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Towards a statistical theory of a magnetized accretion disk corona

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Abstract. We present a statistical description of a stochastic magnetic field in the force-free corona of a turbulent accretion disk. We represent the field by an ensemble of magnetic loops tied to the disk, which is described by the distribution function of loops over their sizes. Each loop evolves under several physical processes, e.g., Keplerian shearing, random walk of the footpoints due to disk turbulence, and reconnection with other loops. To represent these processes statistically, we construct a loop kinetic equation for the evolution of the distribution function, similar to Boltzmann's kinetic equation, with a binary collision integral representing reconnection between loops. We solve the equation numerically to obtain a statistical steady state. Once the loop distribution function is known, one can calculate important integral characteristics of the coronal magnetic field, such as the overall magnetic energy and the magnetic dissipation rate; their distribution with height above the disk; and the rate of angular momentum transfer by the coronal loops. We also access the efficiency of the reconnective inverse cascade in producing a population of very large loops.

1. Introduction

Many galactic black-holes and active galactic nuclei show high-energy power-law tails in their X-ray spectra, usually attributed to Comptonization of thermal soft X-ray or UV accretion disk photons by an overlying hot corona (Bisnovatyi-Kogan & Blinnikov 1976; Liang & Price 1977). Most of the papers devoted to accretion disk coronae (ADC) focus on modeling these observed spectra and hence on the properties of the coronal gas and radiation. Dynamically, however, the ADC is expected to be magnetically-dominated and thus it makes sense to first study the corona's magnetic structure (Tout & Pringle 1996).

The general physical picture of ADC formation is similar to that of the solar corona and can be summarized as follows (Galeev, Rosner, & Vaiana 1979; Miller & Stone 2000). The disk is turbulent, just as the solar convection zone. Magnetic fields are generated by the turbulent dynamo in the disk and then buoyantly escape into the overlying low-density regions. Thus, magnetic loops constantly emerge from the disk; some of the turbulent energy is then carried as Poynting flux up into the corona, where it is dissipated episodically, via discrete reconnection events. This magnetic dissipation leads to coronal heating and particle acceleration.

The magnetic field above a turbulent disk is probably highly complex, with constantly-evolving structures on various spatial scales. Therefore, we seek to describe a magnetized ADC statistically. Here are the fundamental specific questions we want to address: How

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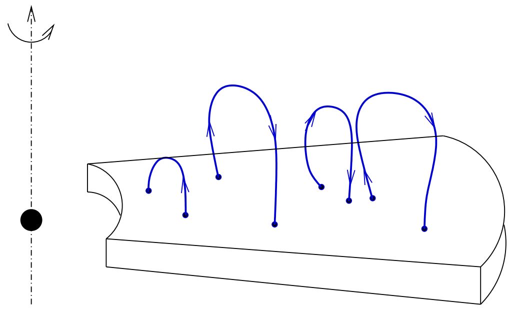

Fig. 1. Accretion-Disk Corona represented by an ensemble of magnetic loops.

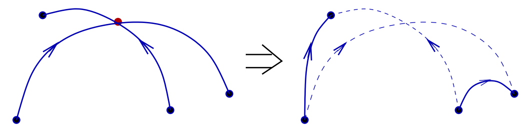

Fig. 2. Reconnection between two loops

much magnetic energy is stored in the corona and how is it distributed with height? What determines the magnetic scale-height? Or, if it's a power law, then what is the power-law exponent? Next, what is the distribution of magnetic dissipation with height? What is the distribution of magnetic loops in sizes, strengths, etc.? How non-potential is the magnetic field? What is fraction of the open flux? Is there a magnetic "inverse cascade", leading to a population of very large loops? How much angular momentum is transferred by the coronal magnetic field? How do all these things depend on the efficiency of reconnection?

2. Statistical Theory of ADC

To build this theory, we represent the corona by an ensemble of magnetic loops (see Fig. 1) carrying the same fixed amount magnetic flux, $\Delta\Psi$. Each loop in our model is characterized by only two parameters, the radial and azimuthal footpoint separations, Δr and Δy, or, equivalently, by the total footpoint separation L and the orientation angle θ. We then introduce the *distribution function* of loops, $F(L,\theta)$ and derive the *loop kinetic equation* (LKE) for this function, similar to the Boltzmann kinetic equation for a gas; we then solve the LKE to obtain a *statistical steady state*. When doing this, we are interested in spatial scales larger than the disk thickness H but smaller than its radius R and in temporal scales longer than the orbital period but shorter than the overall accretion time.

To derive the loop kinetic equation, we first analyze the main physical processes that govern the evolution of coronal loops, such as: **(1)** *emergence* of small loops into corona: modeled in the LKE by a source term $S(\mathcal{A})$ or via boundary conditions at small scales ($l \sim H$); **(2)** *random footpoint motions* due to the disk turbulence: modeled by an anisotropic diffusion operator, $D_r\partial^2_{\Delta r} + D_y\partial^2_{\Delta y}$; **(3)** *Keplerian shear*, stretching loops azimuthally and thereby also making them grow in height: modeled by an advection term, $1.5\Omega_K \Delta r\, \partial_{\Delta y} F(\mathcal{A})$; **(4)** *reconnection between individual loops*, analogous to binary collisions between atoms in a gas (see Fig. 2): modeled by a nonlinear integral operator, $\dot{F}_{\rm rec}(\mathcal{A})$, analogous to Boltzmann's collision integral.

Note that processes (1)–(3) on average pump energy from the disk into the corona, creating a stressed non-potential force-free field. In contrast, reconnection (4) relaxes the accumulated magnetic stresses and dissipates the free magnetic energy. Overall, a magnetically-active ADC can be described as a **Boiling Magnetic Foam**.

In our model, there are two types of interaction between loops. One is reconnection, representing episodic *binary interaction* between individual loops. The other is the lateral confinement of each loop by the *collective* magnetic pressure $\bar{B}^2(z)/8\pi$ of many nearby loops. We regard $\bar{B}(z)$ as a self-consistent *mean field*, related to F via $\bar{B}(z) = \Delta\Psi \int d\mathcal{A}\, F(\mathcal{A})\, H[Z(\mathcal{A})-z]$, where $Z(\mathcal{A})$ is the height of loops of type $\mathcal{A}$, and $H(Z-z)$ is the step-function. In turn, this mean field controls loop thickness, $d(z) \sim \bar{B}^{-1/2}(z)$, which affects the reconnection cross-section (see below).

We emphasize that reconnection is very important in our model. In particular, it controls the coronal magnetic scale-height H_B. If it is too efficient, the coronal field is nearly potential and $H_B \sim H$; then, the free magnetic energy stored in the corona is small, as is the magnetic dissipation rate. On the other hand, if reconnection is inhibited, magnetic loops grow in height until $H_B \sim R \gg H$. Then, it turns out, the power pumped into the corona goes down, even though the free magnetic energy is large. Similarly, the torque due to coronal loops also goes down. Strong magnetic dissipation and large torque thus require an intermediate reconnection efficiency. Another reason why reconnection is important is that it may lead to the formation of large loops — a coronal *inverse cascade* [e.g., Tout & Pringle (1996)], which may enhance the angular-momentum transfer via the coronal magnetic field.

As mentioned above, we treat reconnection between loops as a binary collision (c.f. Tout & Pringle 1996): two loops reconnect forming two new loops (see Fig. 2). This is described by two nonlinear integral terms in the LKE: a source and a sink. The general form of the sink term, for example, is $\dot{F}_{\mathrm{rec},-}(\mathcal{A}) = -\int d\mathcal{B}\, Q(\mathcal{A},\mathcal{B})F(\mathcal{A})F(\mathcal{B})$, where $Q(\mathcal{A},\mathcal{B}) = q\Omega\,\sigma_{AB}$ is the rate of such reconnection events. Here, the dimensionless q parametrizes the importance of reconnection relative to Keplerian shear, Ω is the disk rotation rate — the typical rate for rearranging the coronal field, and σ_{AB} is the *reconnection cross-section*; it accounts for a higher probability of larger loops to overlap in space and hence to interact. Thus, σ_{AB} is roughly proportional to the smaller-loop's length and to the sum of loop thicknesses at the interaction point.

In the actual numerical treatment of the reconnection term, we go through all pairs of loops and, for each pair, integrate over the impact parameter b of the particular reconnection event. Assuming for simplicity that all the loops are semicircular in shape, we formulate explicit *reconnection rules* that determine the two product loops in terms of the two incoming loops and b (similar to using conservation laws to calculate the new velocities of two colliding particles). Also, we compute the height of the reconnection site and thus determine the corresponding loop thickness, $d(z) \sim \bar{B}^{-1/2}(z)$, which enters the reconnection cross-section.

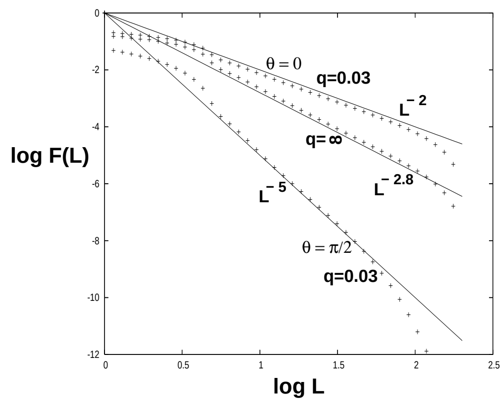

Fig. 3. Steady-state loop-size distributions $F(L)$ for purely azimuthal ($\theta = 0$) and purely radial ($\theta = \pi/2$) loops for case without shear ($q = \infty$) and for a relatively strong shear ($q = 0.03$), for $d(z) = \text{const}$.

3. Results and Prospects for Future Work

We integrated the LKE numerically and obtained steady-state solutions for the cases with and without the height-dependence of the loop thickness and with various relative levels of shear, described by q. We always get orientation-dependent power laws: $F(L,\theta) \sim L^{-\alpha(\theta)}$. Fig. 3 shows the case in which the variation of $d(z)$ with height z is ignored. When shear is absent ($q = \infty$), $F(L,\theta)$ is isotropic, $\alpha(\theta) = \text{const}$. Increasing shear (decreasing q) flattens the distribution of azimuthal loops and steepens that of radial loops. Also, taking into account the $d(z)$-dependence steepens the power laws.

This work is still in progress. We are now assessing the energy-distribution of flares as well as the overall magnetic dissipation and coronal angular momentum transfer. We also plan to incorporate open field lines and to apply our model to the solar corona.

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References

Bisnovatyi-Kogan, G.S. & Blinnikov, S. I. 1976, Sov. Astr. Lett., 2, 191

Galeev, A.A., Rosner, R., & Vaiana, G.S. 1979, ApJ, 229, 318
Liang, E.P.T. & Price, R.H. 1977, ApJ, 218, 247
Miller, K.A. & Stone, J.M. 2000, ApJ, 534, 398
Tout, C.A. & Pringle, J.E. 1996, MNRAS, 281, 219