

# Phase closure image reconstruction for infrared interferometry

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**Abstract.** Much of the analysis in optical and infrared interferometry has relied on modeling of the visibility data, which can lead to ambiguous interpretation. However, next generation interferometric instruments are being designed to provide imaging capabilities in the optical and near-infrared.

In order to test the imaging potential of a next generation near-infrared interferometer – the VLTI-spectro-imager (VSI) – we have embarked on a study of image reconstruction and analysis of key science cases. Our main aim was to test the influence of the number of telescopes, observing nights and distribution of the visibility points on the quality of the reconstructed images. Our results show that observations using six ATs during one complete night yield the best results in general and is critical in most science cases.

An optical, six telescope, ~200 meter baseline configuration will achieve 4 mas spatial resolution in the K band, which is comparable to ALMA and almost 50 times better than JWST will achieve. Our results show that such an instrument will be capable of imaging, with unprecedented detail, a plethora of sources, ranging from complex stellar surfaces to microlensing events.

**Key words.** Techniques: interferometric – Techniques: image processing

## 1. Introduction

Given an incoherent source, the observed brightness distribution is given by:

$$I^{obs}(x, y) = PSF(x, y) * I^{true}(x, y) + N(x, y)$$

where  $x$  and  $y$  are angular displacements on the plane of the sky with the phase center as origin,  $PSF(x, y)$  is the point spread function,  $I^{true}(x, y)$  is the true object brightness distribution and  $N(x, y)$  is the noise.

In practice, interferometry does not make measurements in the image plane but in Fourier

space. The relevant quantity is called the complex visibility and is measured at each  $uv$  point, the position vector of the baseline projected on a plane perpendicular to the source direction, which together define the  $uv$  plane:

$$V^{obs}(u, v) = V^{true}(u, v) \times S(u, v) + N'(u, v)$$

where  $S(u, v)$  is the sampling function,  $V^{true}(x, y)$  is the true visibility and  $N'(u, v)$  is the noise in Fourier space.

The van Cittert-Zernike theorem states that the brightness distribution can be recovered by the inverse Fourier transform and deconvolution of the observables:

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$$I^{true}(x, y) * PSF(x, y) = \int \int V^{true}(u, v) \times S(u, v) e^{-2\pi i(ux+vy)} du dv$$

Closure phase imaging is the standard method for imaging in optical interferometry (Weigelt et al. 1998). This technique relies on two quantities related to the complex visibility - the squared visibility or power spectrum:

$$V(u_i, v_i) V^*(u_i, v_i) = |V(u_i, v_i)|^2$$

and the triple product or bi-spectrum:

$$T(u_i, v_i, u_j, v_j) = V(u_i, v_i) V(u_j, v_j) V^*(u_i + u_j, v_i + v_j)$$

to perform image reconstruction. The triple product is the product of the complex visibilities on the baselines forming a closed loop joining any three telescopes -  $u_i+u_j=u_k$ ,  $v_i+v_j=v_k$ ,  $u_i, v_i, u_j, v_j$ . The modulus of the triple product is the triple product amplitude and the argument is the closure phase. The closure phase is independent of atmospheric-induced phase variations.

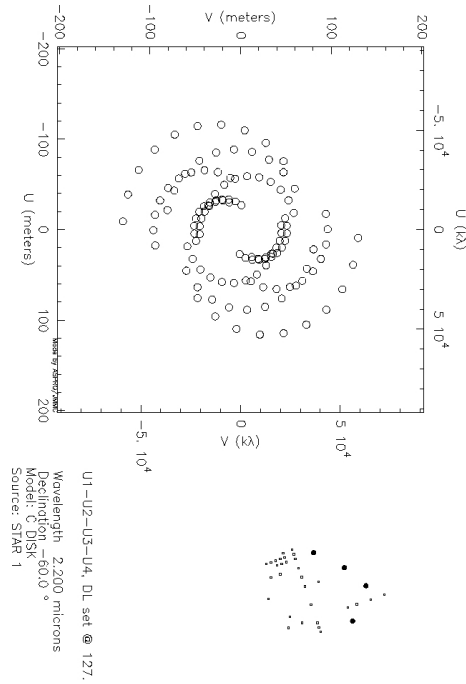
### 2. Array configuration

In order to test the importance of the number, type and configuration of the telescopes and the number of observing nights on the image reconstruction, we have adopted three telescope setups. The setups were chosen to optimize the  $uv$  plane coverage, while taking into consideration such limitations as the VLTI delay lines.

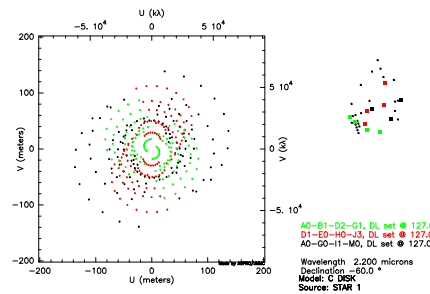
### 3. OIFITS file generation

Synthetic images were provided by the science case groups (Garcia et al. 2007) and imported into ASPRO, a model and simulation tool for AMBER and MIDI. Assuming the objects were located at a  $-60$  degree declination angle and using the configurations above, typical object magnitudes were used to generate noisy, squared visibilities and closure phases in an OIFITS file.

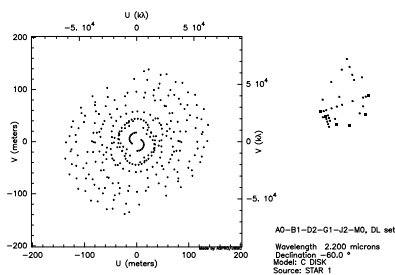
We assumed an one hour per calibrated measurement. The actual on-source integration time is, however, 10-15 minutes per hour due to overheads. The total integration time assumes an entire transit.



**Fig. 1.** 4 UT × 1 night configuration (U1-U2-U3-U4) – faint source configuration.



**Fig. 2.** 4 AT × 3 night configuration (A0-B1-D2-G1, D1-E0-H0-J3, A0-G0-I1-M0) - the minimum imaging  $uv$  coverage consistent with the current VLTI capabilities.



**Fig. 3.** 6 AT  $\times$  1 night configuration (A0-B1-D2-G1-J2-M0) – has same baselines as 4 AT  $\times$  3 night configuration but less uv points.

#### 4. Phase closure image reconstruction

The role of image reconstruction is to obtain the best approximation –  $I^{model}(x,y) \sim I^{true}(x,y)$  – to the true brightness distribution. For phase closure image reconstruction the solution involves minimizing the value of a penalty term subject to some condition (positivity and normalization):

$$P(z) = P_L(z) + P_{prior}(z) = P_L(z) + \mu R(z)$$

where  $z$  represents the intensity of the pixels,  $P_L$  is the likelihood penalty,  $P_{prior}$  is the prior penalty,  $R(z)$  is the regularization term and  $\mu$  is a multiplier (hyperparameter) tuned so that at the solution, the likelihood terms are equal to their expected values.  $P_L$  enforces agreement with the data, while  $P_{prior}$  provides information where the data fail to do so;  $P_{prior}$  is responsible for the so-called regularization of the inverse problem.

The solution is therefore found by minimizing the likelihood penalty term for squared visibilities and triple products under a prior constraint:

$$P_L = P_{V^2} P_T \propto -\exp \frac{\chi_{V^2}^2 + \chi_T^2}{2}$$

where  $\chi_{V^2}^2$  is the likelihood term with respect to the squared visibility data and  $\chi_T^2$  to the triple product. Typically, the data penalties are defined assuming the measurements follow Gaussian statistics:

$$\chi_{V^2}^2(z) = \sum \frac{1}{\sigma_{V^2}^2} (V_{data}^2 - V_{model}^2)^2$$

$$\chi_T^2(z) = \sum \frac{1}{\sigma_T^2} \left| e^{i\phi_T^{data}} - e^{i(\varphi_1 + \varphi_2 - \varphi_3)} \right|^2$$

where  $V^2$  refers to the squared visibilities (data and model),  $\phi_T^{data}$  to the measured closure phase,  $\varphi_1$ ,  $\varphi_2$ ,  $\varphi_3$  to three individual phases and  $\sigma$  to the respective standard deviations.

Regularization can enforce agreement with some preferred and/or exact properties of the solution. Because we want to favor compact sources, we have implemented smoothness as the regularization constraint:

$$R(z) = \sum w |z|^2$$

where the regularization weights  $w$  are chosen in order to ensure spectral smoothing, that is, it enforces smoothness of the Fourier spectrum of the image and enforces compactness of the brightness distribution of the field of view.

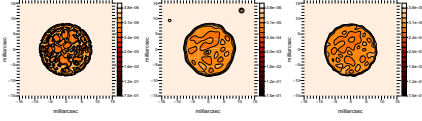
We have implemented the Multi-Aperture Reconstruction Algorithm (MIRA; version 0.7; April 2008) as developed by Eric Thiebaut with support from the Jean-Marie Mariotti Center (JMMC). MIRA works in the YORICK and C platform and is optimized to handle optical interferometric data with sparse  $uv$  coverage (Thiebaut 2005).

#### 5. Image reconstruction analysis

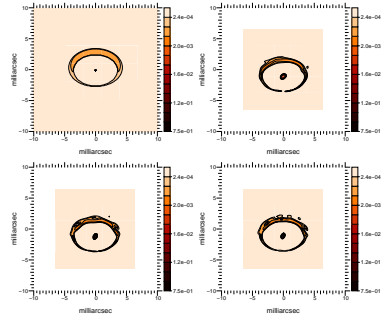
Because MIRA typically super-resolves in the image reconstruction, we have compared our reconstructed images with the synthetic images provided by the science groups. Relative astrometry information was obtained for the images using ds9. Photometry of the image components was performed using IRAF procedure PHOT.  $SNR_{V^2}$  and  $SNR_T$  correspond to the range of signal-to-noise ratios in the squared visibilities and closure phases measured in the data. We give the example of two of the science cases below.

#### 6. Conclusions

Next generation optical interferometric instrumentation like the VSI (Malbet et al. 2008) will



**Fig. 4.** A simulated stellar surface of a M giant (top left); MIRA reconstruction 4 AT  $\times$  3 nights configuration,  $\text{SNR}_{V^2}=1.5\text{-}541$ ,  $\text{SNR}_T=0.01\text{-}05$  (top right); MIRA reconstruction 6 AT  $\times$  1 night configuration,  $\text{SNR}_{V^2}=0.1\text{-}600$ ,  $\text{SNR}_T=0.1\text{-}115$ .



**Fig. 5.** A simulated stellar surface of the structure of inner disks surrounding YSOs (top left); MIRA reconstruction 4 UT  $\times$  1 night configuration (top right); MIRA reconstruction 4 AT  $\times$  3 nights configuration (bottom left); MIRA reconstruction 6 AT  $\times$  1 night configuration (bottom right).

be designed to offer superb imaging capabilities in order to bypass ambiguities in the interpretation of visibility data. It will be particularly novel in that it will allow spatial resolutions of milliarcseconds, comparable to ALMA and even better the future JWST.

In order to test the VSI, we have run several tests on key scientific cases. Our results show that the use of the 6 AT  $\times$  1 night configuration generally yields better reconstructed images and is crucial in some science cases. Such diverse objects as AGN, stellar

**Table 1.** YSO. Radii units are in pixels.

	Image	MIRA 4 UT
flux star	18.1%	18.7%
flux disk	81.9%	81.3%
ratio	0.2	0.2
hline inner diameter	$65 \times 45$	$65 \times 45$
outer diameter	$85 \times 60$	$80 \times 56$
$\text{SNR}_{V^2}$	-	2055-5215
$\text{SNR}_T$	-	0.04-0.2

	MIRA 4 AT 3	MIRA 6 AT
flux star	18.6%	18.6%
flux disk	81.4%	81.4%
ratio	0.2	0.2
inner diameter	$65 \times 45$	$65 \times 45$
outer diameter	$82 \times 60$	$85 \times 60$
$\text{SNR}_{V^2}$	89-1385	12-1363
$\text{SNR}_T$	0.04-0.5	0.2-12

surfaces and disks are shown possible to image with future VLTI instrumentation.

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