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Ten years of cepheids theoretical studies

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Abstract. I review our theoretical analysis of Classical Cepheids, as based on updated pulsation and evolutionary models computed in the last ten years, emphasizing main results and open issues.

Key words. Stars: evolution – Stars: pulsation – Stars: Classical Cepheids

1. Introduction

GRAPES means "*GR*oup for *A*dvancement of *P*ulsation and *E*volution *S*tudies" and it is a project, started roughly ten years ago, aimed at the interpretation of the properties of radially pulsating stars (RR Lyrae, Cepheids, etc.). It involves several researchers of the Astronomical Observatories in Naples, Rome and Teramo, at the Physics Department of the Pisa University, as well as from foreign institutions.

In the following, I'll review the studies concerning Classical Cepheids.

2. The theoretical route: main papers

As general comments, let me recall that the pulsation models adopt a modern nonlinear time-dependent convective approach which provides pulsation period and amplitude, as well as firm constrains on the blue and red edge of the pulsation region. The models are computed as one parameter families with constant chemical composition, mass and luminosity, by varying the effective temperature. For each chemical composition and mass, the adopted luminosity refers to Mass-Luminosity (*ML*) relations based on canonical evolutionary tracks or deals with higher luminosity levels as produced by a convective core mild overshooting and/or mass loss. The entire pulsation region is covered by varying the effective temperature T_e by steps of 100K. This yields that increasing (decreasing) by 100K the effective temperature of the computed bluest (reddest) fundamental model yields non-pulsating structures. Accordingly, the effective temperature of the computed bluest model, increased by 50K, and the effective temperature of the reddest model, decreased by 50K, are adopted as the effective temperature of the fundamental blue edge (FBE) and red edge (FRE), respectively. The same procedure is used for first-overtone models to get the first-overtone blue edge (FOBE) and red edge (FORE).

The bolometric light curves provided by the nonlinear approach are transformed into $UBVRIJK$ bands by means of the model

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atmospheres by Castelli, Gratton & Kurucz (1997a,b). Then, after a time integration over the whole pulsation period, the pulsator predicted mean magnitudes (both magnitude-averaged (M_i) and intensity-averaged $\langle M_i \rangle$) are derived.

In order to illustrate the stages in the progress of our research, let me mention:

- **Physical Structure (Bono & Stellingwerf 1994; Bono, Marconi & Stellingwerf 1999)**: Fundamental and first-overtone nonlinear models with nonlocal and time-dependent treatment of convection are computed adopting various masses, effective temperatures and luminosities, and for three different chemical compositions (Y=0.25, Z=0.004; Y=0.25, Z=0.008; Y=0.28, Z=0.02) taken as representative of the variables in the Small Magellanic Cloud (SMC), Large Magellanic Cloud (LMC) and the Milky Way, respectively.
- **Predictable Scenario (Bono, Castellani & Marconi 2000)**: Predictions concerning the location of the instability strip in the HR diagram, the pulsation equation, pulsation amplitudes in luminosity, radius, velocity, and effective temperature are derived.
- **Predicted Period-Luminosity (*PL*), Period-Color (*PC*), and Period-Luminosity-Color (*PLC*) Relations (Bono, Caputo, Castellani & Marconi 1999)**: Bolometric light curves of models adopting a canonical ML relation are transformed into visual and near-infrared bands and the intensity-weighted mean magnitudes $\langle M_V \rangle$ and $\langle M_K \rangle$ are derived. Both the zero point and the slope of the predicted PL_V and PL_K relations appear to be dependent on metallicity, with the amplitude of this effect decreasing at the longer wavelength. In particular, at fixed period, the metal-rich pulsators should be fainter than the metal-poor ones.
- **Mean Magnitudes and Colors (Caputo, Marconi & Ripepi 1999)**: Predicted differences between static and mean quantities turn out to be in agreement with the observed trend of Galactic Cepheids. It is also shown that estimates of the Cepheid mean reddening, distance, and metallicity may be derived if three-filter data (e.g., BVK) are taken into account.
- **Multiwavelength Relations (Caputo, Marconi & Musella 2000)**: Predicted Period-Luminosity, Period-Color, Color-Color and Period-Luminosity-Color relations in the $BVRIJK$ bands are derived. All the theoretical relations are, in various degrees, metallicity dependent.
- **The Hertzsprung Progression (Bono, Marconi & Stellingwerf 2000)**: For models with a LMC chemical composition (Y=0.25, Z=0.008) and various masses, the theoretical light and velocity curves reproduce the observed Hertzsprung progression (HP), with the predicted period at the HP center ($P(HP)$ = 11.24±0.46 d) in very good agreement with the empirical value 11.2±0.8 d. Moreover, light and velocity amplitudes are in good qualitative agreement with observational evidence.
- **Metallicity Effects on the Cepheid Distance Scale (Caputo, Marconi, Musella & Santolamazza 2000)**: Predicted PL_V and PL_I relations at Z=0.004 and 0.02 suggest a metallicity correction to LMC-based distances as given by $\delta\mu_0/\delta \log Z \sim -0.27$ mag dex^{-1}, where $\delta \log Z$ is the difference between the metallicity of the Cepheids and the LMC value Z=0.008. This result disagrees with the empirical result $\delta\mu_0/\delta \log Z \sim +0.24$ mag dex^{-1} suggested by Cepheid observations in two different fields of the galaxy M101 (Kennicutt et al. 1998, 2003). However, it is shown that such a theoretical correction appears supported by an existing, although weak, correlation between the Cepheid distance and the oxygen abundance of external galaxies within a given group or cluster, as well as by a similar correlation between the *Key Project* H_0 estimate and the [O/H] parameter of the galaxies which calibrate the SNIa luminosity. On the contrary, the empirical metallicity correction seems to be excluded.
- **Intermediate-Mass Star Models with Different Helium and Metal**

Contents (Bono, Caputo, Cassisi, Marconi, Piersanti & Tornambe' 2000): Evolutionary tracks are constructed by adopting a wide range of stellar masses (3-15$M_{\odot}$) and chemical compositions. A new updated canonical *ML* relation for Classical Cepheids is derived.

- **Theoretical Models for Bump Cepheids (Bono, Castellani & Marconi 2002)**: The observed light curve of bump Cepheids is closely reproduced by the theoretical one.
- **Effects of Helium and Heavy-Element Abundance (Fiorentino, Caputo, Marconi & Musella 2002)**: New fundamental pulsation models are computed at supersolar metallicity (Z=0.03, 0.04) and selected choices of the helium-to-metal enrichment ratio $\Delta Y/\Delta Z$. It is shown that the location into the HR diagram of the Cepheid instability strip depends on both metal and helium abundance, moving toward higher temperatures with decreasing metal content (at fixed Y) or with increasing helium content (at fixed Z). The contributions of helium and metals to the predicted relations are discussed as well as the implications on the Cepheid distance scale. It is shown that the adoption of empirical PL_V and PL_I relations based on Cepheids at the LMC to get distance moduli with an uncertainty of ±0.10 mag is fully justified for variables in the short-period range ($P \leq 10$ days), at least with $Z \leq 0.04$ and $\Delta Y/\Delta Z$ in the range of 2-4. Conversely, at longer periods ($P > 10$ days) a correction to LMC-based distance moduli may be needed, whose sign and amount depend on the helium and metal content of the Cepheids. On this ground, the empirical metallicity correction suggested by Cepheid observations in two fields of the galaxy M101 may be accounted for, provided that the adopted helium-to-metal enrichment ratio is reasonably high (∼3.5).
- **Effects of Input Physics (Petroni, Bono, Marconi & Stellingwerf 2003)**: The role of the input physics (equation of state and opacity tables) and of the spatial resolution across the hydrogen and the helium partial ionization regions are investigated. As a whole, the nonlinear models are marginally affected by these physical and numerical assumptions.
- **Evolutionary Models for Magellanic Clouds (Castellani, Degl'Innocenti, Marconi, Prada Moroni & Sestito 2003)**: Evolutionary tracks for Magellanic Cloud stars are computed without and with a convective core mild overshooting.
- **Mass Loss in NGC 1866 Cepheids (Brocato, Caputo, Castellani, Marconi & Musella 2004)**: Comparison of pulsation models with observed magnitudes yields firm evidence for pulsation masses smaller than predicted by stellar evolution.
- **BASTI project (Pietrinferni, Cassisi, Salaris & Castelli 2004, 2006)**: A large and updated stellar evolution database for masses between 0.5 and 10$M_{\odot}$, metal content [Fe/H] from −2.27 to +0.40, with both scaled solar and α-enhanced metal distributions. The evolutionary models are computed without (canonical models) and with mild overshooting from the convective cores and with two different choices for the efficiency of mass loss.
- **The Period-Age Relation (Bono, Marconi, Cassisi, Caputo, Gieren & Pietrzynski 2005)**: The Period-Age and Period-Age-Color relations for fundamental and first-overtone classical Cepheids are studied. It is found that both relations present a mild dependence on metal content and may provide estimates of individual stellar ages in the Galaxy and in external Galaxies, independently of the distance.
- **Varying Metallicity and $\Delta Y/\Delta Z$ (Marconi, Musella & Fiorentino 2005)**: Additional pulsation models at Z=0.01 to 0.04 and various $\Delta Y/\Delta Z$ ratios are computed. Previous results concerning the theoretical metallicity correction to the Cepheid distance scale are confirmed.
- **Effects of Convection and *ML* Relation (Fiorentino, Marconi, Musella & Caputo 2007)**: The effects of the assumed value of the mixing length parameter l/H_p on the pulsation properties are investigated. New

predicted relations are derived by releasing the assumption of a fixed *ML* relation.

3. Status of art: main theoretical results

The major sets of pulsation models[1] computed by our group are listed in Table 1, where the adopted luminosity refers to a canonical Mass-Luminosity ("can") or deals with higher luminosity levels as produced by a convective core mild overshooting and/or mass loss ("over") .

It is well known that a restatement of the Stefan's law for pulsating stars is a mass-dependent Period-Luminosity-Color relation for which the pulsator absolute magnitude in a given photometric band is uniquely determined by the period, mass and intrinsic color. Luckily enough, the color coefficients are not too different from the extinction-to-reddening ratios suggested by optical and near-infrared reddening laws and for this reason the so-named Wesenheit functions are widely used to reduce the effects of the finite width of the instability strip and, at the same time, to bypass interstellar extinction problems (see Madore 1982; Madore & Freedman 1991; Tanvir 1999; Caputo, Marconi & Musella 2000). In particular, I will discuss the Wesenheit functions $WBV = V - 3.3(B-V)$, $WVI = V - 2.52(V-I)$, $WVK = V - 1.09(V-K)$ and $WJK = K - 0.575(J-K)$ for fundamental pulsators.

On the other hand, by adopting as reference level the luminosity L_c=$f(M,Z,Y)$ provided by the *ML* relation determined by Bono et al. (2000), one can estimate the ratio L/L_c for each pulsation model. Then, by a linear interpolation through all the models, one derives the "evolutionary" Period-Wesenheit (*PW*) relations listed in Table 2 for fundamental pulsators. According to these relations, once the $\log L/L_c$ is adopted, we can estimate the true distance modulus μ_0 of individual Cepheids, provided that the metal content is known. Note that, aside from minor discrepancies among the canonical *ML* relations available in the literature, the occurrence of both a convective core mild overshooting and mass-loss before or during the pulsation evolutionary phase yields *positive* $\log L/L_c$ values. Consequently, the *PW* relations at $L = L_c$ provide the *maximum* value of the Cepheid distance. Of importance, is also to note that the predicted *P-WBV* relation shows an opposite metallicity dependence with respect to *P-WVI*, *P-WVK* and *P-WJK* (see also Fiorentino et al. 2007). Such a result suggests a quite plain method to estimate the Cepheid metal content and, at the same time, it warns against averaging the μ_0 values inferred by different *PW* relations.

Closing this section, let me summarize in Table 3 some selected results of synthetic *PL* relations (see Fiorentino et al. 2007) based on the predicted instability strip inferred by the pulsation models and the *BASTI* evolutionary tracks computed adopting no convective core overshooting and a mass-loss Reimers parameter η=0.4. As a whole, three points are worthy of mention:

1. the slope and the intrinsic dispersion of the predicted Period-Magnitude distribution at fixed metal abundance decrease moving from visual to near-infrared bands, in agreement with well-established empirical results;
2. the metal-poor relations have a steeper slope than the metal-rich ones, with the effect decreasing at the longer wavelengths;
3. at periods longer than $\sim$ 5 days, the metal-poor pulsators have brighter magnitudes than the metal-rich ones, with the effect again decreasing from *V* to *K* magnitudes.

4. Pulsation models versus observed Cepheids

The *BVI* magnitudes collected by Udalski et al. (1999) and the *JK* data of Persson et al. (2004) yield the *PL* and *PW* relations listed in Table 4 in the form m_i=a + $b\log P$ and $W = \alpha + \beta \log P$.

As a first result, one has that the slope of the observed *PL* relations appears consistent with those predicted at Z=0.008, which is the widely adopted metal content of LMC Cepheids. Using the synthetic linear relations given in Table 3, one derives

[1] Other models, not included in Table 1, have been computed for specific studies.

Table 1. Basic parameters of pulsation models.

Z	Y	$M/M_\odot$	$\log L/L_\odot$	l/H_p
0.004	0.25	3.5-11.0	can, over	1.5, 1.8
0.008	0.25	3.5-11.0	can, over	1.5, 1.8
0.01	0.26	5.0-11.0	can	1.5, 1.8
0.02	0.25, 0.26, 0.28, 0.31	5.0-11.0	can, over	1.5, 1.7, 1.8
0.03	0.275, 0,31, 0,335	5.0-11.0	can	1.5
0.04	0.25, 0.29, 0.33	5.0-11.0	can	1.5

Table 2. Selected evolutionary *PW* relations for fundamental pulsators with Z=0.004 to 0.04, as based on intensity-averaged magnitudes.

W	α	β	γ	δ
		W=α+βlog$P$+γlogL/L_c+δlog$Z$		
WBV	−4.11±0.07	−3.83±0.01	+0.68±0.02	−0.67±0.01
WVI	−2.76±0.11	−3.28±0.01	+0.84±0.04	+0.05±0.02
WVK	−2.44±0.10	−3.31±0.01	+0.84±0.04	+0.14±0.02
WJK	−2.52±0.09	−3.36±0.01	+0.85±0.03	+0.13±0.01

Table 3. Synthetic *PL* relations for fundamental pulsators with log$P$ ≥0.4 in the form M_i = a + $b$log$P$.

Z	M_i	a	b
0.008	M_V	−1.31±0.23	−2.84±0.04
0.008	M_I	−1.87±0.17	−3.08±0.04
0.008	M_K	−2.50±0.08	−3.38±0.02
0.02	M_V	−1.41±0.11	−2.51±0.04
0.02	M_I	−2.09±0.09	−2.80±0.03
0.02	M_K	−2.63±0.05	−3.22±0.02

$\mu_{0,V}$=18.46±0.28 mag, $\mu_{0,I}$=18.55±0.20 mag and $\mu_{0,K}$=18.54±0.10 mag, with and average value of $\mu_0(LMC)$=18.52±0.23 mag.

Regarding the observed *PW* relations, their slopes are in a quite close agreement with the values listed in Table 2 and the use of the predicted *PW* relations yields

$$\mu_{0,WBV} = 20.12 - 0.68 \log L/L_c + 0.67 \log Z \quad (1)$$

$$\mu_{0,WVI} = 18.64 - 0.84 \log L/L_c - 0.05 \log Z \quad (2)$$

$$\mu_{0,WJK} = 18.42 - 0.85 \log L/L_c - 0.13 \log Z \quad (3)$$

with a standard dispersion of ±0.20, ±0.10 and ±0.08 mag, respectively. It is of interest to note that the mutual agreement among the three *PW*-based distance moduli is actually achieved at Z=0.008 and that at this metal abundance we get $\mu_0(LMC)$=18.72 mag with logL/L_c=0 (canonical evolutionary scenario) or 18.55 mag with logL/L_c=0.20 (mild overshooting or mass-loss).

Concerning the Galactic variables, the absolute VIK magnitudes determined by Benedict et al. (2007) on the basis of HST trigonometric parallaxes of ten Cepheids yield the *PL* and *PW* relations reported in Table 5. In spite of the small number of variables and

Table 4. Observed PL and PW relations for LMC fundamental Cepheids.

m_i	a	b	W	α	β
V_0	17.07±0.16	−2.78	WBV	16.01±0.22	−3.83
I_0	16.59±0.11	−2.98	WVI	15.88±0.08	−3.29
K_0	16.04±0.06	−3.26	WJK	15.90±0.05	−3.36

Table 5. Absolute PL and PW relations for fundamental Galactic Cepheids with HST trigonometric parallaxes.

M_i	a	b	W	α	β
M_V	−1.62±0.10	−2.43	–	–	–
M_I	−1.97±0.08	−2.81	WVI	−2.37±0.05	−3.48
M_K	−2.39±0.05	−3.32	WVK	−2.34±0.05	−3.49

bearing in mind that the PL relations are statistical relations which reflect the distribution within the instability strip, the observed slope and zero-point listed in Table 5 appear comfortably consistent with the predicted values at a solar chemical composition, as listed in Table 3. Of importance, is to note that, compared with the LMC results given in Table 4, the Galactic PL_V and PL_I relations have a milder slope, *in agreement with the predicted behavior*. In order to test this consistency, let me adopt the observed LMC relation WVI=15.88−$\mu_0(LMC)$−3.29log$P$ to derive the intrinsic distance modulus of each Galactic Cepheid from its measured WVI function. Once the apparent V and I magnitudes are corrected for reddening, this whould yield the absolute Galactic PL_V and PL_I relations. Using the VI data by Berdnikov, Dambis & Vozyakova (2000) and the $E(B-V)$ reddening listed by Fernie et al. (1995), I show in Fig. 1 that the resulting Galactic relations (solid lines)

$$M_V = 17.14 - \mu_0(LMC) - 2.56 \log P \qquad (4)$$

$$M_I = 16.71 - \mu_0(LMC) - 2.83 \log P \qquad (5)$$

are not only *shallower* than the LMC PL_V and PL_I relations given in Table 4 (dashed lines), but even *fainter* (at log$P \geq 0.5$), whichever is the LMC distance (see Table 4).

5. Main conclusions

Although the pulsation models are still under analysis, some major results seem worthy of notice:

1. the slopes of the observed Period-Magnitude relations of the hundreds of LMC Cepheids observed by Udalski et al. (1999) and Persson et al. (2004) agree with the those predicted at $Z \sim 0.008$, the classical value of LMC variables;
2. a similar agreement is found between the observed and the predicted slope of the Period-Wesenheit relations. On this ground, one derives $\mu_0(LMC)$ ∼18.72 mag with logL/L_c=0 (canonical evolutionary scenario) or 18.55 mag with logL/L_c=0.20 (mild overshooting or mass-loss);
3. the predicted flattening of the optical Period-Magnitude relations when passing from the metal-poor to metal-rich variables is supported by the HST-based distances to a selected sample of Galactic Cepheids, as well as by observational evidences based on the straight application of the *empirical* LMC P-WVI relation to all the Galactic Cepheids with measured VI magnitudes;
4. moreover, at log$P \geq 0.5$, the Galactic PL_V and PL_I relations are fainter than the LMC ones whichever is the LMC distance, again

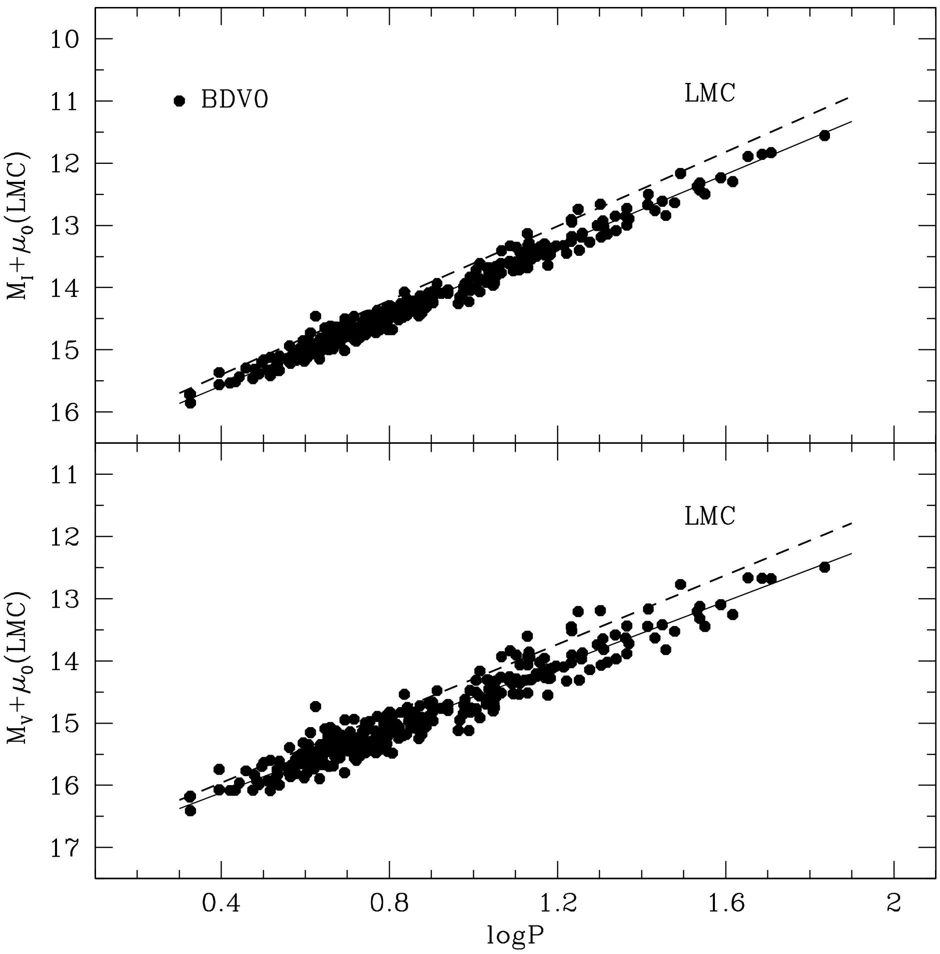

Fig. 1. PL_V and PL_I relations of Galactic Cepheids (solid lines) with measured VI magnitudes (Berdnikov, Dambis & Vozyakova 2000 [BDV0]), in comparison with LMC relations (dashed lines).

in agreement with the theoretical predictions.

Acknowledgements. To Vittorio.

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