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The effects of coupling variations on BBN

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Abstract. The effect of variations of the fundamental nuclear parameters on big-bang nucleosynthesis are modeled and discussed in detail taking into account the interrelations between the fundamental parameters arising in unified theories. Considering only ^{4}He, strong constraints on the variation of the neutron lifetime, neutron-proton mass difference are set. We show that a variation of the deuterium binding energy is able to reconcile the ^{7}Li abundance deduced from the WMAP analysis with its spectroscopically determined value while maintaining concordance with D and ^{4}He.

1. Introduction

Big bang nucleosynthesis (BBN) is one of the most sensitive available probes of physics beyond the standard model. The concordance between the observation-based determinations of the light element abundances of D, ^{3}He, ^{4}He, and ^{7}Li (Walker et al. 1991; Olive et al. 2000; Fields & Sarkar 2008), and their theoretically predicted abundances reflects the overall success of the standard big bang cosmology. Many departures from the standard model are likely to upset this agreement, and are tightly constrained (Malaney & Mathews 1993; Sarkar 1996; Cyburt et al. 2005).

There is rather excellent agreement between the predicted abundance of D/H as compared with the determined abundance from quasar absorption systems (Pettini et al. 2008) (and references therein). Indeed, what is often termed the success in cosmology between BBN and the CMB is in reality only the concordance between theory and observation for D/H at the WMAP value of η. Currently, there is no discrepancy between theory and observation for ^{4}He. But this success is tempered by the fact that ^{4}He is a poor baryometer, it varies only logarithmically with η, and the observational uncertainty it rather large (Olive & Skillman 2001, 2004; Fukugita & Kawasaki 2006).

It has also become generally accepted that there is a problem concerning the abundance of ^{7}Li. WMAP (Komatsu et al. 2009) has accurately fixed the value of the baryon-to-photon ratio, $\eta = (6.23\pm0.17)\times10^{-10}$ corresponding to $\Omega_B h^2 = 0.02273 \pm 0.00062$ where $\Omega_B = \rho_B/\rho_c$ is the fraction of critical density in baryons, $\rho_c = 1.88 \times 10^{-29} h^2$ g cm^{-3}, and h is the Hubble parameter scaled to 100 km/Mpc/s. At that value, the predicted abundance of ^{7}Li is approximately 4 times the observationally determined value (Cyburt et al. 2008). Several attempts at explaining this discrepancy by adjusting some of the key nuclear rates proved unsuccessful (Coc et al. 2004; Angulo et al. 2005; Cyburt et al. 2004; Cyburt & Pospelov 2009).

In this contribution, I will first consider the limits from BBN on the variation of couplings.

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Then I will briefly review the problem with ^{7}Li. Then I will consider in more detail the impact on variable couplings and a possible resolution to the ^{7}Li problem.

2. BBN limits on α

As noted in the theoretical overview, the success of BBN relies on a fine balance between the overall expansion rate of the Universe and the weak interaction rates which control the relative number of neutrons to protons at the onset of nucleosynthesis. Among the most sensitive probes of new physics using BBN is the ^{4}He abundance. The ^{4}He abundance can be estimated simply from the ratio of the neutron-to-proton number densities, n/p, by assuming that essentially all free neutrons are incorporated into ^{4}He. Thus,

$$Y = \frac{2(n/p)}{[1 + (n/p)]} \tag{1}$$

The neutron to proton ratio is determined by

$$(n/p) \sim e^{-\Delta m_N/T_f} \tag{2}$$

where Δm_N is the neutron-proton mass difference, and $T_f \sim 0.8$ MeV is the temperature at which the weak interaction rate for interconverting neutrons and protons falls below the expansion rate of the Universe. The value of n/p must then be adjusted to account for free neutron decays which occur prior to the onset of nucleosynthesis at $T \sim 0.1$ MeV.

The balance between the expansion rate and the weak interaction rate can be expressed as

$$G_F^2 T_f^5 \sim \Gamma_{wk}(T_f) \sim H(T_f) \sim \sqrt{G_N N} T_f^2 \tag{3}$$

where T_f is the freeze out temperature defined by this equation. Changes in the expansion rate, characterized by the Hubble parameter H which is proportional to $\sqrt{G_N N}$ where N is the number of relativistic particles, or changes in the weak rates, which may result from changes in fundamental parameters, affect the neutron-to-proton ratio and ultimately the ^{4}He abundance, Y. Thus one can use the concordance between the theory and the observational determination of the light element abundances to constrain new physics (Cyburt et al. 2005).

As one can see from eq. 1, changes in Y will be induced by changes in (n/p).

$$\frac{\Delta Y}{Y} \simeq \frac{1}{1 + (n/p)} \frac{\Delta(n/p)}{(n/p)} \tag{4}$$

Furthermore, changes in (n/p) are induced from changes in T_f and Δm_N

$$\frac{\Delta(n/p)}{(n/p)} \simeq \frac{\Delta m_N}{T_f} \left(\frac{\Delta T_f}{T_f} - \frac{\Delta^2 m_N}{\Delta m_N} \right) \tag{5}$$

Changes in the fine structure constant affect directly the neutron-proton mass difference which can be expressed as

$$\Delta m_N \sim a\alpha\Lambda_{QCD} + bv, \tag{6}$$

where $\Lambda_{QCD} \sim O(100)$ MeV is the mass scale associated with strong interactions, $v \sim O(100)$ GeV is the Higgs vacuum expectation value (vev) which determines the weak scale. A discussion on the contributions to Q can be found in Gasser & Leutwyler (1982). The constants a and b are numbers which fix the final contrbution to Δm_N to be -0.8 MeV and 2.1 MeV, respectively. As one can see, changes in α directly induce changes in Δm_N, which affects the neutron-to-proton ratio and hence the helium abundance (Kolb, Perry, & Walker 1986; Bergstrom, Iguri, & Rubenstein 1999; Campbell & Olive 1995; Nollett & Lopez 2002; Ichikawa & Kawasaki 2004).

^{4}He is observed in extragalactic HII regions, and unfortunately there are significant systematic uncertainties in abundance determinations. The analysis of Olive & Skillman (2001, 2004) found a systematically higher Y_p, with significantly increased errors, suggesting that previous analyses had underestimated their systematics. The resulting ^{4}He abundance was found to be

$$Y_p = 0.249 \pm 0.009. \tag{7}$$

In spite of the uncertainties, using the relatively good agreement between theory and observation, one can still obtain a reasonable limit on $\Delta\alpha/\alpha$ from BBN. If the dominant contribution to $\Delta\alpha$ comes from changes in Δm_N, then we have

$$\frac{\Delta Y}{Y} \approx \frac{\Delta\alpha}{\alpha} \tag{8}$$

Thus the current uncertainty in the observationally determined value of Y_p leads to a bound of $|\Delta\alpha/\alpha| < 0.04$. Since this limit is applied over the age of the Universe, we obtain a limit on the rate of change $|\dot{\alpha}/\alpha| \lesssim 3 \times 10^{-12}$ yr^{-1} over the last 13.7 Gyr. As will be discussed in more detail below, if the variations in α are coupled to other gauge and Yukawa couplings, this limit improves by about two orders of magnitude (Campbell & Olive 1995; Ichikawa & Kawasaki 2002; Müller et al. 2004; Coc et al. 2007; Dent et al. 2007)

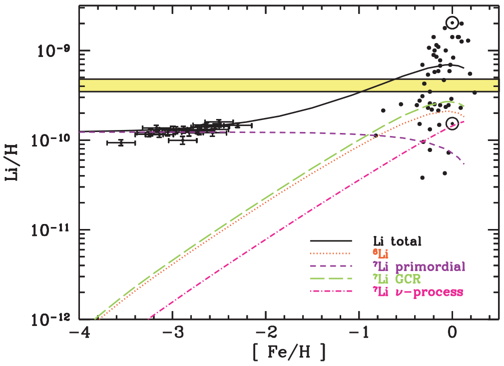

Fig. 1. Li abundances as a function of [Fe/H] adapted from Ryan et al. (2000). The various curves show the evolution of ^{6}Li and ^{7}Li in standard galactic cosmic ray nucleosynthesis models. The vertical band shows the BBN calculated abundance of ^{7}Li assuming the WMAP value for η.

3. The ^{7}Li Problem

The ^{7}Li problem is exemplified in Fig. 1 where data from Ryan et al. (2000) is shown in comparison to the predicted value of ^{7}Li assuming the WMAP value of η. Indeed a recent analysis of ^{7}Li in BBN (Cyburt et al. 2008) finds that the predicted value

$$^7\mathrm{Li/H} = (5.24^{+0.71}_{-0.62}) \times 10^{-10} \qquad (9)$$

at the WMAP value of $\eta = 6.23 \times 10^{-10}$. This represents a 23% increase in ^{7}Li over previous calculations. The increase is primarily due to an increase in the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ cross section. Newer data (Cyburt & Davids 2008) implies 17% increase in this reaction leading to a 16% increase in ^{7}Li. In addition, the 1.5% increase in η from the 3-year to 5-year WMAP data (Komatsu et al. 2009) leads to a 3% increase in ^{7}Li and finally another 1% increase is due to updated *pn* rates. In addition, the uncertainty the BBN ^{7}Li abundance is roughly a factor of 2 times smaller than previous determinations.

The BBN predicted value of ^{7}Li/H is significantly larger than observational determinations of the ^{7}Li abundance in metal-poor halo stars. Most observations lead to a ^{7}Li abundance in the range $(1 - 2) \times 10^{-10}$, consistent with the original determination by Spite & Spite (1982). Extrapolating the data to zero metallicity one arrives at a primordial value (Ryan et al. 2000) $\mathrm{Li/H}|_p = (1.23 \pm 0.06) \times 10^{-10}$, though the systematic uncertainties were recognized to be large and an abundance of

$$(\mathrm{Li/H}) = (1.23^{+0.68}_{-0.32}) \times 10^{-10} \qquad (10)$$

was derived at 95% confidence.

An important source for systematic error lies in the derived effective temperature of the star. $[\mathrm{Li}] = \log(^7\mathrm{Li/H}) + 12$ is very sensitive to the temperature, with $\partial[\mathrm{Li}]/\partial T_{\mathrm{eff}} \simeq 0.065 - 0.08$. Unfortunately there is no standard for determining effective temperatures, and for a given star, there is considerable range depending on the method used. This spread in temperatures was made manifest in the recent work of Meléndez & Ramírez (2004) using the infra-red flux method (IRFM) which showed differences for very low metallicities ([Fe/H] < -3) by as much as 500 K, with typical differences of ~ 200 K with respect to that of Ryan et al. (1999). As a consequence the derived ^{7}Li abundance was significantly higher with $\mathrm{Li/H}|_p = (2.34 \pm 0.32) \times 10^{-10}$ (Meléndez & Ramírez 2004; Fields et al. 2005).

A dedicated set of observations were performed with the specific goal of determining the effective temperature in metal-poor stars (Hosford et al. 2009). Using a large set of Fe I excitation lines (~ 100 lines per star), the Boltzmann equation was used with the excitation energies, χ_i to determine the temperature through the distribution of excited levels. Again, there was no evidence for the high temperatures reported in Meléndez & Ramírez

(2004), rather, temperatures were found to be consistent with previous determinations. The mean ^{7}Li abundance found in Hosford et al. (2009) was Li/H = $(1.3 - 1.4 \pm 0.2) \times 10^{-10}$, consistent with the bulk of prior abundance determinations.

There are of course other possible sources of systematic uncertainty in the ^{7}Li abundance. It is possible that some of the surface ^{7}Li has been depleted if the outer layers of the stars have been transported deep enough into the interior, and/or mixed with material from the hot interior; this may occur due to convection, rotational mixing, or diffusion. It is also possible that the lithium discrepancy is a sign of new physics beyond the Standard Model. One possibility is the cosmological variation of the fine structure constant (Coc et al. 2007) as will be discussed below. Another potential solution to the lithium problem is particle decay after BBN which could lower the ^{7}Li abundance (and produce some ^{6}Li as well) (Jedamzik 2004). This has been investigated in the framework of the constrained minimal supersymmetric Standard Model if the lightest supersymmetric particle is assumed to be the gravitino (Feng et al. 2004; Ellis et al. 2005) and indeed, some models have been found which accomplish these goals (Jedamzik et al. 2006; Cyburt et al. 2006; Pospelov et al. 2008).

4. Varying Constants and the ^{7}Li Problem

As discussed in the theoretical overview (in these proceedings), in unified theories of particle interactions, one generally expects that a change in the fine structure constant would directly imply a change in other gauge couplings (Campbell & Olive 1995), as well as and perhaps more importantly, variations in the QCD scale Λ_{QCD}. In addition, one might expect variations in the Yukawa couplings and Higgs vev as well. In what follows, I will assume possible changes in all gauge and Yukawa couplings as well as changes in Λ_{QCD} and v.

From the low energy expression for Λ_{QCD},

$$\Lambda = \mu \left(\frac{m_c\, m_b\, m_t}{\mu^3} \right)^{2/27} \exp\left(-\frac{2\pi}{9\alpha_s(\mu)} \right), \qquad (11)$$

we can determine the relation between the changes in Λ and our other variable constants (Campbell & Olive 1995; Langanker, Segre, & Strassler 2002; Dent & Fairbairn 2003; Calmet & Fritzsch 2002; Damour, Piazza, & Veneziano 2002),

$$\begin{aligned} \frac{\Delta\Lambda}{\Lambda} &= R\frac{\Delta\alpha}{\alpha} \qquad (12) \\ &\quad + \frac{2}{27}\left(3\frac{\Delta v}{v} + \frac{\Delta h_c}{h_c} + \frac{\Delta h_b}{h_b} + \frac{\Delta h_t}{h_t} \right). \end{aligned}$$

Typical values for R are of order 30 in many grand unified theories, but there is considerable model dependence in this coefficient (Dine et al. 2003).

Furthermore, in theories in which the electroweak scale is derived by dimensional transmutation, changes in the Yukawa couplings (particularly the top Yukawa) leads to exponentially large changes in the Higgs vev. In such theories,

$$\frac{\Delta v}{v} \sim S\frac{\Delta h}{h} \qquad (13)$$

with $S \sim 160$, though there is considerable model dependence in this value as well. This dependence gets translated into a variation in all low energy particle masses (Dixit & Sher 1988; Scherrer & Spergel 1993; Yoo & Scherrer 2003).

The quantities of interest in this calculation are (Coc et al. 2007), the neutron-proton mass defference, $Q = \Delta m_N$, the neutron mean life, τ_n, and the deuterium binding energy, B_D. The variation of Q will then scale as

$$\begin{aligned} \frac{\Delta Q}{Q} &= -0.6\left[\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right] \qquad (14) \\ &\quad + 1.6\left[\frac{\Delta(h_d - h_u)}{h_d - h_u} + \frac{\Delta v}{v} \right]. \end{aligned}$$

The neutron lifetime can be well approximated by

$$\begin{aligned} \tau_n^{-1} &= \frac{1}{60}\frac{1 + 3\, g_A^2}{2\pi^3} G_F^2\, m_e^5 \,[\\ &\quad \sqrt{q^2 - 1}(2q^4 - 9q^2 - 8) \\ &\quad + 15\ln(q + \sqrt{q^2 - 1}) \Big], \qquad (15) \end{aligned}$$

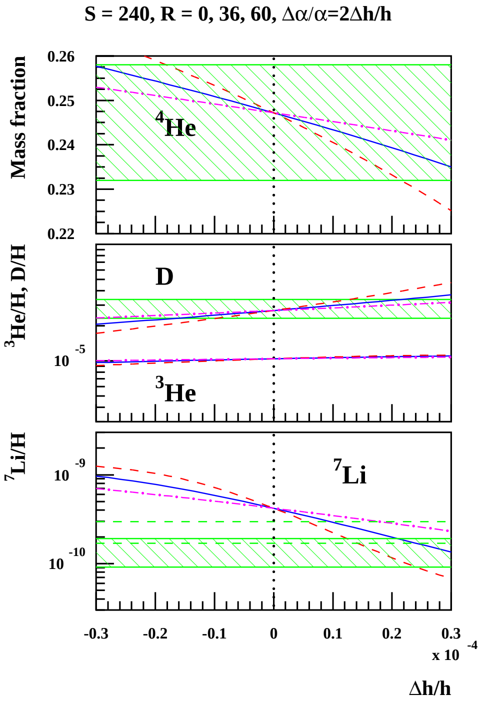

Fig. 2. Primordial abundances of ^{4}He, D, ^{3}He and ^{7}Li as a function of $\Delta h/h = (1/2)\Delta\alpha/\alpha$ when allowing a variation of the fine structure constant for three values of the R parameter: $R = 0$ (red lines), $R = 36$ (blue lines) and $R = 60$ (magenta lines).

where $q = Q/m_e$. Since $G_F = 1/\sqrt{2}v^2$ and $m_e = h_e v$. We then have for the relative variation of the neutron lifetime,

$$\frac{\Delta\tau_n}{\tau_n} = -4.8\,\frac{\Delta v}{v} + 1.5\,\frac{\Delta h_e}{h_e} - 10.4\,\frac{\Delta(h_d - h_u)}{h_d - h_u} + 3.8\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right). \quad (16)$$

Finally, based on a potential model for the nucleon mass including meason exchanges, we can write (Dmitriev & Flambaum 2003; Dmitriev et al. 2004; Flambaum & Shuryak 2002, 2003; Coc et al. 2007)

$$\frac{\Delta B_D}{B_D} = 18\,\frac{\Delta\Lambda}{\Lambda} - 17\left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s}\right). \quad (17)$$

Using the relations in eqs. 12 and 13, we can write

$$\frac{\Delta B_D}{B_D} = -13(1+S)\,\frac{\Delta h}{h} + 18R\,\frac{\Delta\alpha}{\alpha} \quad (18)$$

$$\frac{\Delta Q}{Q} = 1.5(1+S)\,\frac{\Delta h}{h} - 0.6(1+R)\,\frac{\Delta\alpha}{\alpha}, \quad (19)$$

$$\frac{\Delta\tau_n}{\tau_n} = -(8+4S)\frac{\Delta h}{h} + 3.8(1+R)\frac{\Delta\alpha}{\alpha}. \quad (20)$$

If in addition, we relate the gauge and Yukawa couplings through $\delta h/h = (1/2)\delta\alpha/\alpha$, we can further write,

$$\frac{\Delta B_D}{B_D} = -[6.5(1+S) - 18R]\frac{\Delta\alpha}{\alpha} \quad (21)$$

$$\frac{\Delta Q}{Q} = (0.1 + 0.7S - 0.6R)\frac{\Delta\alpha}{\alpha} \quad (22)$$

$$\frac{\Delta\tau_n}{\tau_n} = -[0.2 + 2S - 3.8R]\frac{\Delta\alpha}{\alpha}, \quad (23)$$

These relations can then be implemented in a BBN calculation. Some results (Coc et al. 2007) are shown in Fig. 2 where the parameter $S = 240$ is held fixed and results are shown for three values of R. For $R = 36$, the deuterium abundance can be used to set a limit

$$-1.6\times10^{-5} < \frac{\Delta h}{h} < 2.1\times10^{-5}. \quad (24)$$

5. Conclusions

While there is overall concordance between BBN calculations of D/H and Y and their observational determination, there is at present a problem with ^{7}Li. It is entirely possible that the fundamental constants of nature have varied across cosmological timescales, though there are many constaints on such variations. When the constant variations are coupled as one might expect in a unified theory, BBN limits are of order a few $\times10^{-5}$. It is possible that a variation of this order would decrease the primordial abundance of ^{7}Li without overproducing D/H. As such, one can wonder if this is a viable solution to the ^{7}Li problem.

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