consistent relative shifts between an anchor and different Fe π lines can in principle lead to an accurate measure of $\Delta\alpha/\alpha$ (see Fig. 1).

The expected shifts are very small when $\Delta\alpha/\alpha$ is less than 10^{-5} and in reality the absorption profiles are complex. Therefore, direct detection of relative shifts will be difficult and we have to rely on statistical detections. The accuracy at which the variation can be measured depends very much on how well the absorption line centroids are measured through detailed modelling of the absorption line profiles. It is usual to approximate any absorption line profile as a combination of Voigt profiles that are convolved with the instrumental profile and characterized by column density (N), velocity dispersion (b) and redshift (z) in addition to the rest-wavelength of the species. In real data, in addition to wavelength calibration inaccuracies, small relative shifts can be introduced due to various systematic effects such as ionization and metal inhomogeneities in the absorbing region, isotopic abundances, and atmospheric dispersion effects, etc. However, one hopes, most of the random systematic effects may be canceled by using a large number of measurements.

While most of the initial attempts provided constraints on the variation of α , MM method applied to large heterogeneous samples of QSO absorption lines resulted in the claim for smaller value of α in the past, $\Delta\alpha/\alpha = (-0.574 \pm 0.102) \times 10^{-5}$ for $0.2 \leq z \leq 3.7$ (Murphy et al. 2003). A confirmation of this claim by an independent analysis of a different set of data could have dramatic consequences

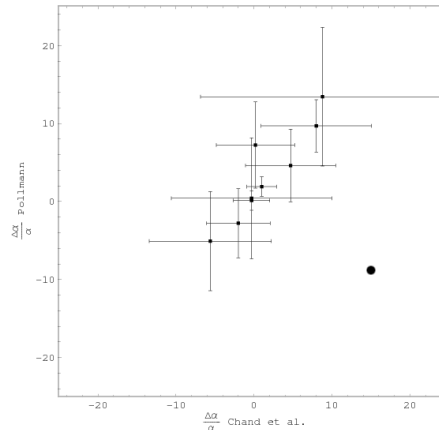


Fig. 2. Comparison of individual $\Delta\alpha/\alpha$ measurements (using AD method) of Chand et al. (2005) and independent measurement by Quast et al. (private communications).

on our understanding of fundamental physics. This has motivated different groups to undertake detailed analysis of QSO spectra to probe the variations of α and other dimensionless constants. Below, we summarise various results on the α variations obtained by our group over the years using UVES/VLT.

2.1. The alkali doublet (AD) method

Alkali doublets are conspicuous in astrophysical spectra both in emission (for example the [O III] $\lambda\lambda 4969, 5007$ doublet) and in absorption (for example the Si IV $\lambda\lambda 1393, 1402$). The method, although less sensitive than the Many-multiplet method, has the advantage of using only one species there by avoiding systematics related to chemical and ionization inhomogeneities. Also Si IV doublets can be detected to vary large redshifts thereby providing a good probe of α variations at high redshifts. Bahcall et al. (1967) were the first of a long list to apply this technique to QSO spectra.

More recently, Murphy et al. (2001) analysed a KECK/HIRES sample of 21 Si IV doublets observed along 8 QSO sight lines and derived $\Delta\alpha/\alpha < 3.9 \times 10^{-5}$. The analysis of 15 Si IV doublets selected from a ESO-UVES

Table 1. Summary of various constraints on $\Delta\alpha/\alpha$ using AD method

Number of system	Spectral resolution	S/N	$\Delta\alpha/\alpha$	References
10	≥ 36000	...	$\leq 1.1 \times 10^{-4}$	Cowie & Songaila (1995)
16	~ 15000	15	$+(2.0 \pm 7.0) \times 10^{-5}$	Varshalovich et al. (1996)
21	≥ 36000	15-40	$-(0.5 \pm 1.3) \times 10^{-5}$	Murphy et al., (2001)
15	≥ 45000	60-80	$+(0.15 \pm 0.43) \times 10^{-5}$	Chand et al., (2005)
21+15			$-(0.02 \pm 0.55) \times 10^{-5}$	Chand et al., (2005)+Murphy et al.(2001)

sample yielded the strongest constraint obtained with this method: $\Delta\alpha/\alpha < 1.3 \times 10^{-5}$ (3σ level) over the redshift range $1.59 \leq z \leq 2.92$ (Chand et al. 2005). Independent Analysis of the same data by Quast and his collaborators have confirmed this result (see Fig. 2). In Table 1 we summarize all the results till date based on AD method.

The AD method can be applied to emission as well as absorption lines. However emission lines are usually broad as compared to absorption lines. Errors are therefore larger on individual measurements and must be beaten by large statistics. As a result, the constraints obtained from emission lines are not as strong as those derived from absorption lines. Bahcall et al. (2004) have recently found $\Delta\alpha/\alpha < 4.2 \times 10^{-4}$ using O III emission lines from SDSS QSOs.

2.2. The Many-Multiplet Method

The power of the Many-Multiplet Method is to use a large number of transitions to constrain the variation of α . At least five transitions are used, usually from different species and some time from different ionization states also. The transitions are chosen so that their sensitivities to a change in α (i.e q values) are different.

We have applied the MM method to a sample of very high quality (S/N \sim 60 – 80, $R \geq 44,000$) UVES/VLT data. In view of the numerous systematic errors involved in the MM method, we have carried out detailed simulations to define proper selection criteria to choose suitable absorption systems in order to perform the best analysis (see Chand et al. 2004 for details). Application of these

selection criteria to the full sample of 50 Mg II/Fe II systems lead us to restrict the study to 23 Mg II/Fe II systems over a redshift range $0.4 \leq z \leq 2.3$. The weighted mean of the individual measurements from this analysis is a non detection with a 3σ upper limit of $\Delta\alpha/\alpha < 0.20 \times 10^{-5}$ (Srianand et al. 2004, Chand et al. 2004). In Table 2 we summarize various available measurements using MM method. In these analysis, like Murphy et al. (2003), we have used rest wavelengths of different species as measured in the laboratory for the isotopic combinations observed on earth. When using the rest wavelength of the dominant isotope we find a possible negative shift in the mean value of $\Delta\alpha/\alpha$ with respect to zero. *Thus, it was pointed out that unknown isotopic abundances may limit the effective use of MM method to achieve better constraints in future.*

All further analysis performed with UVES spectra (concentrating only on 6 Fe II transitions in a couple of good systems) fail to confirm any variation in α (Quast et al. 2004; Levshakov et al. 2005). In particular, Chand et al. (2006) analyse spectra of the bright quasar HE 0515–4414 taken with two different instruments, UVES at the VLT and HARPS at the 3.6 m telescope in La Silla. They show that the results of a non-evolving α reported in the literature based on UVES/VLT data should not depend strongly on wavelength calibration uncertainties and multiple component Voigt profile decomposition. Interestingly, usage of a combination of negative and positive shifters from a single ion avoid complications due to inhomogeneities and isotopic abundance differences. However, number of systems suitable for such measurements is unfortunately

Table 2. Summary of various constraints on $\Delta\alpha/\alpha$ using AD method

Number of system	z-range	$\Delta\alpha/\alpha$ (10^{-5})	References
128	0.2–3.7	$-(0.54 \pm 0.12)$	Murphy et al. 2003
23	0.4–2.3	$-(0.06 \pm 0.06)$	Srianand et al. 2004
	„	$-(0.64 \pm 0.43)$	Murphy et al. 2008
21	„	$+(0.01 \pm 0.14)$	Srianand et al. 2007

very small and limits achieved are of the same order of magnitude. *Thus independent analysis by different groups using VLT/UVES data fail to confirm the claimed variations in $\Delta\alpha/\alpha$ by Murphy et al. (2003). It is to be remembered that UVES spectra used for all these studies are a factor 1.5 better resolution and typically have a factor 3 higher S/N than the KECK/HIRES data used in Murphy et al. (2003).*

2.3. Analysis of LP data using VPFIT:

Murphy et al. (2008) using the same sample but with spectra having different error arrays, claimed that the UVES data analysis should lead to $\Delta\alpha/\alpha = (-0.64 \pm 0.36) \times 10^{-5}$ and attributed the difference between this value and the one we reported in Srianand et al. (2004) and Chand et al. (2004) to flawed χ^2 procedure. To address this criticism, we have refitted all our systems using VPFIT (the same code used by Murphy et al. (2003) for their analysis). In Fig. 3 we compare our old measurements (open circles) with the new measurements using VPFIT (filled circles). Our reanalysis using VPFIT keeping constant resolution across the spectrum and with identical initial guess parameters leads to $\Delta\alpha/\alpha = (0.01 \pm 0.15) \times 10^{-5}$ for 21 systems (excluding two systems that deviate at more than 3σ level). Like our original measurements the new ones also show very little scatter ($\chi^2_\nu \sim 1$) around the mean, contrary to the claims by Murphy et al. (2008). The apparent discrepancy claimed by Murphy et al (2008) is mainly due to two deviant systems that show variation in $\Delta\alpha/\alpha$ at more than 4σ level (see detailed discussion in Srianand et al. 2007).

3. Constraints using 21-cm absorption

As the energy of the hyperfine H I 21-cm transition is proportional to the combination of three fundamental constants, $x = \alpha^2 g_p / \mu$, high resolution optical and 21-cm spectra can be used together to probe the combined cosmological variation of these constants (Tubbs & Wolfe 1980). In the definition of x , α is the fine-structure constant, μ is the proton-to-electron mass ratio and g_p is the dimensionless gyromagnetic factor of the proton (see e.g. Tzanavaris et al. 2005). Recently, Tzanavaris et al. (2005) obtained, $\Delta x/x = (0.63 \pm 0.99) \times 10^{-5}$, using this technique but on a very poor sample of absorbers: 21-cm spectra were digitally scanned from the literature and some of the UV data were obtained using 4-m class telescopes ($R \sim 10000$ and $S/N \sim 15-20$). In addition, for some systems the radio source is extended. It is to be also noted that it is impossible to improve the spectra of few known $z > 1$ 21-cm absorbers used in the analysis, detected before 1987, due to the presence of man made radio frequency interference in the redshifted 21-cm frequencies. All this makes the available constraints based on 21-cm at $z \geq 1$ unreliable and not stringent.

To apply this technique, the redshift of the 21-cm line must be compared to that of UV lines of C I, Si II, C II and/or Mg II. Two difficulties arise: (i) the radio and optical sources must coincide: as QSOs in the optical can be considered as point like sources, it must be checked on VLBI maps that the corresponding radio source is also point like which is not true for all quasars; (ii) the gas that is responsible for 21-cm and UV absorption lines must be co-spatial: it is likely to be the case if the lines are narrow.

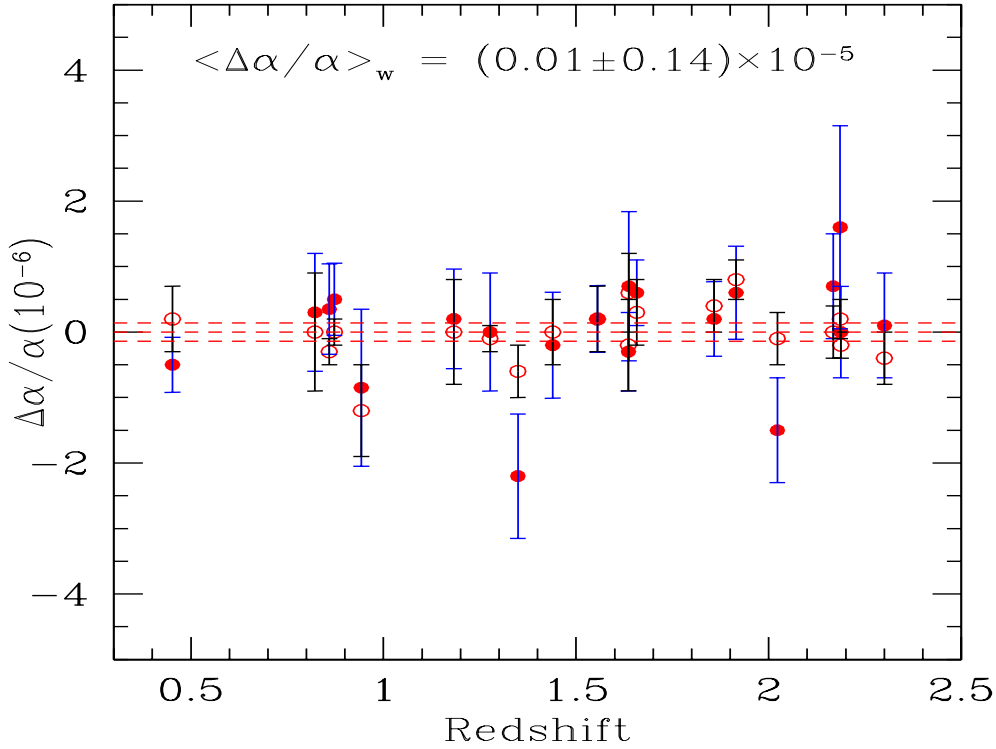


Fig. 3. Comparison of individual $\Delta\alpha/\alpha$ MM method measurement of Chand et al. (2004) (open circle) and the one obtained for the same data with VPFIT (filled circles).

Therefore systems in which the measurement can be performed must be selected carefully. Since the overall number of suitable systems is very small, a systematic survey is needed to enlarge the sample.

We have just completed a systematic survey of 21-cm absorption from strong SDSS Mg II systems using 400 hrs of GMRT 610 MHz observations. This has resulted in 9 new 21-cm absorption systems (see Gupta et al. 2007; Srianand et al. 2008; Gupta et al. 2009), more than doubling the number of such systems at $z > 1$. Unlike previous detections, we are able to follow these sources at high resolution (~ 0.9 km/s per channel) with GMRT with adequate S/N. This makes our complete sample of 21-cm absorption systems ideally suited for various follow-up analysis such as (i) probing the time variation of fundamen-

tal constants, (ii) deriving the physical conditions in the absorbing gas through spin temperature (T_S) measurements, (iii) studying the effect of metallicity and dust on the detectability of 21-cm absorption, and (iv) performing VLBA follow-up to study the morphology of the absorbing gas.

For any experiment on the variation of fundamental constants it is important to use narrow unblended absorption lines in order to derive precise positions of the lines. Therefore it is required that the 21-cm absorber has a single narrow component. Then high spectral resolution optical observations are essential to deblend absorption line into narrower components and identify the one that is responsible for the 21-cm absorption line. Covering a large number of species that trace the physical conditions of the cold neutral medium that pro-

duces the 21-cm absorption will confirm the physical association of the components.

In order to understand the milliarcsec structure of radio sources in our sample, we have just completed a detailed VLBA observations of all the sources in our sample (Gupta et al. in preparation). Using these low frequency VLBA images and high resolution spectroscopy we have identified 4 systems in our sample that satisfy the above conditions. High spectral resolution UVES/VLT observations of two of these systems are completed. The analysis of these systems has resulted in $\Delta x/x = (z_{UV} - z_{21})/(1 + z_{21}) = (0.0 \pm 1.5) \times 10^{-6}$ at $z = 1.3$. Another 21-cm absorption system at $z = 3.1$ that also shows H_2 in the UVES spectra provide $\Delta x/x = -(0.2 \pm 0.5) \times 10^{-5}$. These results are consistent with the absence of variation in α and μ noted by various authors using UVES data. We expect to provide the strongest constraint on $\Delta x/x$ by the time we complete different observations of our GMRT sample.

4. Conclusion

In conclusion, various samples we have analysed using VLT/UVES have failed to confirm the claimed variation in $\Delta\alpha/\alpha$. This is also true for other constants like μ and x where almost all the groups find only upper limits. Thus it boils down to the question why KECK/HIRES data has shown significant positive results. While UVES data has been analysed by various independent groups such analysis have not been carried out for the spectra obtained with KECK. Recently, HIRES observations with iodine cell have raised some concerns regarding the wavelength stability of KECK/HIRES data (Griest et al. 2009). What effect this has on Murphy et al's measurements needs to be understood. Similarly there were critical investigations of wavelength stability of UVES through various comparisons (to name few Chand et al. 2005, Molaro et al 2008, Thompson et al. 2009). It will be also interesting to investigate the iodine cell experiment results for UVES. In any case it appears the present day spectroscopy with standard calibration techniques provide a conservative 3σ upper limit of 0.3×10^{-5} at $0.5 \leq z \leq 2.5$.

This limit is at least a factor 10 higher than what is obtained in the terrestrial experiments. It is now important to push this limit to less than 10^{-6} . This should be possible with 30m class telescopes with better wavelength calibrations (using laser combs), higher resolution spectroscopy at higher S/N than what is achieved with 10m class telescopes today. In order to avoid uncertainty due to isotopic abundances it will be better to use systems with strong Ni II, Mn II and Fe II transitions so that single species based measurements should be possible. As pointed out before such systems are rare and efforts should be made to identify them through systematic searches for example based on SDSS data base.

Studies based on molecular absorption lines seen in the radio/mm wavelength range are more sensitive than that based on optical/UV absorption lines only. They provide constraints on the variation of a combination of α , μ and the proton g-factor (G_p). Murphy et al. (2001) have obtained $\Delta\alpha/\alpha = (-0.10 \pm 0.22) \times 10^{-5}$ at $z = 0.2467$ and $\Delta\alpha/\alpha = (-0.08 \pm 0.27) \times 10^{-5}$ at $z = 0.6847$, assuming a constant G_p . $\Delta\alpha/\alpha = (0.6 \pm 1.0) \times 10^{-5}$ is obtained from an OH system at $z = 0.247$ toward PKS 1413+135 (Kanekar et al. 2004). Such studies using radio observations have not been performed yet at $z \geq 1$ due to the lack of molecular absorption systems. In addition, most of the interesting extra-galactic systems that are used for the analysis are either gravitationally lensed or towards red blazars. In both cases covering factor of absorbing region becomes a issue. Future radio facilities (ALMA and SKA) will allow us to find typical systems that can be used for constraining the variation of fundamental constants. Here again there is a growing need of systematic survey to identify good systems at $z \geq 1$ for detailed spectroscopic studies.

Acknowledgements. This work is based on observations collected during several observing programmes at the European Southern Observatory with the Ultra-violet and Visible Echelle Spectrograph mounted on the 8.2 m KUEYEN telescope operated at the Paranal Observatory, Chile and at the Giant Meter-wave Radio Telescope in India. We gratefully acknowledge support from the Indo-French Centre for

the Promotion of Advanced Research (Centre Franco-Indien pour la Promotion de la Recherche Avancée).

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