Mem. S.A.It. Vol. 79, 716

Memorie della

Influence of a global magnetic field on stellar structure

V. Duez, A.S. Brun, S. Mathis, P.A.P. Nghiem, and S. Turck-Chièze

CEA/DSM Laboratoire AIM - DAPNIA/SAp, CNRS, Univ. Paris Diderot, F-91191 Gif-sur-Yvette Cedex, France
e-mail: vincent.duez@cea.fr

Abstract. The theoretical framework we have developed to take into account the influence of a global axisymmetric magnetic field on stellar structure and evolution is described. The prescribed field, possibly time-dependent, is expanded in the vectorial spherical harmonics basis. Hydrostatic equilibrium and energetic balance are consequently modified. Convection's efficiency and onset are also revised. Finally, our numerical strategy and the results one can expect from the implementation of those theoretical results are discussed.

Key words. Magnetohydrodynamics (MHD) – Stars: magnetic fields – Stars: evolution

1. Introduction

Over the last decade, various attempts to implement magnetic field in stellar structure equations have been pursued (see *e.g.* Lydon & Sofia 1995). However, these descriptions rest mostly on modifying the structure equations by adding terms such as magnetic pressure to the gas pressure or magnetic energy density to the total energy. Moreover, these quantities are defined with the help of arbitrary parameters whose values are intended to mimic the geometry of a three-dimensional magnetic field.

The present work depicts the method one can adopt to properly preserve the geometrical nature of the magnetic field, even after its implementation in a unidimensional stellar evolution code. The field is expanded in the vectorial spherical harmonics basis, virtually up to an infinite order. Therefore, the contribution of each mode is naturally taken into account in its projection along the radial direction.

This approach is a first step in an attempt to implement the coupling of magnetic field with rotation in stellar evolution codes through the Lorentz force's torque. The aim is to improve the so-called Standard Model in order to reproduce internal dynamical properties of the stars, and especially of the Sun. As a matter of fact, other physical processes than hydrodynamical processes have to be invoked to explain the angular momentum transport which is responsible of the quasi-uniform rotation profile observed in the solar radiative zone.

2. Modification of stellar structure equations

2.1. Magnetic field expansion

The axisymmetric magnetic field is written in terms of magnetic stream-functions through the divergenceless form

$$\boldsymbol{B}\,(r,\theta) = \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\left[\xi_P(r,\theta)\widehat{\mathbf{e}}_r\right] + \boldsymbol{\nabla}\times\left[\xi_T(r,\theta)\widehat{\mathbf{e}}_r\right], \tag{1}$$

where the poloidal and toroidal functions, ξ_P and ξ_T are developed in the spherical harmonics basis (*cf.* Bullard & Gellman 1954)

$$\xi_P\,(r,\theta) = \sum_{l=1}^{\infty} \xi_0^l\,(r)\,Y_l^0\,(\theta)\,, \tag{2}$$

$$\xi_T\,(r,\theta) = \sum_{l=1}^{\infty} \chi_0^l\,(r)\,Y_l^0\,(\theta)\,. \tag{3}$$

They can be chosen so that they verify stationary magneto-hydrostatic configurations, as was chosen *e.g.* in Rashba et al. (2007). That gives, once expanded in the vectorial spherical harmonics basis

$$\boldsymbol{B}\,(r,\theta) = \underbrace{\sum_{l=1}^{\infty}\left\{\left[l(l+1)\frac{\xi_0^l}{r^2}\right]\boldsymbol{R}_l^0(\theta) + \left[\frac{1}{r}\partial_r\xi_0^l\right]\boldsymbol{S}_l^0(\theta)\right\}}_{\text{poloidal (meridional) part}} + \underbrace{\sum_{l=1}^{\infty}\left\{\left[\frac{\chi_0^l}{r}\right]\boldsymbol{T}_l^0(\theta)\right\}}_{\text{toroidal (azimuthal) part}}, \tag{4}$$

where $\boldsymbol{R}_l^0(\theta) = Y_l^0(\theta)\widehat{\mathbf{e}}_r$, $\boldsymbol{S}_l^0(\theta) = \boldsymbol{\nabla}_{\mathcal{S}}Y_l^0(\theta)$, $\boldsymbol{T}_l^0(\theta) = \boldsymbol{\nabla}_{\mathcal{S}}\times\boldsymbol{R}_l^0(\theta)$ with $\boldsymbol{\nabla}_{\mathcal{S}} = \widehat{e}_\theta\partial_\theta + \widehat{e}_\varphi\frac{1}{\sin\theta}\partial_\varphi$.

2.2. Associated terms

Following the method outlined by Mathis & Zahn (2005), and taking the latitudinal average, we express the mean radial physical quantities associated with the magnetic field $\boldsymbol{B}$ in terms of the magnetic stream functions ξ_0^l and χ_0^l:

- the Lorentz force $\mathcal{F}_{\mathcal{L}} = \boldsymbol{j}\times\boldsymbol{B} = \frac{1}{\mu_0}\left(\boldsymbol{\nabla}\times\boldsymbol{B}\right)\times\boldsymbol{B}$;
- the magnetic pressure $P_{\rm mag} = \frac{1}{2\mu_0}\boldsymbol{B}^2$;
- the magnetic tension $\frac{1}{\mu_0}\left(\boldsymbol{B}\cdot\boldsymbol{\nabla}\right)\boldsymbol{B}$;
- the ohmic heating
  $Q = (1/\mu_0)||\eta||\otimes(\boldsymbol{\nabla}\times\boldsymbol{B})\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$;
- the Poynting's flux $\boldsymbol{\Pi} = (1/\mu_0)\,(\boldsymbol{E}\times\boldsymbol{B})$
  $= \boldsymbol{\nabla}\cdot\left[(||\eta||\otimes(\boldsymbol{\nabla}\times\boldsymbol{B}))\times\boldsymbol{B} - (\boldsymbol{v}\times\boldsymbol{B})\times\boldsymbol{B}\right]/\mu_0$

where $\boldsymbol{E}$ is the electric field. One can refer to Mathis & Zahn (2005) for most of the detailed expressions. $||\eta||$ is the magnetic diffusivity tensor while μ_0 is the magnetic permeability of vacuum.

2.3. Structure modifications

2.3.1. Energetic balance

The energetic balance is modified according to

$$\frac{\partial E_{\rm tot}}{\partial t} = -\boldsymbol{\nabla}\cdot\boldsymbol{F}_{\rm tot} + \varepsilon_{\rm tot}, \tag{5}$$

where the total energy production rate per unit mass is given by:

$$\varepsilon_{\rm tot} = \varepsilon_{\rm nuc} + Q/\rho, \tag{6}$$

$\varepsilon_{\rm nuc}$ being the nuclear one, and where the total flux is the sum of the radiative, convective and Poynting's fluxes:

$$F_{\rm tot} = F_{\rm rad} + F_{\rm conv} + \Pi. \tag{7}$$

The Poynting's flux is thus not involved directly in the heat equation but it contributes to the luminosity $L = 4\pi R^2 F_{\rm tot}$.

2.3.2. Structure equations

The five modified structure equations are then, with $P = P_{\rm mag} + P_{\rm gas}$:

$$\frac{\partial P}{\partial M} = -\frac{GM}{4\pi R^4} + \mathcal{F}^{\rm T}_{\mathcal{L};r}\,, \tag{8}$$

$$\frac{\partial T}{\partial M} = \frac{\partial P}{\partial M}\frac{T}{P}\nabla\,, \tag{9}$$

$$\frac{\partial R}{\partial M} = \frac{1}{4\pi R^2 \rho}\,, \tag{10}$$

$$\frac{\partial L}{\partial M} = \varepsilon - \frac{\partial U}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} + \frac{1}{\rho}Q\,, \tag{11}$$

$$\frac{\partial X_i}{\partial t} = -\frac{\partial F_i}{\partial M} + \Psi_i(P_{\rm gas}, T; X) \quad (1 \le i \le n_{\rm elem}) \tag{12}$$

where the stellar structure classical notations have been used while $\mathcal{F}^{\rm T}_{\mathcal{L};r}$ is the radial component of the magnetic tension.

3. Influence on convection

3.1. Convection's efficiency

The four gradients are defined by: $\left\{\begin{array}{lcl} \nabla & = & \frac{d\ln T}{d\ln P} \\ \nabla' & = & \frac{d\ln T'}{d\ln P} \end{array}\right.$ and $\left\{\begin{array}{lcl} \nabla_{\rm ad} & = & \frac{\Gamma_2 - 1}{\Gamma_2} \\ \nabla_{\rm rad} & = & \frac{3\kappa\rho\lambda_P}{4acT^4}F_{\rm tot} \end{array}\right.$ where the prime stand for the physical quantities of the convective element.

We introduce the magnetic pressure, as was done by Brun et al. (1998) when introducing the turbulent pressure and Poynting's flux respectively through

$$\nabla^*_{\rm ad} = \nabla_{\rm ad}\left(\frac{d\ln P_{\rm gas}}{d\ln P}\right) \quad \text{and} \quad \nabla^* = \frac{3\kappa\rho\lambda_P}{4acT^4}\Pi. \tag{13}$$

Defining convection's efficiency by

$$\Gamma \equiv \frac{\text{“Excess heat content” just before dissolving}}{\text{Energy radiated during element lifetime}} \tag{14}$$

and following Böhm-Vitense's prescription, we deduce the cubic equation:

$$\Phi_0\Gamma^3 + \Gamma^2 + \Gamma - A^2\left(\nabla_{rad} - \nabla^*_{ad} - \nabla^*\right) = 0, \tag{15}$$

with, taking Cox & Giuli (1968) notations, $\Phi_0 = \frac{9}{4}$, $A = \frac{C_p\kappa g Q^{1/2}\rho^{5/2}\Lambda^2}{12\sqrt{2}acT^3P^{1/2}}$.

Convection's efficiency will therefore be modified by the introduction of the magnetic field with pressure modification (through A) by lowering the radiative gradient (through ∇^*) and by modifying the difference between the adiabatic gradient of the convective element and the radiative gradient (through ∇^*_{ad}).

3.2. Modification of Schwarzchild's criterion

The Schwarzchild criterion for stability of a compressible fluid

$$\frac{1}{\gamma} - \frac{P}{\rho}\frac{d\rho}{dP} < 0 \tag{16}$$

is modified when including magnetic field according to Gough & Tayler (1966) prescription

$$\frac{1}{\gamma} - \frac{P}{\rho}\frac{d\rho}{dP} < \frac{B^2}{B^2 + \gamma P}, \tag{17}$$

where γ is the ratio of specific heats. This should tend to rise the threshold at which convection is triggered, what could lowers the extension of the convective zone.

4. Numerical implementation strategy

1. Choose the magnetic stream functions;
2. Elaborate routines computing physical quantities associated to the magnetic field;
3. Introduce the Lorentz Force in the *magneto*-hydrostatic equilibrium;
4. Compute the total pressure $P = P_{\rm gas} + P_{\rm mag}$ before solving transport and energy equations;
5. Introduce ohmic heating in the energy equation;
6. Solve the cubic to deduce the real gradient;
7. Modify Schwarzchild's criterion by adding the right-hand-side term.

5. Perspectives

Once implemented in a stellar evolution code, we will then be able to describe the influence of a global magnetic field on evolution of the stars. As a first application, the impact of a fossil magnetic field in the core and the radiative zone of the Sun will be modeled. The main structural modifications generated by this field will thus be identified. For example, we will see if it has any influence on the extension of the convection zone, or if obvious gradients or sound-speed modifications do appear. On the other hand, we will be able to quantify the oscillation's frequencies shifts due to a global magnetic field by comparing the magnetic model with the standard model. A further step will be the coupling with rotational transport in order to access to a self-consistent picture of MHD dynamical processes occurring in stellar interiors.

References

Brun, A.S., Turck-Chiéze, S., & Zahn, J.P. 1998, in ESA Special Publication, ed. S. Korzennik, Vol. 418, 439
Bullard, E.C., & Gellman, H. 1954, in Phil. Trans. R. Soc. A, Vol. 247, 213
Cox, J.P., & Giuli, R.T. 1968, Principles of stellar structure, (New York, Gordon and Breach)
Gough, D.O., & Tayler, R.J. 1966, MNRAS, 133, 85
Lydon, T.J., & Sofia, S. 1995, ApJS, 101, 357
Mathis, S., & Zahn, J.-P. 2005, A&A, 440, 653
Rashba, T.I., Semikoz, V.B., Turck-Chiéze, S., & Valle, J.W.F. 2007, MNRAS, 377, 453