



The Tip of the Red Giant Branch

M. Bellazzini

Istituto Nazionale di Astrofisica – Osservatorio Astronomico di Bologna, Via Ranzani 1,
I-40127 Bologna, Italy e-mail: michele.bellazzini@oabo.inaf.it

Abstract. I review the latest results on the calibration of the Tip of the Red Giant Branch as a standard candle, in the optical and in the near infrared. The agreement among different and independent empirical calibrations is rather good, if all the uncertainties are taken into account. The possible extension of the calibration to SDSS photometric bands (i, z) is also discussed.

Key words. Stars: Red Giant – Galaxy: globular clusters – Cosmology: distance scale

1. Introduction

The use of the Tip of the Red Giant Branch (TRGB) as a standard candle is a mature technique that is currently used to reliably estimate distances to galaxies of all morphological types, from the Local Group (see, f.i., McConnachie et al. 2005, and references therein) up to the Virgo cluster and beyond (see, f.i., Ferrarese et al. 2000; Durrell et al. 2007, and references therein). The underlying physics is well understood (Salaris, Cassisi & Weiss 2002; Madore & Freedman 1998, hereafter MF98) and the observational procedure is operationally well defined and robust to contamination by fore/background stars and/or stars brighter than the TRGB (MF98, Sakai et al. 1996). The key observable is the sharp cut-off occurring at the bright end of the Red Giant Branch (RGB) Luminosity Function (LF) that can be easily detected with the application of either non-parametric (Madore & Freedman 1995; Sakai et al. 1996) or parametric

(see, for example Mendez et al. 2002; McConnachie et al. 2005) methods. The necessary condition for a safe application of the technique is that the observed RGB LF be well populated, with more than ~ 100 stars within 1 mag from the TRGB (Madore & Freedman 1995; Bellazzini et al. 2002). Under these conditions, the typical uncertainty on the estimate of the apparent magnitude of the tip I^{TRGB} is of order of a few hundreds of mag (≤ 0.05), and the uncertainties on M_I^{TRGB} dominates the error budget of distance estimates obtained with the TRGB technique. In general, the magnitude of the TRGB depends quite weakly on the age of the considered population (Barker, Sarajedini & Harris 2004; Salaris, Cassisi & Weiss 2002) and, in the Cousins' I band, it depends weakly also on the metallicity, at least for relatively metal poor systems (Da Costa & Armandroff 1990; Lee, Freedman & Madore 1993). In Near Infra Red (NIR) passbands the dependence on metallicity is strong but it can be accounted for with very simple models (see below, and also Ferraro et al. 2000; Ivanov & Borissova

Send offprint requests to: M. Bellazzini

2002; Valenti, Ferraro & Origlia 2004; Bellazzini et al. 2004, and references therein).

Here, I will briefly review the status of the calibration of this standard candle and the perspectives for future applications. For obvious reasons of space this review cannot be and is not intended to be exhaustive or complete; for example, I will not report on the hundreds of applications of the method, I will focus on the most recent results, I will deal more with calibrations in the optical than in NIR passbands, and, also, I will not discuss the differences between the prediction of the various theoretical models (Salaris, Cassisi & Weiss 2002). More detailed discussions and references can be found in MF98, Salaris & Cassisi (1998), Bellazzini, Ferraro & Pancino (2001, hereafter B01), Bellazzini et al. (2004, hereafter B04), and Rizzi et al. (2007, hereafter R07). The TRGB method in the broader context of distance scales is discussed by Walker (2003) and Alves (2004), among the others. The behaviour of the TRGB in complex stellar populations is investigated by Barker, Sarajedini & Harris (2004) and Salaris & Girardi (2005). A detailed and up-to-date report of stellar evolution on the Red Giant Branch is given by Salaris, Cassisi & Weiss (2002).

Nature provides serious limitations in our possibility of studying the behaviour of the TRGB as a function of metallicity, age, elemental abundance pattern, etc., since we lack populous star clusters covering the whole extension of the space of parameters. For this reason, to have an idea of - at least - the differential changes of the TRGB magnitude in a given passband in response to variations of the above quoted parameters we must recur to grids of theoretical models (which may suffer from several problems and inadequacies, see Salaris, Cassisi & Weiss 2002). Also in this case a complete overview of all the available sets of tracks/isochrones would be impossible: here I use mainly BASTI models (Pietrinferni et al. 2004; Cordier et al. 2007), but also Padova models (Girardi et al. 2000, 2004), Yale-Yonsei (Y^2) models (Yi et al. 2001), the old-ages isochrones by Straniero, Chieffi & Limongi (1997), and,

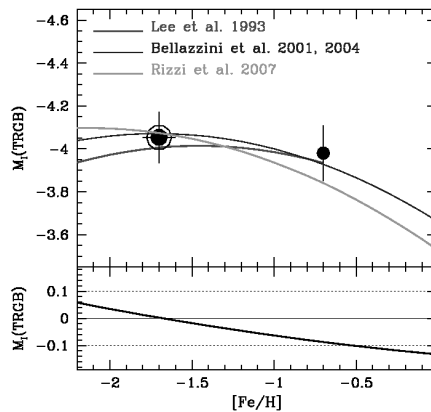


Fig. 1. Comparison between different empirical calibrations of M_I^{TRGB} as a function of metallicity (upper panel). The large filled circle is the observed tip level of ω Cen, the smaller circle corresponds to the tip of 47 Tuc, according to B04. The R07 calibration, as a function of $(V - I)_0$ color has been expressed as a function of $[Fe/H]$ using the $(V - I)_0^{TRGB}$ vs. $[Fe/H]$ relation by B01. The lower panel shows the difference between the B04 and R07 calibrations as a function of $[Fe/H]$.

finally, the recently published Dartmouth models (Dotter et al. 2007).

2. Empirical calibrations

Until now, any empirical calibration of the TRGB assumed a model having the form:

$$M_I^{TRGB} = f([Fe/H]) + ZP \quad (1)$$

where $f([Fe/H])$ is a simple function of the metallicity (or of a proxy for metallicity), typically a polynomial, and ZP is the corresponding Zero Point¹. The $[Fe/H]$ range of validity of the calibration must be specified. This kind of models implicitly assume that the impact of other parameters (like age, Helium abundance, abundance pattern etc.) on the TRGB magnitude can be neglected. In fact

¹ Here I make the case for the Cousins' I passband, but the same kind of model is used also for calibrations in other bands.

uncertainties in these parameters can introduce errors in the determination of M_I^{TRGB} of order ± 0.1 mag that we are currently renouncing to account for, also given the difficulty in obtaining observational constraints on these parameters, in particular for distant galaxies. For instance, the uncertainty on the metallicity scale alone (Zinn & West (1984) versus Carretta & Gratton (1997)) may introduce errors in M_I^{TRGB} as large as ± 0.05 mag; stellar populations having the same total content of heavy elements Z but differing by 0.3 dex in $[\alpha/Fe]$ may display differences in M_I^{TRGB} as large as ± 0.1 mag, depending on the metallicity². On the other hand, for ages larger than ~ 4 Gyr and $[M/H] \leq -0.5$ there is essentially no (reasonable) variation of a physical parameter other than metallicity that seems able to produce variations of M_I^{TRGB} larger than ± 0.1 (Barker, Sarajedini & Harris 2004; Salaris & Girardi 2005, MF98,B04). In summary, the adoption of the family of models of Eq. 1 seems justified, at the present stage, but the user should be aware of all the (possibly) associated uncertainty. Finally, it must be recalled that the Zero Point itself is known with an uncertainty of ± 0.12 , in the best case (B04); TRGB estimates of distance moduli with error bars smaller than this figure neglect part of the actual error budget.

2.1. The calibrating systems

The natural calibrators of the TRGB method are Galactic Globular Clusters (GGC): they are well studied systems with known age and metallicity (Da Costa & Armandroff 1990; Lee, Freedman & Madore 1993). The main drawback of globulars is tied to their nature of low luminosity stellar systems, that means, in most cases, an upper part of the RGB LF that is too poorly populated to provide reliable detection of the LF cut-off (Crocker & Rood 1984; Bellazzini et al. 2002). In practice there is just a handful of GCs that are (potentially) suit-

² For this reason it is advisable to use some form of global metallicity, when possible, like for example $[M/H]$ (Salaris, Chieffi & Straniero 1993; Ferraro et al. 1999, and references therein); see B04.

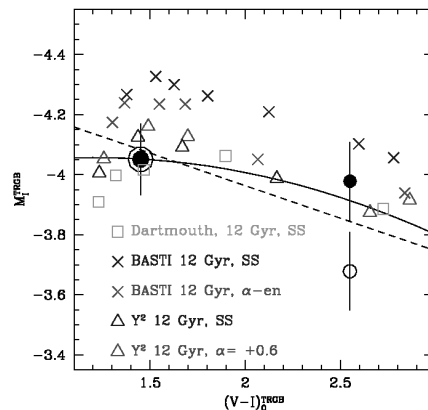


Fig. 2. Comparison of the B01 (continuous line) and R07 (dashed line) calibrations of M_I^{TRGB} as a function of the color of the Tip, and the theoretical predictions of different sets of theoretical models, with solar scaled (SS) chemical composition and various $[\alpha/Fe]$ ratios. The filled circles are the same as in Fig. 1, above; the open circle is the level of the tip of 47 Tuc if the very recent distance estimate by McLaughlin et al. (2006) is adopted (see B04 for a discussion on the distance of this cluster). The difference between the distance of 47 Tuc adopted by B04 and the one estimated by McLaughlin et al. (2006) gives a quantitative idea of the uncertainties that are still affecting even the *local* distance scale.

able as calibrating pillars (B04). B01 tried to circumvent this limitation by taking the form of $f([Fe/H])$ from a large set of GCCs from Ferraro et al. (1999, F99) and the Zero Point from the clean detection of the Tip obtained in the most populous GGC, i.e. ω Cen (B01).

A general limitation of any currently available calibration resides in the uncertainty of the distance to the calibrating systems, usually obtained from RR Lyrae or (equivalently) from the Horizontal Branch level (F99); this approach makes the TRGB method a tertiary distance indicator. By adopting the distance modulus for ω Cen obtained by Thompson et al. (2001) from a double-lined detached eclipsing binary, B01 tied the ZP of their calibration to a semi-geometrical distance estimate, thus virtually making the TRGB a primary dis-

tance indicator. The GAIA astrometric satellite (Turon et al. 2005) will measure trigonometric parallaxes of a large number of ω Cen stars, thus providing an exquisitely accurate distance to the cluster. This will provide an iron-clad ZP for the B01 calibration and will definitely turn the TRGB into a primary distance indicator. The B01 calibration has been extended also to NIR passbands in B04.

R07 introduced two very interesting novelty in their approach to the empirical calibration. While their distance scale fully relies on the HB, with all the associated limitations (see B01), they (a) adopt well resolved nearby galaxies as calibrating systems, and, (b) they calibrate M_I^{TRGB} as a function of the color of the tip $(V - I)_0^{TRGB}$, instead of metallicity or of other less well defined/behaved proxies (Lee, Freedman & Madore 1993). Point (a) resolve the problem of poorly populated RGB LF that affects GCs, since the calibrating systems are much larger systems, plenty of RGB stars; moreover they easily cover a large range in metallicity, in particular reaching solar and super-solar values, out of the reach of GGCs. The RGB of a large galaxy can be so rich of stars that it can be divided in color strips, obtaining more than one estimate of the TRGB level for the same galaxy, at different $(V - I)_0^{TRGB}$ (see R07). Averaging these estimates would mitigate possible systematics associated with the actual Star Formation History (SFH) of the considered system (see Salaris & Girardi 2005). Point (b) allows the simultaneous measure of the actual observable (the RGB LF cut-off) and a very sensitive metallicity proxy (the color at the cut-off) from the same Color Magnitude Diagram (CMD). In all cases in which an estimate of the metallicity of the stellar system under consideration must be obtained from the CMD a TRGB calibration based on $(V - I)_0^{TRGB}$ should be preferred as it uses a direct observable that does not depend on the distance, (see Lee, Freedman & Madore 1993; Sakai et al. 1996; Bellazzini et al. 2002, and references therein, for different approaches to the same problem).

As the B01/B04 calibration is provided as a function of $[Fe/H]$ and $[M/H]$ while R07 one is provided as a function of $(V - I)_0^{TRGB}$, some

transformation is needed to actually compare the two calibrations. We use the relations from B01 to transform both calibrating relations. In particular the B01 calibration becomes:

$$M_I^{TRGB} = a(V - I)_0^2 - b(V - I)_0 - c \quad (2)$$

where $a = 0.080$, $b = -0.194$, and $c = -3.939$, and $(V - I)_0$ is the color at the tip, i.e. $(V - I)_0^{TRGB}$.

In Fig. 1 different empirical calibrations of M_I^{TRGB} (expressed as a function of $[Fe/H]$) are compared. The overall agreement is reasonably good. In particular B01/B04 and R07 calibration are completely independent and are based on completely different sets of calibrating systems. Still, the two calibrations are in excellent agreement (within $\approx \pm 0.05$ mag for $[Fe/H] \leq -1.0$; the difference becomes larger than ≈ 0.1 mag only for $[Fe/H] \geq -0.5$). In the M_I^{TRGB} vs. $(V - I)_0^{TRGB}$ plane (Fig. 2) the agreement is even better. The comparison with various sets of models shown in Fig. 2 suggests two main conclusions: (1) the observed slope is reasonably reproduced by all the models, over a very large color range; (2) there is a large variation in the predicted ZP, that may be partly due to the color transformations adopted in the various models (see B04 for a discussion). A deeper discussion of differences among the various theoretical models is clearly beyond the scope of this paper (see Salaris, Cassisi & Weiss 2002). Limited preliminary experiments suggests that different models may provide significantly different predictions for the variations of M_I^{TRGB} in response to variations of $[\alpha/Fe]$; the issue is not treated by existing theoretical studies and it probably deserves a deeper investigation.

2.2. Helium abundance

In recent years it has emerged the possibility that the early chemical evolution of (at least some) stellar system may lead to the generation of stars with a very high abundance of Helium, up to $Y \sim 0.40$ (see Norris 2004; D'Antona et al. 2005; Piotto et al. 2005; Sollima et al. 2005, and these proceedings), triggering a renewed interest in the study of

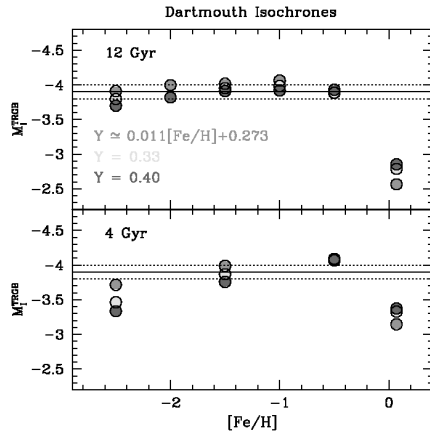


Fig. 3. M_I^{TRGB} as a function of $[Fe/H]$ for models with different Helium abundances Y . Upper panel: 12 Gyr old models; Lower panel: 4 Gyr old models.

the effects of He abundance on remarkable features of CMDs.

In Fig. 3 I use Dartmouth models to explore the behaviour of M_I^{TRGB} in response to *large* variations of Y . For old populations, the effect is relatively small. It is interesting to note the inversion of the sense of the dependence at the extremes of the metallicity range: at $[Fe/H] = -2.5$ the population with the lowest Y has the brightest Tip, the opposite is true at solar metallicity.

Fig. 3 suggests that at intermediate ages (~ 4 Gyr) the effect of Helium on M_I^{TRGB} may be very strong. The issue certainly deserves to be investigated in deeper detail with other sets of theoretical models. In any case, it should be recalled that Fig. 3 explores quite extreme regimes of He abundances (far larger than the Sun, for instance), and a conclusive case for the existence of such extremely-He-rich stars (at least in large numbers) is yet to be done.

2.3. Age, high metallicities and NIR.

In Fig. 4 BASTI models of different ages are superposed to the B01/B04 and R07 calibrations, after a $+0.24$ mag shift to match the ZP of the empirical calibrations. The lower panel

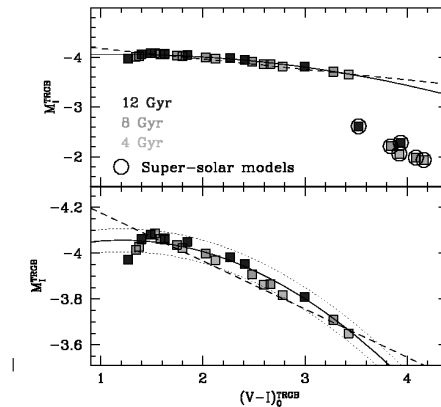


Fig. 4. Upper panel: M_I^{TRGB} as a function of the color of the tip $(V-I)_0^{TRGB}$ for solar-scaled BASTI models of different ages (4 - 8 - 12 Gyr, corresponding to light - medium - dark grey filled squares; all models have been shifted by $+0.24$ mag in M_I to match the empirical calibrations, see Fig. 2, B01 and B04). The large empty circles marks models with solar and super-solar metallicity. The continuous line is the B01 calibration in the form of Eq. 2, the dashed line is the R07 calibration. Lower panel: the same as above but zoomed on the models with sub-solar metallicity. The symbols are the same as in the upper panel. The dotted lines enclose the ± 0.05 mag range around the B01 calibration.

shows that, for $(V-I)_0^{TRGB} \leq 3.5$, the effect of age variations on M_I^{TRGB} is within $\leq \pm 0.05$ mag, thus confirming the weakness of the age dependency, in the range 4 Gyr - 12 Gyr. The upper panel shows that for solar and super-solar populations the dependency of M_I^{TRGB} on color (and metallicity) becomes strong and may also require more complex models.

This problem is largely removed by passing to NIR photometry. In Fig. 5 the empirical calibrations of the absolute J,H,K magnitude of the tip as a function of $(J-K)_0^{TRGB}$ obtained from B04 data are compared to BASTI models of different ages and $[\alpha/Fe]$ ratios. While the dependence on color (metallicity) is quite strong, a simple linear model provide a good description of the TRGB magnitude to $\approx \pm 0.1$ mag, up to super-solar metallicities. As the

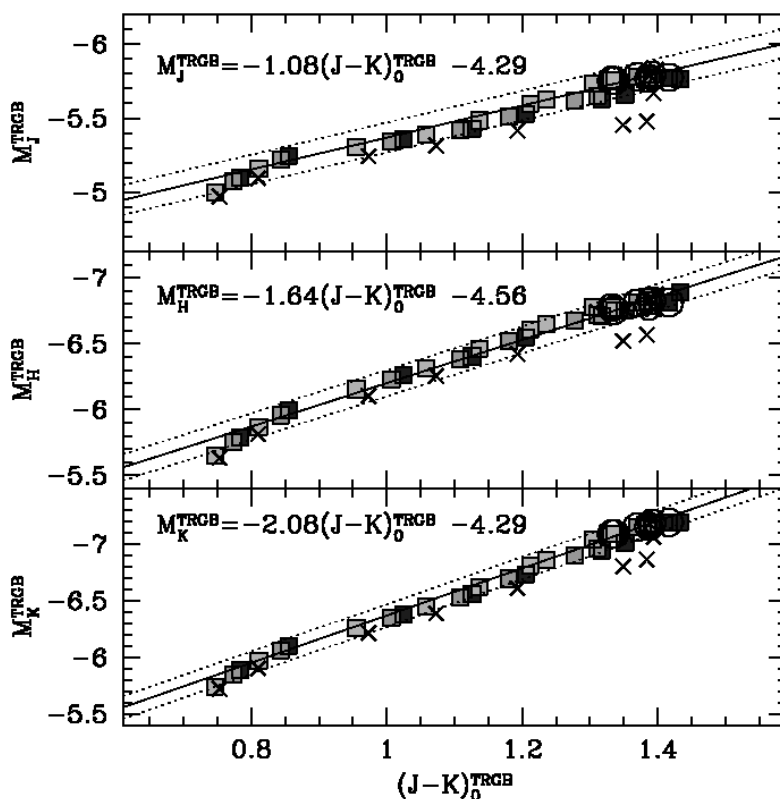


Fig. 5. J,H,K absolute TRGB magnitudes as a function of $(J - K)_0^{TRGB}$ color from the BASTI models for different ages; the symbols are the same as in Fig. 4, with the addition of the \times symbols that are α -enhanced, 12 Gyr old models. The continuous lines are derived from the empirical calibrations by B04; the dotted lines enclose the ± 0.1 mag range around the B04 calibration.

most powerful and innovative telescopes of the future (JWST³, ELT⁴, TMT⁵, etc.) will be dedicated to / optimized for infrared observations and they will probably allow us to resolve the RGB of very metal rich elliptical galaxies, the importance of NIR calibrations of the TRGB will be ever growing in the next couple of decades.

In Fig. 5 the calibrating equations for the absolute magnitude of the Tip as a function of

$(J - K)_0^{TRGB}$ color instead of $[M/H]$ are also reported, as derived from B04 data.

2.4. The TRGB in the SDSS system

The Sloan Digital Sky Survey is providing accurate photometry of a very wide portion of the Northern sky in its own specific (ugriz) photometric system (Fukugita et al. 1996; Adelman-McCarthy et al. 2007). Other currently ongoing (or planned) large surveys will further extend the sky coverage in the same (or very similar) photometric system

³ www.jwst.nasa.gov/

⁴ www.eso.org/projects/e-elt/

⁵ www.tmt.org/

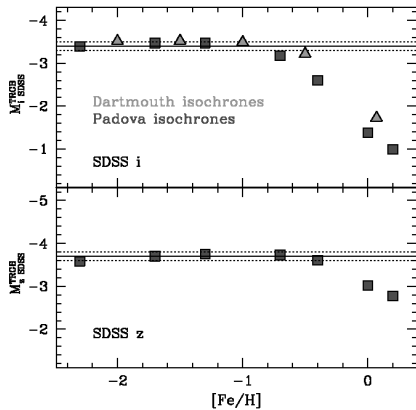


Fig. 6. *i* and *z* absolute TRGB magnitudes as a function of metallicity for Dartmouth (triangles) and Padova (squares) models. The horizontal lines marks $M_i^{TRGB} = -3.44 \pm 0.1$ (continuous and dotted lines, respectively), and $M_z^{TRGB} = -3.67 \pm 0.1$.

(SEGUE⁶, PanSTARRS⁷, LSST⁸). It is very likely that most photometric studies in the future will be performed in this system, as a large number of (relatively faint) secondary standard stars will be present in any given field of the sky. In Fig. 6 we use two sets of models to show that in both the *i* and *z* SDSS passbands, the magnitude of the TRGB has a very weak dependence on metallicity over wide metallicity ranges (as in Cousins' I, or even better). As a preliminary calibration, $M_i^{TRGB} = -3.44 \pm 0.1$ for $[\text{Fe}/\text{H}] \leq -1.0$, and $M_z^{TRGB} = -3.67 \pm 0.1$ for $[\text{Fe}/\text{H}] \leq -0.4$ can be adopted, from Fig. 6; it is quite clear that an empirical calibration is needed, also for these passbands. It is worth noting that PanSTARRS will observe also in a band (*Y*) that is intermediate between *z* and *J*, that may even turn out to be the most indicated for the calibration of the TRGB as a standard candle (see R07).

⁶ segue.uchicago.edu/

⁷ pan-starrs.ifa.hawaii.edu/public/

⁸ www.lsst.org

3. Conclusions

The future of the TRGB as a distance indicator looks very promising, as new telescopes and surveys will give access to more distant systems, larger areas of the sky and more favourable wavelength windows. In general applications to distant galaxies, the lack of knowledge of many parameters that may influence the TRGB magnitudes will probably prevent the method to reach exquisite levels of accuracy (i.e., better than 10% uncertainties); on the other hand the *weakness* of all these dependencies ensures a good level of robustness and reliability of the method. The Zero Point of current empirical calibrations is uncertain at the ± 0.1 level, but the results the GAIA astrometric mission should reduce such uncertainty virtually to zero. An extension of the empirical calibration to passbands like SDSS *i* and *z* and a detailed comparison between the prediction of the different sets of theoretical models are suggested as the most interesting problems to afford, at the present stage.

Acknowledgements. M.B acknowledges the financial support to this research by INAF, through the grant CRA 1.06.08.02., assigned to the project *A hierarchical merging tale told by stars: motions, ages and chemical compositions within structures and substructures of the Milky Way*.

References

- Adelman-McCarthy, J.K., et al. 2007, ApJS, 172, 634
- Alves, D.R. 2004, New Astr. Rev., 48, 659
- Barker, M.K., Sarajedini, A., & Harris, J. 2004, ApJ, 606, 869
- Bellazzini, M., Ferraro, F.R., & Pancino, E. 2001, ApJ, 556, 635 (B01)
- Bellazzini, M., Ferraro, F. R., Origlia, L., et al. 2002, AJ, 124, 3222
- Bellazzini, M., Ferraro, F.R., Sollima, A., Pancino, E., & Origlia, L. 2004, A&A, 424, 199 (B04)
- Carretta, E., & Gratton, R. 1997, A&AS, 121, 95
- Cordier, D., Pietrinferni, A., Cassisi, S., & Salaris M. 2007, AJ, 133, 468

- Crocker, D. A., & Rood, R. T. 1984, in *Observational Tests of the Stellar Evolution Theory*, IAU Symp., 105, 159
- D'Antona, F., Bellazzini, M., Caloi, V., Fusi Pecci, F., Galletti, S., & Rood, R. T. 2005, *ApJ*, 631, 868
- Da Costa, G.S., & Armandroff, T.A. 1990, *AJ*, 100, 162
- Dean, J. F., Warren, P. R., & Cousins, A. W. J. 1978, *MNRAS*, 183, 569
- Dotter, A., Chaboyer, B., Jevremovic, D., Baron, E., Ferguson, J.W., Sarajedini, A., & Anderson, J., 2007, *AJ*, 134, 376
- Durrell, P.R., et al. 2007, *ApJ*, 656, 746
- Ferrarese, L., et al. 2000, *ApJS*, 128, 431
- Ferraro, F. R., Messineo, M., Fusi Pecci, F., de Palo, M. A., Straniero, O., Chieffi, A., & Limongi, M. 1999, *AJ*, 118, 1738
- Ferraro, F. R., Momtegriffo, P., Origlia, L., & Fusi Pecci, F. 2000, *AJ*, 119, 1282
- Fukugita, M., Ichikawa, T., Gunn, J. E., Doi, M., Shimasaku, K., Schneider, D. P. 1996, *AJ*, 111, 1748
- Girardi, L., et al. 2000, *A&AS*, 141, 371
- Girardi, L., et al. 2004 *A&A*, 422, 205
- Ivanov, V. D., & Borissova, J. 2002, *A&A*, 390, 937
- Lee, Y.-W., Freedman, W.L., & Madore, B. 1993, *ApJ*, 417, 553 (L93)
- Madore, B.F., & Freedman, W.L. 1995, *AJ*, 109, 1645
- Madore, B.F., & Freedman, W.L. 1998, in *Stellar astrophysics for the Local Group*, ed. A. Aparicio, A. Herrero, and F. Sanchez (Cambridge: Cambridge University Press), 305
- McConnachie, A. W., Irwin, M. J., Ferguson, A. M. N., Ibata, R. A., Lewis, G. F., & Tanvir, N. 2005, *MNRAS*, 356, 979
- McLaughlin, D.E., Anderson, J., Meylan, G., Gebhardt, K., Pryor, C., Minniti, D., & Phinney, S. 2006, *ApJS*, 166, 249
- Mendez, B., Davis, M., Moustakas, J., et al. 2002, *AJ*, 124, 213
- Norris, J.E. 2004, *ApJ*, 612, L25
- Pietrinferni, A., Cassisi, S., Salaris, M., & Castelli F. 2004, *ApJ*, 612, 168
- Piotto, G. 2005, *ApJ*, 621, 777
- Rizzi, L., Tully, R.B., Makarov, D., Makarova, L., Dolphin, A.E., Sakai, S., & Shaya, E.J. 2007, *ApJ*, 661, 815 (R07)
- Sakai, S., Madore, B.F., & Freedman, W.L. 2002, *ApJ*, 461, 713
- Salaris, M., Chieffi, A., & Straniero, O. 1993, *ApJ*, 414, 580
- Salaris, M., & Cassisi, S. 1998, *MNRAS*, 298, 166
- Salaris, M., Cassisi, S., & Weiss, A. 2002, *PASP*, 114, 375
- Salaris, M., & Girardi, L. 2005, *MNRAS*, 357, 669
- Sollima, A., Pancino, E., Ferraro, F. R., Bellazzini, M., Straniero, O., & Pasquini, L. 2005, *ApJ*, 634, 332
- Straniero, O., Chieffi, A., & Limongi, M. 1997, *ApJ*, 490, 425
- Thompson, I. B., Kaluzny, J., Pych, W., Burley, G., Krzeminski, W., Paczyński, B., Persson, S. E., & Preston, G. W. 2001, *AJ*, 121, 3089
- Turon, C., O'Flaherty, K. S., & Perryman, M. A. C., (Eds.) 2005, *ESA SP 576*
- Valenti, E., Ferraro, F.R., & Origlia, L. 2004, *MNRAS*, 354, 815
- Ventura, P., Zeppieri, A., Mazzitelli, I., & D'Antona, F. 1998, *A&A*, 334, 953
- Zinn, R., & West, M.J. 1984, *ApJS*, 55, 45
- Walker, A. R. 2003, in *Stellar candles for the extragalactic distance scale*, ed. D. Alloin, and W. Gieren (Springer), *Lect. Notes Phys.*, 635, 265
- Yi, S., Demarque, P., Kim, Y.-C., Lee, Y.-W., Ree, C.H., Lejeune, T., & Barnes, S. 2001, *ApJS*, 136, 417