

# Theoretical uncertainties on white dwarf luminosity functions

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**Abstract.** White dwarf stars play an important role in many fields of modern astrophysics. In the present work we discuss the limits of the available theoretical studies of cooling sequences. We analyze the variation of the age of globular clusters derived from the observed white dwarf sequence caused by different assumptions about the conductive opacity as well as that induced by changing the carbon abundance in the core. The former causes a global uncertainty of the order of 10% and the latter of about 5%. We discuss different choices of the initial-to-final mass relation, which induces an uncertainty of 8% on the globular cluster age estimate.

**Key words.** Stars: white dwarfs – Stars: luminosity function, mass function

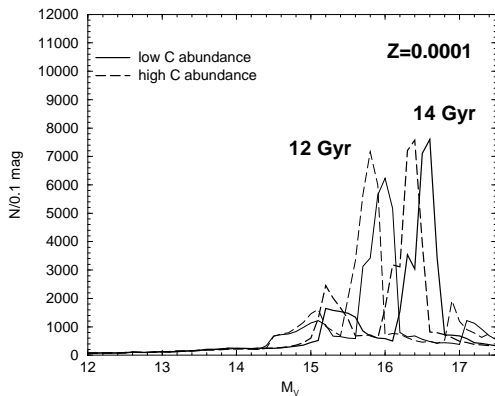
## 1. Introduction

The potential of white dwarf (WD) cosmochronology was realized almost half a century ago by Schmidt (1959), but the utility of this method has been hampered by the extreme faintness of WDs. This situation is now changing and the sample of cold, faint WDs is improving dramatically both in quality and quantity (Calamida et al. 2007; Cool et al. 1996; De Marchi et al. 2004; Hansen et al. 2002, 2007; Monelli et al. 2005; Paresce et al. 1995; Renzini et al. 1996; Richer et al. 1997; Zoccali et al. 2001)

The growing amount of data has prompted a renewed interest in both the theoretical studies of WD evolution and cosmochronology

(Benvenuto & Althaus 1999; Chabrier et al. 2000; Hansen 1999; Prada Moroni & Straniero 2002, 2007; Salaris et al. 2000).

In this context it is clearly necessary to check the reliability of the current generation of cooling models. In a first paper (Prada Moroni & Straniero 2002), we showed that there are still sizeable differences in the predicted cooling ages at low luminosity among the most recent models. For a  $0.6 M_{\odot}$  CO WD, the discrepancy at  $\log(L/L_{\odot}) = -5.5$  reaches 4 Gyr, which means a relative discrepancy larger than 25%. As a sample case, we analyzed some of the main sources of uncertainty in the evolution of a CO WD of  $0.6 M_{\odot}$ , discussing the effects of both the pre-WD history and the physical input. A more precise estimate of this uncertainty



**Fig. 1.** Theoretical LFs for 12 (thin lines) and 14 Gyr (thick lines) for WDs with low (solid lines) and high (dashed lines) C abundance.

should be based on WD luminosity functions (LFs) instead of tracks for a given mass, since this is the best WD chronometer. In fact, the peak of the WD LF shifts by about 0.3 mag per Gyr in the V band, while the main sequence turn-off luminosity, the classically used cluster clock, shifts only about 0.1 mag (Prada Moroni & Straniero 2007). This implies that the WD age estimate is significantly less affected by the uncertainty in the distance modulus, which is the main source of error in dating GCs.

We have calculated several grids of models of DA C-O WDs, with mass in the range  $0.5\text{--}0.9 M_{\odot}$ , under different assumptions for the conductive opacity and the C-O profile (Prada Moroni & Straniero 2007). The derived isochrones and LFs are compared to determine the main uncertainties. We have also analyzed the effect on the WD LFs, and thus on the age estimate, of adopting different initial-to-final mass relationships.

## 2. Carbon and Oxygen abundances in the core

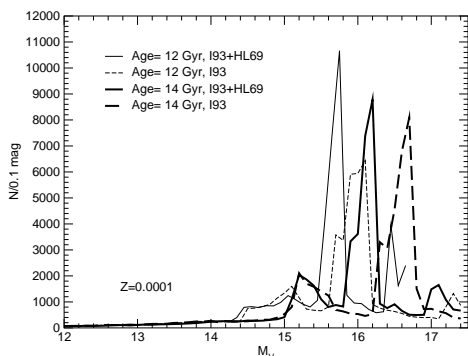
The evolution of a WD can be roughly described as a cooling process in which the luminosity is supplied by the thermal energy stored in the C-O core. Carbon has a higher specific heat than oxygen, thus, the cooling rate

depends sensitively on the C/O ratio: the larger the carbon abundance in the core, the slower the cooling of the WD. The predicted chemical profiles in the core of a CO WD is still quite uncertain due to two poorly constrained ingredients of the He-burning star models, the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  cross section and the efficiency of convective mixing, mainly during the final phase of core He-burning (Imbriani et al. 2001; Straniero et al. 2003). We have computed two sets of WD cooling tracks for two C-O chemical stratifications in the core, one with a central carbon mass fraction of about 0.2 and the other with 0.5, a range that spans the present uncertainty in the theoretical prediction (Prada Moroni & Straniero 2007). As shown in the figure 1, the poor determination of the C abundance in the core translates directly into an uncertainty of about 0.2 mag in the predicted peak position of the WD LFs, roughly corresponding to 0.6 Gyr in the inferred GC age.

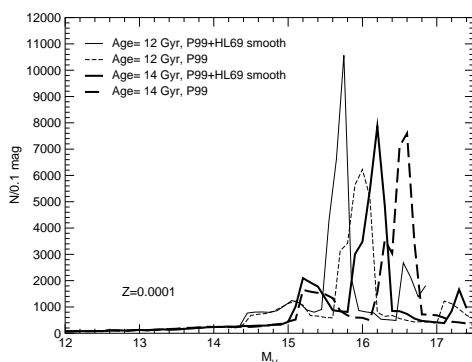
## 3. Conductive opacity

In the core of a WD, where the electrons are fully degenerate, the energy transfer is largely dominated by electron thermal conduction whose efficiency is high enough to keep the core almost isothermal. In contrast, in the He-rich mantle and in the H-rich envelope the electrons are only partially or non degenerate, and thermal conduction is less efficient. The thin envelope is the most opaque region of the WD, a kind of insulating layer which regulates the temperature decrease of the core. Thus, the adopted opacity in this very thin region critically affects the cooling evolution of WDs.

We have computed five sets of cooling tracks with masses in the range  $0.5\text{--}0.9 M_{\odot}$  and the related set of isochrones and LFs for different conductive opacities. Figure 2 shows the comparison between the WD LFs of 12 and 14 Gyr obtained with two different sets of models: one computed adopting the opacity computed by Itoh and coworkers (Itoh et al. 1983; Mitake et al. 1984; Itoh et al. 1993) (hereafter I93) throughout the star, and the other adopting I93 only in the fully degenerate regions ( $\theta = T_F/T < 0.1$ ) and Hubbard & Lampe



**Fig. 2.** Theoretical LFs for 12 (thin lines) and 14 Gyr (thick lines) for WD models computed adopting the conductive opacity of Itoh and coworkers (I93) in the fully degenerate regime ( $\theta = T_F/T < 0.1$ ) and the Hubbard & Lampe (1969, HL69) in the partially degenerate one ( $\theta > 0.1$ ) (I93+HL69, solid lines) and the I93 in the whole structure (I93, dashed lines).



**Fig. 3.** Theoretical LFs for 12 (thin lines) and 14 Gyr (thick lines) for the "P99+HL69 smooth" WD models (solid lines), computed adopting the conductive opacity of Potekhin (P99) and HL69 in the regions where, respectively,  $\theta < 0.1$  and  $\theta > 1$ , and a linear interpolation in the transition zone and the ones computed adopting the P99 conductive opacity in the whole structure (dashed lines).

1969 (hereafter HL69) in the partially degenerate zones.

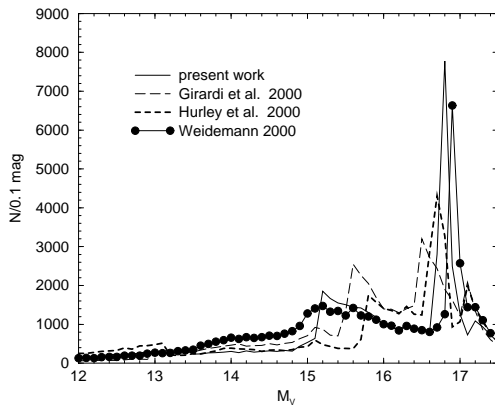
Note that the two sets differ only in the treatment of the conductive efficiency in the partially degenerate regime, corresponding to the very thin external layer, whose mass is less than 1% of the total. Despite this tiny difference, the effect is very large, causing a shift in the position of the LF peak of about 0.3 mag for 12 Gyr and 0.5 mag for 14 Gyr, corresponding to a difference in the inferred GC age of 1 Gyr and 1.6 Gyr, respectively. Figure 3 shows a similar prescription where the I93 opacities have been substituted by those of Potekhin 1999, (hereafter P99, see also Potekhin et al. 1999). Once again, the effect is quite large: the WD LFs peak shifts of about 0.4 mag, corresponding to a difference in the inferred age of about 1.3 Gyr. These results demonstrate the extreme sensitivity of the computed cooling times - and thus of the theoretical WD LFs - to the treatment of conductive opacity in the very thin envelope, affecting the age estimate of GCs at about the 10% level (Prada Moroni & Straniero 2007).

#### 4. Initial-to-final mass relationship

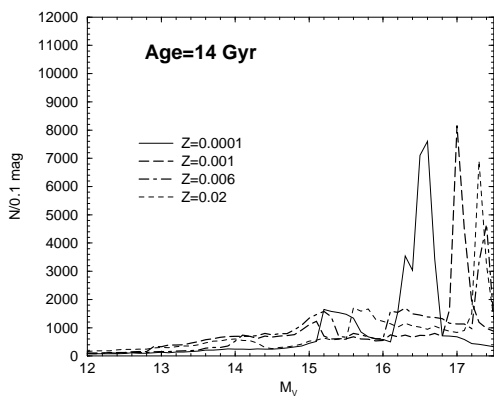
In order to compute WD isochrones and LFs, the age of the progenitor at the start of the cooling sequence must be added to the cooling time. This quantity depends on the mass and the chemical composition of the progenitor.

The relationship between the main sequence mass and the final WD mass is still quite uncertain. In fact, it is sensitive to many poorly known phenomena, such as the extension of the H-burning convective core - which affects the stellar lifetime - and the efficiency of the mass loss mechanism, particularly during the TP-AGB phase when the star loses the largest fraction of its mass.

We computed four set of WD LFs adopting the same set of cooling models but different initial-to-final mass relationships. The substitution of our relation (see e.g. table 1 in Prada Moroni & Straniero 2007) for that of Weidemann (2000) produces, as can be seen in figure 4, a shift of the peak of about 0.1 mag, corresponding to a difference in the age



**Fig. 4.** Theoretical WD LFs for 12 Gyr and different initial-to-final mass relations for the solar chemical composition from: Weidemann (2000, filled circles); Girardi et al. (2000, dashed line); our relation (solid line); Hurley et al. (2000, short dashed line).



**Fig. 5.** Theoretical LFs of 14 Gyr for the four labeled metallicities.

estimate of about 0.3 Gyr. The same shift, but in the opposite direction, is produced by substituting our relation with that provided by Hurley et al. (2000). A larger effect is obtained by exchanging our relation with Girardi et al. (2000): in this case the peak shifts of about 0.3 mag, which translates in an age error of 1 Gyr.

## 5. Metallicity

The cooling evolution of a WD is almost insensitive to the original metallicity of the progenitor star (Prada Moroni & Straniero 2002), but the WD isochrones and LFs are strongly affected by metallicity variations because the pre-WD evolutionary time scale is a sensitive function of the original chemical composition. In addition, also the initial-to-final mass relation changes with metallicity.

In order to estimate the effect of the assumed metallicity, we computed four sets of isochrones and the related LFs for:  $Z=0.0001$ ,  $0.001$ ,  $0.006$  and  $0.02$ . As shown in figure 5, the position of the peak of the WD LF is a sensitive function of the metal content of the progenitor stars. Between  $Z=0.0001$  and  $Z=0.001$ , there is a difference in the visual magnitude of the peak of about 0.4 mag and the same between  $Z=0.001$  and  $Z=0.006$ .

Thus, in order to use WDs to obtain GC ages, one should compute theoretical isochrones and LFs self-consistently using the cluster metallicity. In this case, since typical error on  $[Fe/H]$  is of the order of 0.1 dex, the effect on the WD age determination is negligible.

## 6. Conclusions

In this work we analyzed the accuracy of the predicted WDs ages by computing several sets of WD isochrones and LFs under different prescriptions for the theoretical ingredients.

The main contributory uncertainty is the adopted conductive opacity, in particular in the partially degenerate regime characteristic of the outermost layer.

For the range of age typical of galactic GCs the conductive opacity produces 10% uncertainty in the ages.

Concerning the C-O amount in the core, the uncertainty in the inferred age is less than 5%.

Moreover, let us recall that this contribution will further decrease in the next future thanks to the experiments devoted to the measure of the  $^{12}C(\alpha, \gamma)^{16}O$  reaction rate at the energy of interest.

The initial-to-final mass relation plays an important role in dating GCs by means of

WDs. The uncertainty on the age can reach the 8%. Finally, given the sensitive dependence of the evolution of the WD progenitors on the metal content, in order to use WDs to date GCs, one has to compute WD isochrones and LFs for the metallicity of the cluster. In fact, between  $Z=0.0001$  and  $0.001$  and between  $Z=0.001$  and  $0.006$ , the difference in the inferred age is, for the range of interest, of the order of 10%.

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