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Radiative accelerations in stellar evolution

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Abstract. A brief review of various methods to calculate radiative accelerations for stellar evolution and an analysis of their limitations are followed by applications to Pop I and Pop II stars. Recent applications to Horizontal Branch (HB) star evolution are also described. It is shown that models including atomic diffusion satisfy Schwarzschild's criterion on the interior side of the core boundary on the HB without the introduction of overshooting. Using stellar evolution models starting on the Main Sequence and calculated throughout evolution with atomic diffusion, radiative accelerations are shown to lead to abundance anomalies similar to those observed on the HB of M15.

Key words. Stars: abundances – Stars: diffusion – Stars: Population II – Galaxy: globular clusters – Stars: radiative accelerations – Stars: Horizontal Branch

1. Introduction

In his book, Eddington (1926) evaluated the equilibrium concentrations to be expected if atomic diffusion in the presence of differential radiation pressure were efficient. He concluded that some heavy metals should then completely dominate the spectrum. Since light and heavy elements are present in stellar spectra, he concluded (§193, 196) that some mixing process made atomic diffusion inefficient. He suggested that it be meridional circulation (§199). His argument for the importance of meridional circulation was qualitative. In a later paper he evaluated quantitatively the meridional circulation velocity without further commenting on its efficiency in mixing stellar interiors (Eddington 1929). While it was realized in the 1940s that the presence of the giant branch in clusters implied that stars could not be completely mixed, Eddington's argument seems to have had enough weight to prevent the proper calculation of the effect of differential radiation pressure until the late 1960s even if Eddington had partially corrected his argument (see the 1930 correction on page xiii of Eddington 1926). Work however continued on gravitational settling in outer solar layers (Biermann 1937; Wasiutynski 1958; Aller & Chapman 1960); the most important application was made to white dwarfs (Schatzman 1945).

Instead of assuming that equilibrium concentrations were reached, one of us introduced the differential radiative acceleration term, $g_{\rm rad}$, into the diffusion velocity equation of Aller & Chapman (1960) and calculated anomalies to be expected in atmospheric regions (Michaud 1970). Comparison to

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observations of ApBp stars suggested that $g_{\rm rad}$ plays a role in at least some stars. As large atomic data bases started becoming available in the 1980s, we realized that it would be possible to calculate $g_{\rm rad}$ throughout stellar interiors with good accuracy at the same time as the evolution proceeds. All composition changes can then be taken into account self consistently during evolution for the $g_{\rm rad}$ and the Rosseland averaged opacity. It was first tried to make those calculations with TOPBase from the Opacity Project (Alecian et al. 1993; LeBlanc & Michaud 1995; Gonzalez et al. 1995a,b) but using those data we could not reproduce the concentration dependence of the Rosseland averaged OPAL opacities around the solar center (see §5.1 of Turcotte et al. 1998) and we shifted to using OPAL spectra (Richer et al. 1998) to calculate $g_{\rm rad}$. The original spectra they had used to calculate the OPAL opacity tables (Iglesias & Rogers 1993, 1996; Rogers & Iglesias 1992) were kindly made available to us by Iglesias and Rogers.

In this review of $g_{\rm rad}$ in stellar evolution, we will first briefly describe how the calculations are carried out in stellar evolution codes (§ 2) and compare the values of $g_{\rm rad}$ obtained using OP and OPAL data (§ 2.1). A few examples of the effect of $g_{\rm rad}$ in Pop I and Pop II stars are described (§ 3) and, finally, results obtained recently for HB stars are presented (§ 4) showing that $g_{\rm rad}$s play a role in stellar evolution (§ 5).

2. Calculations of radiative accelerations

The particle transport equations are introduced into a standard stellar evolution code descibed in Proffitt (1994), Proffitt & Michaud (1991) and VandenBerg (1985). For each species one adds a force equation (eq. [18.1] of Burgers 1969) and a heat equation (eq. [18.2] of Burgers). Similar equations are written for electrons. It is generally assumed that each atomic species can be treated locally as being in an average state of ionization. One needs to know Z_i, an appropriate mean of the number of lost electrons.

The dominant term for each species contains $g_{\rm rad}(A) - g$ as a factor, where $g_{\rm rad}(A)$ is an appropriate average of the radiative acceleration over the states of ionization of element A. Over most of the stellar interior and for most of the evolution, it dominates transport even though the electric field, "diffusion" and thermal diffusion terms are also included in the calculations and are important for part of the evolution in some stars.

Rosseland opacities, mean ionic charges, and mean radiative forces are calculated using the same interpolation method, based on the principle of corresponding states in § 2.2 of Rogers & Iglesias (1992); see in particular their equations (4) to (6). Interpolation weights are determined for a subset of the data grid, and used to interpolate locally all these variables.

In first approximation, evaluating $g_{\rm rad}(A)$ amounts to calculating the fraction of the momentum flux that each element absorbs from the photon flux. In stellar interiors, it takes the form:

$$g_{\rm rad}(A) = \frac{L_r^{\rm rad}}{4\pi r^2 c}\frac{\kappa_R}{X_A}\int_0^\infty \frac{\kappa_u(A)}{\kappa_u(\text{total})}\mathcal{P}(u)du \qquad (1)$$

where most symbols have their usual meaning. The quantities $\kappa_u(total)$ and $\kappa_u(A)$ are respectively the total opacity and the contribution of element A to the total opacity at frequency u, with u and $\mathcal{P}(u)$ given by:

$$u = \frac{h\nu}{kT} \qquad (2)$$

and

$$\mathcal{P}(u) = \frac{15}{4\pi^4}u^4\frac{e^u}{(e^u-1)^2}. \qquad (3)$$

The calculations of $g_{\rm rad}$(A) involve carrying out the integration over the 10^4 u values for each atomic species, A. This must be repeated at each mesh point (typically 2000 in our models) and at each time step (typically 10^4 up to the HB). The Rosseland average opacity is also continuously recalculated making these calculations fully self consistent with all composition changes. This has some effect on stellar cores (see Fig. [9] and the last two paragraphs of §5.1 of Turcotte et al. 1998 for a discussion of the solar case).

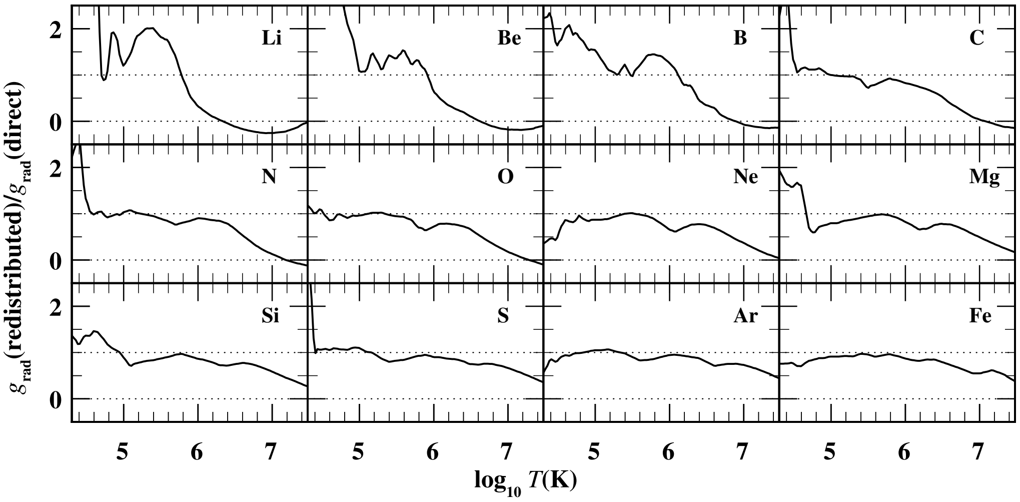

Fig. 1. Correction factors used in evolution calculations except for Fe, see the text. Adapted from Fig. 6 of Richer et al. 1998.

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2.1. Correction factors

The $g_{\rm rad}$s of equation (1) are corrected by two processes. First by taking into account the fraction of momentum that is given to the electron in a photoionization, as first suggested in this context by Michaud (1970) following Sommerfeld (1939), and second by calculating the effect of having many stages of ionization and not only an average one, which has come to be called the effect of redistribution.

The sharing of the momentum between the ion and the ejected electron is caused by the process remembering the direction of the incoming photon and emitting the electron with a distribution which is not exactly spherical. The correction to sphericity is small ($\sim v/c$ where v is the electron velocity) but for a given energy, the momentum of the electron being c/v times that of the photon, the correction to momentum transfer is large. The effect is difficult to calculate except for the ground state of hydrogen but accurate evaluations now exist for a few other cases and confirm the generalization formulas which were used (Massacrier 1996; Massacrier & El-Murr 1996; El-Murr 1999; Seaton 1995). Using their results, Richer et al. (1997) showed that for any shell n of an hydrogenic ion, one could use the simple formula for the effect of momentum sharing (Michaud 1970):

$$f_{\rm ion}(n) = 1 - \frac{8}{5}\left(1 - \frac{\nu_n}{\nu}\right), \tag{4}$$

which has been used for all cases calculated up to now.

When many states of ionization are present, one usually works, for convenience, with an average state of ionization. The simplest "averaging" is to use a single diffusion velocity for each atomic species and calculate it for the average Z of the atomic species,

$$\overline{Z} = \frac{\sum_i X(A_i) Z_i}{\sum_i Z_i}, \tag{5}$$

where the sum i is over the states of ionization of element A. The use of equation (1) is in line with this approach. However each term appearing in a diffusion equation has its own Z dependence and this approach is not always satisfactory. The $g_{\rm rad}$ term is the most sensitive to the averaging process since very often the largest $g_{\rm rad}(A_i)$ is for a very unabundant ion whose lines are desaturated. One could use an average of the form:

$$< g_{\rm rad}(A) >= \frac{\sum_i D(A_i)X(A_i)g_{\rm rad}(A_i)}{\sum_i D(A_i)X(A_i)} \qquad (6)$$

where $g_{\rm rad}(A_i)$ would be due to the fraction of the photon flux momentum absorbed while the atomic species is in state i. However this assumes that one may consider each ionic state of the atomic species A as independent. In practice this is a poor approximation since there are frequent ionizations and recombinations. Consequently one needs to compare the collision and ionization times and take into account the various ionization routes. This rapidly becomes complex and has been solved in detail only for He (Michaud et al. 1979) and Hg (Proffitt et al. 1999) with additional calculations described in Gonzalez et al. (1995a,b) for CNO.

In the evolution calculations these two effects, momentum sharing between ion and electron, and averaging over the ions, are combined into a multiplicative factor (= $[g_{\rm rad}(\text{redistributed})/g_{\rm rad}(\text{not redistributed})]$), which we call a correction factor, applied to the result of equation (1). It was calculated for a number of species (see Fig. [1]) using OP data for solar composition and tabulated as function of R_e ($\equiv N_e/T^3$) and $\log T$. They are used for all compositions. Those shown in Figure 1 were calculated at $\log R_e = 2.5$ which corresponds approximately to densities within main–sequence models. One notes that the correction factors are close to 1.0 for $5 < \log T < 6$. This is the T interval over which $g_{\rm rad}$s play the main role in evolution calculations done up to now.

For $\log T > 6$ the main contribution to the correction factor comes from the sharing of momentum with the electron. For all species shown except Fe the correction close to the center comes from hydrogenic ions for which it is believed to be reasonnably accurate. It was decided not to apply the correction to Fe following the argument of Seaton (1997) that f_{ion} should not be applied to autoionization resonances which often dominate in non hydrogenic ions and especially for Fe. For $\log T < 5$ the correction can be large and should be considered uncertain when they exceed a factor of 2. This usually occurs where the $g_{\rm rad}(A_0)$ contributes a large fraction of the value obtained with equation (6). Since the diffusion coefficient of the neutral state is much larger than those of the ionized states (Michaud et al. 1978) a small concentration of the neutral can lead to a large effect which however depends on the dominant ionization processes (Michaud et al. 1979).

2.2. Radiative accelerations from OP data

In so far as we know, all stellar evoluton calculations done with $g_{\rm rad}$ have been done using OPAL data for equation (1) and the correction factors described above calculated using OP data. However it is possible to do all calculations with OP data using the spectra available from their server (Seaton 2005; Mendoza et al. 2007). Those spectra use 10^4 frequency values just as OPAL spectra but they are equally spaced in a modified frequency which takes the local flux intensity into account (see §2.2.1 of Seaton 2005). It is similar to that proposed by LeBlanc et al. (2000) and should have a similar effect on accuracy as discussed there. This mesh should lead to greater accuracy for $\log T < 5$. However OP has two disadvantages. First it has fewer atomic species than OPAL and the second point is that their data base does not contain the $\overline{Z}$ which is needed to calculate the diffusion velocities. These would need to be calculated separately.

A comparison of $g_{\rm rad}$ calculated with OP atomic data to similar calculations with OPAL data is shown in Figure 2. The agreement is seen to be quite acceptable though one should remember the scale. There are differences by factors of up to 2 which actually appear though they are rare. Similar comparisons were made by Delahaye & Pinsonneault (2005) and Seaton (2007) with data taken from Richer et al. (1998) with similar agreement. See for instance Figure 8 of Seaton (2007) for Si. However those authors note greater disagreement with comparisons they make in solar models with Turcotte et al. (1998). The

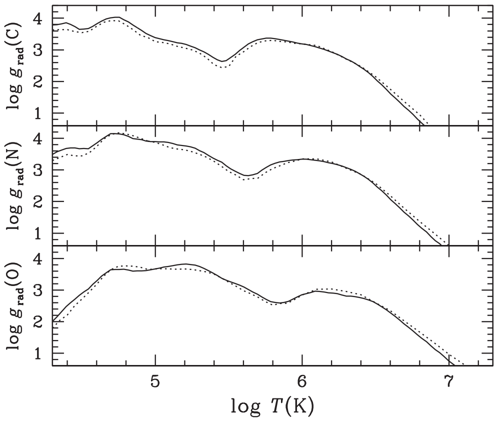

Fig. 2. Comparison of radiative accelerations calculated from OP (dotted line) and OPAL (solid line) atomic data. From Fig. [7] of Richer et al. 1998.

greater differences appear to come mainly from the correction factors discussed above[1]. These are always included in the stellar evolution calculations we made and so are included in the figures of Turcotte et al. (1998) where Delahaye & Pinsonneault (2005) and Seaton (2007) took data for their comparisons. If, for instance, one looks at Figure 11 of Seaton (2007), the difference between OP with mte and OPAL is mostly caused by correction factors shown on Figure 1 above since "mte" contains only part of the corrections we include. The correction factors are *not* included in Figure 1 of Richer et al. (1998) from which figure Delahaye & Pinsonneault (2005) and Seaton (2007) took the data for comparison with that paper.

The OP data has also been used to obtain semianalytic formulas for $g_{\rm rad}$ (Alecian & LeBlanc 2000; Alecian & LeBlanc 2002; Alecian & Artru 1990; LeBlanc & Alecian 2004). These require much less computing power than required by integrating over spectra. They have the further advantage of allowing an evaluation for species for which data is not available by using trends in spectroscopic properties. These are however less accurate than the detailed evaluation from OP or OPAL and do not allow to take into account the effect of individual concentration variations on Rosseland averaged opacity nor on $g_{\rm rad}$s.

[1] There is however one error which we found in Figure 11 of Turcotte et al. 1998: the curve for C was plotted incorrectly. It does not correspond to the values used in the calculations. This error is responsible for the largest discrepancy found by Delahaye & Pinsonneault (2005) and Seaton (2007).

3. Examples of the role of radiative accelerations in Pop I and II stars

Stellar evolution calculations including the effect of $g_{\rm rad}$s have now been done for a large number of stars of both Pop I and Pop II. It is beyond the scope of this brief review to mention all effects of $g_{\rm rad}$s. However we briefly describe one effect found in Pop I stars and one in Pop II stars before giving some more details of recent results for HB stars (§4).

The largest structural effect of $g_{\rm rad}$s in Pop I stars is the appearance of an Fe convection zone in all solar metallicity stars more massive than about 1.5 $M_\odot$ (see Richard et al. 2001). Iron contributes most to opacity at $\log T \sim 5.3$. Its radiative acceleration pushes iron from deeper in the envelope to the point where the $g_{\rm rad}$(Fe) starts decreasing. There Fe accumulates during evolution (see Fig. [4] of Richard et al. 2001) approximately where it contributes most to Rosseland opacity. This leads to an important increase of the radiative gradient which, as the Fe abundance increases, becomes larger than the adiabatic gradient and an Fe convection zone appears. In a 1.5 $M_\odot$ star this takes a significant fraction of the main–sequence life to occur. In stars of 1.7 $M_\odot$ and more, this occurs very early in the main–sequence life. In stars with $Z = 0.01$, Fe convection zones start appearing at 1.3 $M_\odot$.

The detailed treatment of the interaction of metals with H and He in diffusion processes and of the effect of concentration changes on Rosseland opacity has also shown that an accumulation of metals occurs just outside convective cores and causes semi-convection there (see §4 of Richard et al. 2001)

The only requirement is for the region with $\log T \sim 5.3$ to be stable enough for diffusion

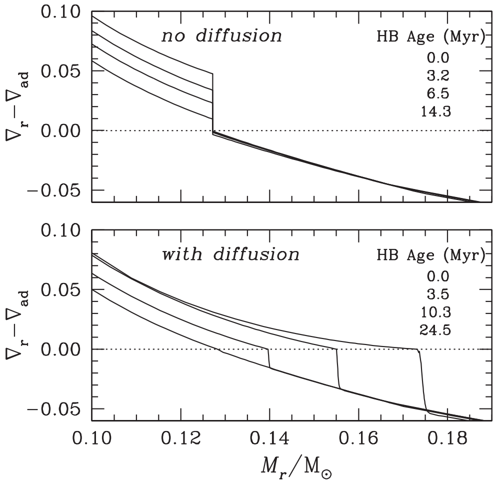

Fig. 3. Difference between the radiative and adiabatic gradient around the He burning core boundary. In both panels, the lowest curve is at zero age HB and the uppermost curve is the final one calculated. In the calculations with diffusion (lower panel) the convective neutrality condition is automatically satisfied while it is not satisfied in the absence of diffusion. Overshooting is not included in these calculations.

processes to occur there. There is no adjustable parameter in those calculations.

In Pop II stars, we were surprised to find that g_{rad}s and gravitational settling cause abundance anomalies by factors of 2–10 in a low metallicity cluster such as M92, much larger than those expected in higher metallicity clusters such as M5, M71 or 47 Tuc. Even in NGC 6397 which is only a factor of 2 more metal rich than M92, the effects are expected to be much smaller though they have apparently been seen in NGC 6397 (Korn et al. 2006). In M92, anomalies might have been seen near the turnoff by King et al. (1998) but the noise was large enough to make this uncertain. This result would need confirmation.

4. Diffusion in HB stars

A 0.8 $M_\odot$ model with $Z = 10^{-4}$ was evolved from the ZAMS through the core expansion phase on the HB. Atomic diffusion was included throughout evolution and no adjustable parameter was involved. The results were compared to a similar model without diffusion (Michaud et al. 2007).

It was found that atomic diffusion had little effect on age determinations using the HB luminosity but that there appeared two interesting effects on the structure of HB stars. After the discussion of instabilities at the junction of He burning cores and radiative zones presented by Paczyński (1970), it became accepted that the mixed C cores of HB stars were extended by overshooting or penetration to maintain convective neutrality at the boundary. It is currently not possible to evaluate reasonably accurately the efficiency of overshooting in HB stars. To quote Sweigart (1994),"Unfortunately the efficiency of convective overshooting under such circumstances is entirely unknown. Canonical HB theory assumes that the convective overshooting is highly efficient...". But as may be seen from Figure 3, it was found that the presence of overshooting during the core expansion phase is made unnecessary by the presence of atomic diffusion. It was not shown that overshooting plays no role but rather that it is not necessary to invoke such a little understood process.

Diffusion also causes an extension of the H burning shell into the He core. Through the outward diffusion of C from the core and the inward diffusion of H into the core there appears an additional H burning region inside the He core which was called diffusion induced H burning (see Fig. [6] of Michaud et al. 2007). This leads to a slight increase in the ZAHB luminosity.

4.1. Surface abundance anomalies

Since HB stars are Pop II stars that just left the giant branch of globular clusters, they are all expected to have the same concentration of metals, at least of those heavier than Al (Gratton et al. 2004). The concentration of CNO and other relatively light species might show small variations but Fe is not expected to be affected. Michaud et al. (1983) however suggested that g_{rad}s should lead to overabundances of at least some metals in those stars where settling causes underabundances of He.

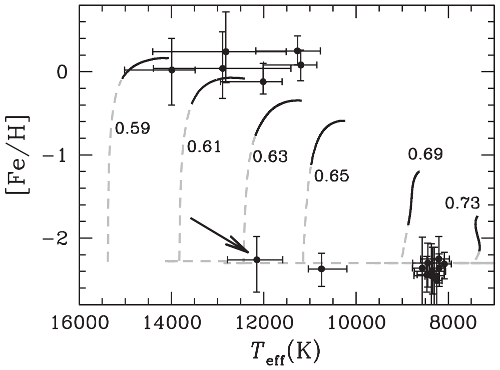

Fig. 4. Concentration of surface [Fe/H] expected in our HB models compared to observations of Behr (2003) for M15. The continuous dark lines cover the interval from 5 to 30 Myr after zero age HB for a number of models whose mass is identified on the figure in $M_{\odot}$. The star marked with an arrow has $V \sin i \sim 16$ km s^{-1}. All other stars with $T_{\rm eff} > 11000$ K have $V \sin i < 8$ km s^{-1}. Adapted from Michaud et al. 2008.

Glaspey et al. (1989) have confirmed the overabundance of Fe in one star of one cluster but at the limit of detection and this observation required confirmation. This prediction has now been confirmed in many clusters (Behr et al. 1999; Moehler et al. 2000; Fabbian et al. 2005; Pace et al. 2006) but in particular by Behr (2003) for M15. Overabundances of Fe by factors of 50–100 are seen in all HB stars with $T_{\rm eff} > 11500$ K while the cooler ones have the same Fe abundance as cluster's giants.

In stellar evolution calculations, the surface concentrations depend on the exterior boundary conditions. In the calculations of Michaud et al. (2007), the simplest assumption was made, that of a mixed outer zone without any mass loss. The mixed mass was adjusted to reproduce approximately the observations of Fe in one of the stars observed by Behr (2003) in M15. The same model reproduced reasonably well the observations in other high $T_{\rm eff}$ stars of that cluster as may be seen in Figure 4. Furthermore, as may be seen in Michaud et al. (2008) the other anomalies are also reasonnably well reproduced. This is a striking confirmation of the role of $g_{\rm rad}$s in HB stars.

5. Conclusions

The availability of large atomic data bases has allowed, as described in § 2, to calculate stellar evolution models for Pop I and II stars up and past the giant branch including all effects of atomic diffusion. They cause abundance anomalies not only in Ap stars, as originally suggested by Michaud (1970), but also in HB stars of clusters (§4.1) and possibly in turnoff stars (§3). They play an essential role in driving pulsations in sdB stars (Fontaine et al. 2003), the field analogue of HB stars. It is furthermore not only the surface region which is affected but 50% of the stellar radius and 10^{-3} of its mass (Richard et al. 2002; Michaud et al. 2007).

Eddington (1926) was right however in suggesting that competing processes also have a role to play. For instance, in M15, rotation plays a role probably through meridional circulation. The arrow in Figure 4 shows the only star with $T_{\rm eff} > 11500$ K which has no abundance anomaly. It is also the only relatively rapidly rotating star. Quievy et al. (2007) have shown that meridional circulation explained that the stars with $T_{\rm eff} < 11500$ K have a normal abundance and not the 5× overabundance that Figure 4 would suggest. While atomic diffusion driven by $g_{\rm rad}$s plays the main role in creating abundance anomalies on the HB, it has to compete with the effects of rotation just as in HgMn or AmFm stars (Charbonneau & Michaud 1991).

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