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On some MHD aspects of star-disc-jets systems

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Abstract. This contribution briefly reviews our current knowledge on the physics of accretion discs driving jets and their connection with the central protostar. Such a connection is now not only probed by modern observations but it is also requested for spinning down protostars, which are known to be both actively accreting and contracting. These two highly energetic processes, jet production from accretion discs and star-disc interactions, are only possible if a large scale magnetic field is present. It is shown that this may indeed be a rather common situation in accretion discs.

Key words. Accretion, accretion discs – Magnetohydrodynamics (MHD) – magnetic fields – ISM: jets and outflows – Stars: pre-main sequence – Stars: rotation

1. Introduction

Actively accreting classical T Tauri stars often display supersonic collimated jets on scales of a few 10-100 AU in low excitation optical forbidden lines (fig. 1, left). Molecular outflows observed in younger Class 0 and I sources may be powered by an inner unobserved optical jet. These jet signatures are correlated with the infrared excess and accretion rate of the circumstellar disc (Cabrit et al. 1990; Hartigan et al. 1995). It is therefore widely believed that the accretion process is essential to the observed jets, although the precise physical connection remains a matter of debate: do the jets emanate from the star, the circumstellar disc or the magnetospheric star-disc interaction?

One argument in favor of accretion-powered disc winds is its universality, since jets are also observed from accreting compact objects. The other two images in figure 1 show bipolar jets observed in radio wavelengths. The middle one is a jet from a radio galaxy or a quasar, emitted from an accretion disc settled around a supermassive black hole. The rightmost image shows bipolar jets coming from the disc around a galactic stellar black hole, looking very similar to quasar jets. Because many characteristics seem to scale with the central black hole mass, such an object has been nicknamed a "microquasar" (Mirabel & Rodriguez 1998).

The basic and universal accretion-ejection mechanism would then be the following. Accretion discs around a central object can, under certain circumstances and whatever the nature of this object, drive jets through the action of large scale magnetic fields. These fields would tap the mechanical energy released by mass accretion within the disc and transfer it to an ejected fraction. The smaller the fraction and the larger the final jet velocity. One thing that must be understood is how the presence of

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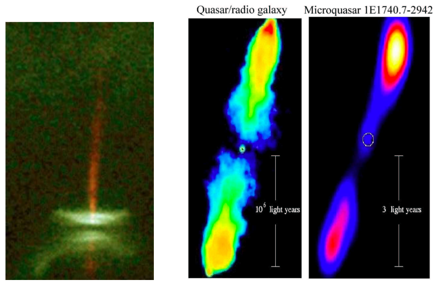

Fig. 1. Jets in astrophysics: young star (left), radio galaxy (middle) and X-ray binary (right).

these jets modifies the nature of the underlying accretion flow. Many papers in the literature actually assume that the accretion disc resembles a standard accretion disc as envisioned by Shakura & Sunyaev (1973).

Thus, although it was soon recognized that ejection and accretion were tightly related (Blandford & Payne 1982; Konigl 1989), a truly self-consistent model appeared only lately (Ferreira & Pelletier 1993; Ferreira 1997; Ferreira & Casse 2004). To date, this is the only published magneto-hydrodynamic (MHD) model that describes in a self-consistent way the physics of an accretion disc threaded by a large scale magnetic field giving rise to self-collimated jets. Such a system was called a Magnetized Accretion-Ejection Structure, hereafter MAES. This model is unique in the sense that it provides both the physical conditions within the disc required to steadily launch jets and the distributions of all quantities in space (although the self-similar assumption used introduces some unavoidable biases).

2. Magnetic fields in accretion discs

The necessary condition for launching a self-collimated jet from a Keplerian accretion disc is the presence of a large scale vertical magnetic field close to equipartition (Ferreira & Pelletier 1995), namely

$$B_z \simeq 0.2 \left(\frac{M}{M_\odot}\right)^{1/4} \left(\frac{\dot{M}_a}{10^{-7} M_\odot/yr}\right)^{1/2} \times \left(\frac{r}{1\ \mathrm{AU}}\right)^{-5/4+\xi/2} \mathrm{G}, \quad (1)$$

where ξ is the disc ejection efficiency as measured by a varying disc accretion rate, namely $\dot{M}_a \propto r^{\xi}$. The value of this magnetic field is far smaller than the one estimated from the interstellar magnetic field assuming either ideal MHD or $B \propto n^{1/2}$ (Heiles et al. 1993; Basu & Mouschovias 1994). This implies some decoupling between the infalling/accreting material and the magnetic field in order to get rid off this field. This issue is still under debate. The question is therefore whether accretion discs can build up their own large scale magnetic field (dynamo) or if they can drag in the interstellar magnetic field? Although no large scale fields have been provided by a self-consistent disc dynamo, this scenario cannot be excluded. But the latter scenario (advection) seems a bit more natural.

Anyway, this remains a major assumption that was critically discussed. Moreover, magneto-centrifugal launching requires a magnetic field at the disc surface with an angle of at least 30^o with the vertical (Blandford & Payne 1982). This condition has been also questioned. Finally, it was argued that this process of jet formation was intrinsically unstable. These three issues cast doubts on the overall accretion-ejection process and it becomes urgent to shed light on them.

2.1. Magnetic field curvature

The Blandford & Payne (1982) criterion can be simply written as $B_r^+ \geq B_z$, where B_r^+ is the radial magnetic component at the disc surface. For a smooth magnetic field, this translates into a magnetic Reynolds number $\mathcal{R}_m = -ru_r/\nu_m$ greater or equal than r/h, where ν_m is the disc magnetic diffusivity and h the disc vertical scale height. Note that $h/r \ll 1$ in a quasi-keplerian accretion disc. This constraint can be readily understood. In order to have comparable magnetic field components at the disc surface, a strong toroidal current must develop, which is what happens when advection is stronger than diffusion.

Now, in turbulent media such as accretion discs, it is usually assumed that all anomalous transport coefficients are comparable. That is, the turbulent magnetic diffusivity ν_m roughly equals the disc viscosity ν_v, namely $\mathcal{R}_m \sim \mathcal{R}_e = -ru_r/\nu_v$. The problem is that in a standard accretion disc (hereafter SAD) viscosity matches the radial inward motion. One therefore has $\mathcal{R}_e \sim 1$ (Shakura & Sunyaev 1973), namely straight magnetic field lines. It is thus clear that no magnetically driven jets can be driven from a SAD (Heyvaerts et al. 1996; Ogilvie & Livio 1998).

However, the torque due to the jet has not been taken into account here. In a jet emitting disc (hereafter JED), the jet torque is roughly r/h times the viscous (turbulent) torque (Ferreira & Pelletier 1993, 1995). As a consequence, the radial accretion velocity is larger in a JED than in a SAD (by this amount) and one gets in a consistent way $\mathcal{R}_m \sim \mathcal{R}_e \geq r/h$. SADs and JEDs are two mutually exclusive solutions.

2.2. Magnetic field dragging in discs

Lubow et al. (1994a) investigated the advection of a large scale magnetic field by a SAD and, indeed, always obtained straight magnetic field lines. They concluded that, unless the magnetic Prandtl number $\mathcal{P}_m = \nu_v/\nu_m$ is unrealistically high, no magneto-centrifugal winds could be launched from a SAD. Again, the only torque taken into account was the viscous torque! Thus, according to the above arguments, they could only obtain $\mathcal{R}_m \sim \mathcal{R}_e \sim 1$, namely straight field lines. But note that $\mathcal{R}_m \sim 1$ does not mean that the magnetic field lags behind while the disc material is accreted. This is what one gets when rigid boundary conditions are applied on potential fields, as was done by these authors.

In fact, even in SAD with $\mathcal{R}_m \sim 1$, a large scale vertical magnetic field can be advected along with the accreting flow and establish a steady-state configuration. Indeed, the diffusion equation in a SAD writes

$$\nu_m \frac{\partial B_z}{\partial r} \simeq u_r B_z \qquad (2)$$

which admits $B_z \propto r^{-\mathcal{R}_m}$ as a solution (Ferreira et al. 2006b). It clearly shows that such a large scale magnetic field can indeed be present in a SAD, as a natural outcome of advection and diffusion. Note that the field strength is increasing towards the center.

Let us assume that the disc material is always ionized enough to allow for some coupling with the magnetic field (and use MHD). The outer parts of the accretion disc will then take the form of a SAD with no jets and almost straight ($\mathcal{R}_m \sim 1$) field lines. Ferreira & Pelletier (1995) showed that an equipartition field would give rise to magnetically-driven jets. We can thus define a transition radius, r_J where the disc magnetization $\mu = B_z^2/\mu_o P$, where P is the total (gas plus radiation) pressure, becomes of order unity. This radius would then mark the transition between the outer SAD and the inner JED.

In a SAD, the disc vertical equilibrium combined with the angular momentum conservation provides

$$P = \frac{\dot{M}_a \Omega_K^2 h}{6\pi\nu_v} \propto r^{-3/2-\delta} \qquad (3)$$

where $\dot{M}_a$ is the disc accretion rate, Ω_K the Keplerian rotation law and $h(r) \propto r^{\delta}$. Since δ is always close to unity in circumstellar discs (and in most discs around compact objects), one gets $\mu \propto r^{-\epsilon}$ with $\epsilon \sim 1$. The disc magnetization is thus naturally increasing towards the disc center!

Thus, one may safely expect JEDs in the innermost disc regions, at least in some objects. It only depends on the strength of the magnetic field. This is very difficult to measure, in any astrophysical object. Donati et al. (2005) did this *tour de force* using the spectropolarimeter ESPadOnS. There is now a clear observational evidence that the protostar FU Ori has indeed such a large scale magnetic field anchored in the disc.

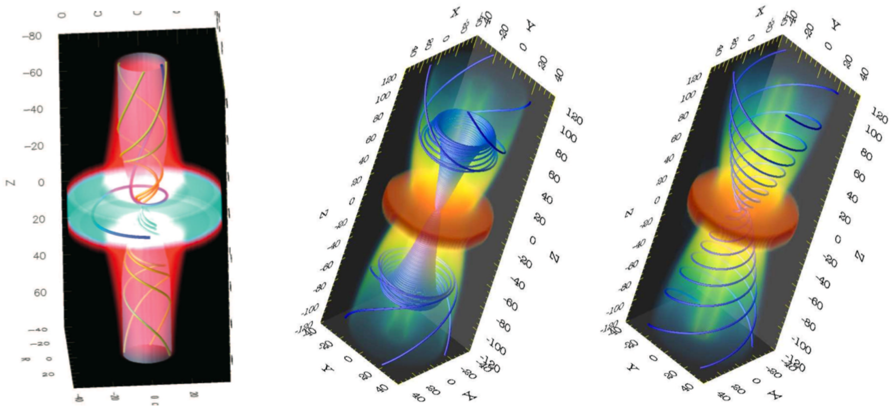

Fig. 2. Left: First MHD simulation of a MAES, done with the MHD code VAC (Casse & Keppens 2002). The numerical experiment has confirmed that only an equipartition field can drive steady jets. **Right:** Two simulations done with the AMR MHD code FLASH (Zanni et al, 2007, A&A, in press). Another important analytical result is confirmed: only a large magnetic diffusivity allows a steady state (rightmost image).

2.3. Is accretion-ejection unstable?

There has been some claims in the literature that the magneto-centrifugal acceleration process was unstable (Lubow et al. 1994b; Cao & Spruit 2002). The idea was the following. Start from a steady picture where the accretion velocity u_r at the disc midplane is due to the jet torque. It leads to a bending of the poloidal field lines described by an angle θ with the vertical. Now imagine a small perturbation δu_r enhancing the accretion velocity. Then, according to these authors, the field lines would be more bent (θ increases) which would lead to lower the altitude of the sonic point. Because the sonic point would be located deeper in the disc atmosphere, where the density is higher, more mass would be henceforth ejected which would then increase the total angular momentum carried away by the jet. This means that the torque due to the jet is enhanced and will, in turn, act to increase the accretion velocity. Thus, according to these authors, the accretion-ejection process is inherently unstable.

But the whole idea of this instability is based upon a crude approximation of the disc vertical equilibrium. A magnetized disc is *not* in hydrostatic equilibrium. In fact, the magnetic field produces a strong vertical compression, comparable to the gravity. As a consequence, as θ is increased, *less* mass is being ejected, not more. This has been pointed out by Königl & Wardle (1996) and Königl (2004) and is indeed verified in full MAES calculations.

3. Jets from accretion discs

3.1. Analytical calculations and numerical experiments

Calculations of the full disc-jets systems were done within a self-similar ansatz (see Ferreira 2002 for more details). So called "cold" solutions (with isothermal or adiabatic magnetic surfaces) are obtained with a typical ejection efficiency $\xi \sim 0.01$. "Warm" solutions assume some heat deposition at the disc surface and allow discs with $\xi \sim 0.1$ (hence jets 3 to 5 times slower but denser).

In both cases, an equipartition magnetic field must be present. A stronger field would compress too much the disc in the vertical direction, forbidding therefore mass to be lifted out by the thermal pressure gradient. On the other hand, a field too low is unable to provide the fast accretion motion, required to allow for a rapid acceleration at the disc surface. Another important constraint is the presence of a strong turbulence within the disc. Indeed, steady-state ejection is only possible if mass can steadily cross the field lines. Such a transport can only be done by some anomalous means, namely waves or local self-sustained turbulence. Within our framework, only turbulence has been accounted for, through an effective magnetic diffusivity ν_m. Its amplitude is controlled by the parameter $\alpha_m = \nu_m / V_A h$, where V_A is the Alfvén speed at the disc midplane. Only large values of $\alpha_m \sim 1$ allow for steady-state solutions (Ferreira 1997).

The physics of MAES, as understood through these semi-analytical works, have been confirmed by two independent groups, using two distinct numerical MHD codes (see figure 2). These are the only works where the mass load is computed in a consistent way with the jet acceleration. Other MHD simulations of jets driven by accretion discs usually do not compute the disc and simply assume this mass load.

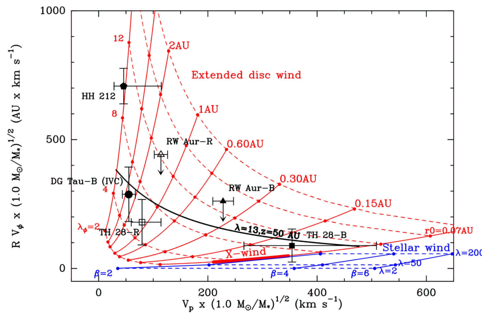

Fig. 3. Comparison of predicted specific angular momentum vs. poloidal velocities with observations of T Tauri microjets. Full and dashed curves show expected theoretical relations for MHD disc and stellar winds. Plotted in symbols are jet kinematics measured at distance $z \simeq 50$ AU in the DG Tau, RW Aur, and Th 28 jets. The infrared HH 212 jet is also shown for comparison (from Ferreira et al. 2006a).

3.2. Constraints from T Tauri jet observations

A consensus seems to have slowly emerged these last years. It is now accepted that most of the ejected mass in jets comes from the accretion disc, even if other ejection events are coexisting (see Ferreira et al. 2006a for more details).

Basically, explaining the jet phenomenon from young stars with only stellar winds faces the quite overwhelming task of finding a means to produce the observed huge mass loss rates. On the other hand, published wide-angle disc winds from the corotation ("X-winds", Shu et al. 1994) have kinematical properties which are inconsistent with observations. This may be seen in fig 3 which is a plot of the specific angular momentum carried by one magnetic surface (anchored at a disc radius r_o) as a function of the jet poloidal speed. In this plot, red solid lines define a constant anchoring radius r_o whereas dashed lines a constant magnetic lever arm $\lambda \sim 1 + 1/2\xi$.

Observations clearly favor self-confined jets launched from some radial extension in the disc (say from 0.1 to 0.5-2 AUs). However, cold models (with $\xi \sim 0.01$, hence $\lambda \sim 50$) are excluded. Only "warm" solutions with $\xi \sim 0.1$ ($\lambda \sim 10$) are fully compatible with current observations. Such models require heat input at the upper disc surface layers in order to allow more mass to be loaded onto the field lines

The origin of this heat deposition remains an open question. It cannot be due solely to illumination by stellar UV and X-ray radiation (Garcia et al., to be submitted). Alternatively, the turbulent processes responsible for the required magnetic diffusivity inside the disc might also lead to a turbulent vertical heat flux leading to dissipation at the disc surface layers. It is interesting to note that in current MHD simulations of the magneto-rotational instability a magnetically active "corona" is quickly established (Stone et al. 1996; Miller & Stone 2000). Although no 3D simulation has been done with open magnetic field lines, this result is rather promising. Indeed, it might be an intrinsic property of the MHD turbulence in accretion discs, regardless of the launching of jets (see arguments developed by Kwan 1997 and Glassgold et al. 2004).

4. The star-disc magnetic interaction

Once they become visible in the optical, T Tauri stars exhibit rotational periods of the order of 10 days, which is much smaller than expected (Bouvier et al. 1997; Rebull et al. 2002). This implies a very efficient mechanism of angular momentum removal from the star during its embedded phase. Moreover, a T Tauri star seems to evolve with an almost constant rotational period although it undergoes some contraction and is still actively accreting disc material for roughly a million years. This is a major issue in star formation, unsolved yet.

One solution to this paradox is the star-disc interaction. Angular resolution is not yet sufficient to directly image this region (of size 0.1 AU or less: it would require optical interferometry) but there are mounting spectroscopic and photometric evidences that the disc is truncated by a stellar magnetosphere (assumed to be a dipole) and that accretion proceeds along magnetic funnels or curtains towards the magnetic poles (see Bouvier et al. 2007 and references therein). This gave rise to the so-called *disc locking paradigm*, where it is assumed that the stellar angular momentum can be transferred to the disc. Unfortunately, this idealized picture can probably not be maintained (see discussion in Matt & Pudritz 2005b).

The simple reason is that accretion onto the star and disc locking are two contradictory requirements. Let Ω_* be the angular velocity of the star. Its magnetosphere will try to make the disc material corotate with the protostar so that the sign of the torque depends directly on the relative angular velocity. Stellar magnetic field lines threading the disc beyond the rotation radius $r_{co} = (GM/\Omega_*^2)^{1/3}$ exert a positive torque, whereas they brake down the disc material below r_{co}. Let us now define the magnetospheric radius r_m below which the stellar magnetic field is strong enough to "truncate" the disc. This is done by enforcing the disc material to flow along the field lines and no longer on the plane of the disc.

Now, one can safely realize that accretion onto the star can only proceed if $r_m < r_{co}$. In this situation, the stellar magnetic field can brake down both the disc and the material accreting in the funnel flows. This implies of course a stellar spin up by the disc material located below r_{co}. The disc locking paradigm assumes that stellar field lines remain anchored beyond r_{co}, giving hopefully rise to some angular momentum balance. But within this paradigm, the disc viscosity must be efficient enough so as to radially transport outwards both the disc and stellar angular momentum! This is unrealistic because the stellar angular momentum is far too large. Moreover, all numerical simulations done so far showed a fast opening of the field lines beyond r_{co} (through numerical reconnection), severing the causal link and thereby strongly reducing this negative torque.

An easy way to brake down an accreting protostar is to launch disc material along magnetic field lines that are anchored onto the rotating star. There are two simplistic ways to achieve this as illustrated in figure 4.

The first way is to assume that both the disc and the stellar fields have the same origin, so that the stellar magnetic moment is parallel to the disc magnetic field (fig. 4, left). Then, both fields cancel each other at some radius in the disc midplane, giving rise to time dependent ejection events above this reconnection zone. It has been proved that such a reconnection X-wind (hereafter ReX-wind) can efficiently brake down a contracting and accreting protostar on the correct time scales (Ferreira et al. 2000). The second way is to consider an anti-parallel magnetic configuration as proposed by Matt & Pudritz (2005a) (fig. 4, right). In this case, magnetic braking is due to a wide open stellar wind unconfined by the outer jet launched from the surrounding disc.

The magnetic star-disc interaction is far too complex to be tackled by analytical means and requires the extensive use of heavy numerical MHD experiments. Although there have

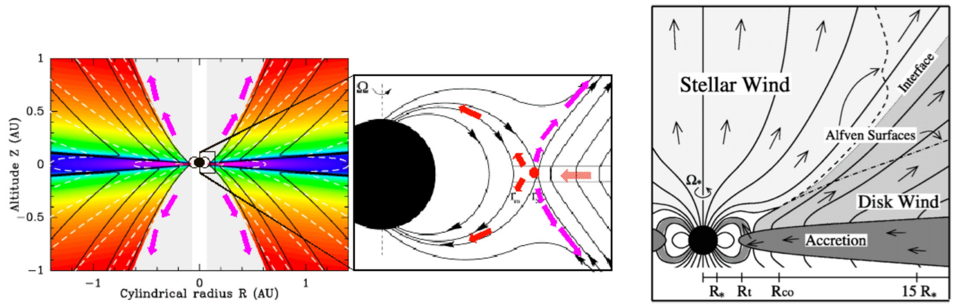

Fig. 4. Left: Star-disc interaction where the stellar magnetic moment is parallel to the disc magnetic field. There are three distinct types of ejection: a stellar wind on the axis, a disc wind (MAES shown in colors) and a sporadic reconnection X-wind at the interface, braking down the protostar (Ferreira et al. 2000). **Right:** Star-disc interaction in the anti-parallel case. Here, the stellar spin down is done by a wide open stellar wind, assuming no strong confinement by the outer disc wind (Matt & Pudritz 2005b).

been already several attempts on these lines none of the results shown can be reliable, either because of a crude description of the circumstellar accretion disc (eg. Romanova et al. 2002; Long et al. 2005), or a too small stellar magnetic field (Küker et al. 2003). The following questions are therefore still debated:

1. **Accretion columns formation:** What are the physical conditions required to allow disc material to flow along stellar field lines? What determines exactly the disc truncation radius? What happens if, as observations clearly suggest, the stellar dipole is inclined?
2. **Magnetic stellar spin down:** What is the dominant mechanism, ReX-wind, stellar wind, something else? Is disc locking discarded for good?
3. **Variability:** What is its origin? Is the star-disc interaction stable? Can accretion and ejection phases coexist? If a stellar dynamo is at work, can the two configurations shown in figure 4 occur in turn?

5. Concluding remarks

The theory of *steady* jet production from Keplerian accretion discs has been completed. The physical conditions required to thermo-magnetically drive jets are known and all the relevant physical processes have been included in the framework of mean field dynamics.

A picture is now gradually emerging, applying to accretion discs around both young stars and compact objects (Ferreira et al. 2006b). A large scale magnetic field is thought to be dragged in by the accretion flow and concentrated in the innermost disc regions. Such a field triggers the magneto-rotational instability (Balbus 2003) in the outer disc regions, producing thereby a standard accretion disc with no ejection (note that a thermally driven or photo-evaporated disc wind is of course clearly possible). When the field becomes at equipartition, the disc starts to drive self-confined jets. The physics of this inner disc is no longer governed by the radial transport of angular momentum, although turbulence is still required in order to provide an anomalous magnetic diffusivity. Finally, some magnetic interaction has to link both the disc material and magnetic field to the central object.

There are many unsolved questions. Can a sustained MHD turbulence provide this diffusivity? What is the origin of the ”coronal heating” required in order to explain the large mass fluxes observed in YSOs? What is the stability of jets and/or of their underlying accretion discs? Indeed, jets do show time dependent features and one must clearly go beyond steady state models. Last but not least, there is the stellar angular momentum removal issue.

It is now accepted that spinning down a star requires ejection from the star itself, so the question is how to create it. Researches are being undergone to probe if each envisioned configuration (parallel or anti-parallel, fig 4) can indeed be established and which is the most effective in spinning down the protostar. Of course, if an active dynamo is at work within those stars, one might also expect some polarity reversal as observed in the Sun. On the same line of thoughts, there is no compelling evidence for dipolar stellar fields. In fact, as Moira Jardine showed in her presentation (this volume), a realistic star-disc interaction is most probably involving a much more complex magnetic configuration.

Undertaking the study of how material flows from the disc onto the star via those complex field structures is for sure a very promising field of research.

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