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Accelerating universe and the time-dependent fine-structure constant

Yasunori Fujii

Advanced Research Institute for Science and Engineering, Waseda University
Tokyo, 169-8555, Japan, e-mail: `fujii@e07.itscom.net`

Abstract. Theoretical background of our proposed relation between the accelerating universe and the time-variability of the fine-structure constant is discussed, based on the scalar-tensor theory, with emphases on the intuitive aspects of underlying physical principles. An important comment is added on the successful understanding of the size of the effective cosmological constant responsible for the acceleration, without appealing to fine-tuning parameters.

1. Introduction

We start with assuming a gravitational scalar field as the dark-energy supposed to be responsible for the accelerating universe. Also from the point of view of unification, a scalar field implies a time-variability of certain constants observed in Nature. In this context we once derived a relation for the time-variability of the fine-structure constant α:

$$\frac{\Delta\alpha}{\alpha} = \zeta \mathcal{Z} \frac{\alpha}{\pi} \Delta\sigma, \tag{1}$$

as was detailed in Chapter 6.6 of(Fujii & Maeda 2003), where σ is the scalar field in action in the accelerating universe. Then we compared the dynamics of the accelerating universe, on one hand, and $\Delta\alpha/\alpha$ derived from QSO, Oklo and atomic clocks, on the other hand Fujii (2008, 2009a). In this article we discuss its theoretical background based on the scalar-tensor theory invented first by Jordan in 1955 (Jordan 1955), focusing upon the underlying physical principles. For details, see our references (Fujii & Maeda 2003; Fujii 2007).

2. Scalar-tensor theory

The basic Lagrangian is given by

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}\xi\phi^2 R - \epsilon\frac{1}{2}\left(\nabla\phi\right)^2 + L_{\rm m} - \Lambda\right), \tag{2}$$

where ϕ is the scalar field, while $(\nabla\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. The two parameters ϵ, ξ are related to the better known symbol ω by $\epsilon = \mathrm{Sgn}(\omega), 4\xi = |\omega|^{-1}$. We also included $-\Lambda$ as the simplest extension from the original scalar-tensor theory. The first term is a well-known nonminimal coupling term defining an effective gravitational constant $8\pi G_{\rm eff} = (\xi\phi^2)^{-1}$.

$L_{\rm m}$ is the matter Lagrangian for which Brans and Dicke (Brans & Dicke 1961) added an assumption that the field ϕ never enters $L_{\rm m}$, because they could save the idea of Weak Equivalence Principle (WEP) only in this way. Since then, their proposal combined with Jordan's original theory has been known widely as the Brans-Dicke theory. We prefer, however, to use a more modest name like the BD model, partly because it seems likely to be replaced ultimately by another model for a better understanding of the accelerating universe, as will be argued later in this article.

Also for the later convenience, let us give an example of $L_{\rm m}$ for a free massive Dirac field as a convenient representative of matter fields;

$$L_{\rm m} = -\overline{\psi}\,(\partial\!\!\!/ + m)\,\psi. \tag{3}$$

Due to the assumed absence of ϕ, the mass of this Dirac particle is simply m, a pure constant. By extending this simple argument we find a general (and traditional) rule that the constancy of masses of matter particles is a unique feature of the BD model.

We use the reduced Planckian units defined by $c = \hbar = M_{\rm P}(= (8\pi G)^{-1/2}) = 1$. As an example, the present age of the universe $t_0 \approx 1.4\times 10^{10}$y can be re-expressed as $\sim 10^{60}$.

We apply what is known as the conformal transformation defined by $g_{\mu\nu} \rightarrow g_{*\mu\nu} = \Omega^2 g_{\mu\nu}$, where $\Omega(x)$ is an arbitrary spacetime function. This allows us to re-express the Lagrangian (2) now in terms of the new transformed metric $g_{*\mu\nu}$. A special choice $\Omega^2 = \xi\phi^2$ leads to the particularly simple result;

$$\mathcal{L} = \sqrt{-g_*}\left(\frac{1}{2}R_* - \frac{1}{2}\,(\nabla_*\sigma)^2 - V(\sigma) + L_{*\rm m}\right), \tag{4}$$

where the R_* term is multiplied by a pure constant, no nonminimal coupling term, hence the same as in the Einstein-Hilbert term. For this reason we say we have moved to the Einstein (conformal) frame. We also say that we come from the Jordan frame, so-called widely. We learn a lesson that the effective gravitational constant can be constant or variable depending on what frame is chosen.

The scalar field ϕ in (2) has been replaced in (4) by σ which are related to each other by $\phi = \xi^{-1/2}e^{\zeta\sigma}$ where $\zeta = (6 + \epsilon\xi^{-1})^{-1/2}$. Notice, unlike ϵ in (2), positivity of the kinetic-energy term of the *diagonalized* σ is assured by $\zeta^2 > 0$ even with a negative ϵ (Fujii & Maeda 2003; Fujii 2007).

Also a constant term Λ in (2) has been converted to a potential $V(\sigma) = \Lambda e^{-4\zeta\sigma}$. Otherwise we put the asterisks almost everywhere. Again for the later convenience in discussing cosmology, we add

$$a_* = \Omega a, \quad \text{and} \quad dt_* = \Omega dt. \tag{5}$$

According to the first relation, the way of the cosmological expansion may differ from frame to frame. The second equation expresses a coordinate transformation of t to the cosmic time t_* re-defined in the Einstein frame.

The matter fields are also transformed according to

$$\psi_* = \Omega^{-3/2}\psi, \quad \text{with} \quad m_* = \Omega^{-1}m, \tag{6}$$

which yields

$$L_{*\rm m} = -\overline{\psi}_*\left(\nabla\!\!\!\!/_* + m_*\right)\psi_*, \tag{7}$$

showing a form-invariance as compared with (3), though we ignored some complications as discussed in Appendix F of (Fujii & Maeda 2003).

We may compare (2) with the effective Lagrangian for the closed strings formulated in higher-dimensional spacetime as shown by Eq. (3.4.56) of (Green Schwarz & Witten 1987);

$$\mathcal{L}_{\rm st} = \sqrt{-g}e^{-2\Phi}\left(\frac{1}{2}R + 2\,(\nabla\Phi)^2 + \cdots\right), \tag{8}$$

with the unique occurrence of a scalar field Φ called dilaton. By introducing ϕ by $\phi = 2e^{-\Phi}$, we re-express this effective Lagrangian, part of which agrees precisely with the first two terms of (2) with the choice; $\epsilon = -1, \xi = 1/4$, or $\omega = -1$. In this sense we might call the Jordan frame as the string frame or the theoretical frame, suggesting that the Jordan frame represents a world in which unification is realized. This might provide a rationale to introduce a constant Λ of the Planckian size in (2). Then how about the observational or physical frame? According to Dicke (Dicke 1962) in this connection, the conformal transformation is a local change of units. Let us emphasize this view.

Suppose we use an atomic clock, measuring time in reference to a frequency of certain atomic transition, in which we have the fundamental unit based on $m_{\rm e}$, the electron mass, ignoring the realistic choice of the reduced mass, for simplicity. Then we find we have no way to detect any change, if any, of $m_{\rm e}$ itself, as far as we continue to use the atomic clock. This situation might be put in a more general term as Own Unit Insensitivity Principle (OUIP) Fujii (2007). We then come to say that using an atomic clock implies that we are in a physical frame in which m_e is kept constant. According to what we discussed about the BD model particularly in connection with the constancy of the masses, we find ourselves in the Jordan frame, which is then identified with the physical frame.

The above simple argument can be extended to astronomical observations based on measuring redshifts of atomic spectra, in which we also use m_e as a fundamental unit in the same way as in using an atomic clock. In this context, we repeat the same statement on the physical frame as the Jordan frame, as far as we accept the BD model. But the situation will be subject to change in the presence of Λ.

3. Cosmology

We now discuss cosmology in the presence of Λ assumed to be positive with the Planckian size, first in the Jordan frame. Under usual simplifying assumptions on the metric also in radiation-dominance, for the moment, we may assume the spatially uniform ϕ depending only on the cosmic time t. We then write down the cosmological equations, obtaining asymptotic solutions,

$$\begin{aligned} a &= \text{const}, \\ \phi &= \sqrt{\frac{4\Lambda}{6\xi+\epsilon}}\, t, \\ \rho &= -3\Lambda\frac{2\xi+\epsilon}{6\xi+\epsilon}, \end{aligned} \tag{9}$$

where a is the scale factor while ρ is the matter density.

Most striking is the first line describing a *static* universe. We emphasize that (9) is a set of the attractor solutions, which any solutions starting with whatever initial values tend to, as confirmed by our recent reanalysis (Maeda & Fujii 2009). In this sense we can hardly accept the Jordan frame as a realistic physical frame, *contrary* to what we concluded toward the end of the preceding section.

We also add that our solution shows no smooth behavior in the limit $\Lambda \to 0$. In other words, in the presence of Λ the solution could be quite different from what had been known in its absence. The presence of Λ may result in a drastic change. So how about in the Einstein frame?

The solution can be obtained either by starting from the cosmological equations derived from (4), or by applying the conformal transformation directly to those solutions given by (9). The result is

$$\begin{aligned} a_* &= t_*^{1/2}, \\ \sigma &= \bar{\sigma} + \frac{1}{2}\zeta^{-1}\ln t_*, \\ \rho_\sigma &= \frac{3}{16}t_*^{-2}, \\ \rho_* &= \frac{3}{4}\left(1-\frac{1}{4}\zeta^{-2}\right)t_*^{-2}, \end{aligned} \tag{10}$$

where $\bar{\sigma}$ is given by $\Lambda e^{-4\zeta\bar{\sigma}} = (16\zeta^2)^{-1}$, while the energy densities ρ_σ and ρ_* are for σ and the matter, respectively. The first line, due to the relations in (5), shows that the universe does expand, precisely in the same way as in the ordinary radiation-dominated universe. We might be tempted to accept the Einstein frame as the physical frame. By comparing the first of (5) and the second of (6), however, we find

$$m_* \sim t_*^{-1/2}. \tag{11}$$

This is not constant in contradiction with OUIP, which would have been respected only if m_* were to be constant. The universe looks again unrealistic; we have no way to accommodate a physical frame which should be realistic.

We trace the origin of this difficulty back to the result of the static universe in the Jordan frame combined with constant mass of matter fields as discussed immediately after (3) thus yielding $am = a/m^{-1}$ = const, implying that the universe expands in the same rate as the microscopic length scale provided by the particle mass. This is also likely carried over to the Einstein frame; $a_* m_*$ = const, which is totally *inconsistent* with current view on today's cosmology.

We naturally wondered if we can find a way out by somehow forcing m_* to stay constant, still maintaining the expansion of $a_* \sim$

$t_*^{1/2}$ as it is so that we could accept the Einstein frame as a physical frame. We finally decided to *leave* the BD model. We revised our previous way to determine L_m, replacing the mass term in (3) by the Yukawa-coupling term $-f\overline{\psi}\psi\phi$. Then we do find a constant m_*, generated spontaneously, hence achieving our expected goal, the Einstein frame identified with the physical frame. The coupling constant f is dimensionless. This simple feature is shared by any other terms in the basic Lagrangian (2), except for the Λ term. In this sense we have a global scale invariance, though partially, so the name, the *scale-invariant* model to *replace* the BD model.

As a price, however, we allowed the ϕ to enter L_m against what BD assumed. So we should expect WEP violating terms, which turn out fortunately unobservable in the classical limit according to the scale-invariant model; constant masses imply their decoupling from σ depriving matter particles of their role to detect the violating effects. But there are quantum effects as well arising from the interactions among matter fields, regenerating the violating terms, which are found to be somewhat suppressed according to the estimate by means of quantum anomalies, a well-established technique in the relativistic quantum field theory. This is precisely the way the relation (1) has been derived.

We point out that the crucially important ingredients were the simple and straightforward arguments on how to define a physical conformal frame in the presence of Λ. Also to be re-iterated is that the entire result hinges upon the static universe encountered in the attractor solution for the radiation-dominated universe in the Jordan frame, though this rather unexpected behavior is found to be somewhat exceptional. In fact we face even more complicated situation for the dust-dominated universe, but with the same final result in terms of the scale-invariant model. For more details, as well as for the ensuing phenomenological analyses, see our references (Fujii & Maeda 2003; Fujii 2007, 2008, 2009a).

This scale-invariant model turns out to show an advantage in the present approach. It allows an interpretaion of σ as a Nambu-Goldstone boson of dilatation symmetry, massless dilaton, to be consistent with the absence of the scalar mass term in the starting Lagrangain (2), assumed widely but rarely on any ground stated explicitly. Quantum effects mentioned above eventually break the symmetry reducing σ to a pseudo-NG boson, still making it easy to understand why WEP violation is supposed to be mediated by a scalar field, likely as light as $\sim 10^{-9}$eV, or the force-range of macroscopic distances $\sim$ 100m, up to a latitude of a few orders of magnitude, as suggested before (Fujii 1971), revisited again in Chapter 6.4 of (Fujii & Maeda 2003). Also in this context, the same quantum anomaly effect is responsible for $\Delta\alpha/\alpha$, which by itself may not look relevant directly to the WEP violation, whereas the same type of technique can be applied to derive the time-dependent mass ratio μ between proton and electron, which is obviously WEP violating as far as σ participates in gravitational phenomena.

4. A comment

Finally we emphasize that the present approach is related closely to the question how successfully we understand the accelerating universe. Of central importance is, as was discussed in (Fujii 2009b), the size of Λ_eff required to fit the observed acceleration, as small as, $\sim 10^{-120}$ in the Planckian units, a well-known number which has symbolized what is known as a fine-tuning problem, a manifestation of our uneasiness if our theory is good enough to the accuracy of as much as 120 orders of magnitudes. But a numerical similarity

$$\Lambda_\mathrm{eff} \sim t_{*0}^{-2}, \tag{12}$$

with $t_{*0} \sim 10^{10}$y $\sim 10^{60}$ appears too remarkable to be dismissed as a mere coincidence. Note that this result had been foreseen in (Fujii 1982; Bertolami 1986) though based on somewhat different interpretation of the scalar-tensor theory including Λ.

In fact our cosmological solution in the Einstein frame contains the third line of (10);

$$\rho_\sigma \sim t_*^{-2}, \tag{13}$$

where LHS is the density of the dark-energy, and can be interpreted as $\Lambda_{\rm eff}$. This might be called Scenario of a Decaying Cosmological Constant, providing with an immediate justification of the relation (12). To be noticed is that today's value of LHS of (13) is small nearly automatically simply because we are old enough as indicated on RHS, requiring no need for an extreme and unnatural fine-tuning processes, somewhat reminiscent of Dirac's argument (Dirac 1938). This simplicity and naturalness deserves to be called a major success of the scalar-tensor theory in its simplest extension by introducing Λ in the Jordan frame, unparalleled by any other phenomenological approaches.

But we concede that this argument applies only to the overall behaviors of the universe. Another kind of mechanism has to be worked out for non-overall behaviors, including the extra acceleration of the universe as we see it. We may have a choice, but in such a way to inherit what we called the success. The present author expects an oscillation of σ (Fujii & Maeda 2003; Fujii 2008, 2009a) then the same in the observed $\Delta\alpha/\alpha$ and $\Delta\mu/\mu$ to a better precision hopefully to be realized in the near future.

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