Mem. S.A.It. Vol. 78, 120

Memorie della

Inversion of coronal Zeeman and Hanle observations to reconstruct the coronal magnetic field

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Abstract. Hanle-effect observations of forbidden coronal line transitions and recently also longitudinal Zeeman-effect measurements of coronal lines show quantitative signatures of the weak coronal magnetic field. The interpretation of these observations is, however, complicated by the fact that they are the result of line-of-sight integrations through the optically thin corona. We study by means of simulated observations the possibility of applying tomographic techniques in order to reconstruct the 3D magnetic field configuration in the solar corona from these observations. The reconstruction problem relates to a family of similar problems termed vector tomography. It is shown that Zeeman data and Hanle data alone obtained from vantage points in the ecliptic plane alone are sensitive only to certain magnetic field structures. For a full reconstruction it is necessary to combine the longitudinal Zeeman and Hanle effect data.

Key words. Sun: Coronal Magnetic Field – Sun: Spectropolarimetry – Sun: Tomography Inversion

1. Introduction

The coronal magnetic field is the main driving force for most plasma processes in the inner corona. To understand the physics of the corona a detailed knowledge of the state of the coronal field is therefore essential. Unlike the photosphere, however, the low density and high temperature of the coronal plasma make direct field measurements in the corona to be rather difficult. Conventionally, the magnetic field of the corona is therefore estimated by means of extrapolations from its photospheric boundary values. Until now, potential field approximations as an extrapolation model were quite popular. The omission of all electrical currents in the corona, however, misses an important part of coronal magnetic field driven physics: as the potential field is the lowest energy state of the field with respect to normal boundary conditions (e.g. Sakurai 1989) it cannot account for the energy stored in the field which is released in dynamical processes such as flares and CMEs.

With the advent of vector magnetograms from the solar surface it became possible to employ more a sophisticated and more realistic model assumption, the force-free condition, for the field extrapolation. There is currently

Send offprint requests to: B. Inhester a considerable interest in algorithms which solve this non-linear and ill-posed extrapolation problem (see, e.g., the contribution by S. Régnier in this issue). It is obvious, however, that extrapolations can only reproduce those structures of the magnetic field which have a measure-able imprint on the surface field. The results are deemed to become less reliable the larger the distance from the surface.

In this paper we investigate therefore whether spectropolarimetric coronal measurements of the magnetic field can be used for its reconstruction. These measurements have a long tradition. Charvin (1965) suggested more than 40 years ago to measure the Hanle effect in the corona. One of the first to perform according observations of the coronal green line at 530.3 nm was Arnaud (1982). But full spectropolarimetric measurements of coronal lines are still today a challenge. For a long time, the observation of the longitudinal Zeeman effect was out of reach. Only recently Lin et al. (2000, 2004) could demonstrate that full spectropolarmetric observations in the corona can be achieved. This success is partly due to improved instrumentation but also profits largely from a change to infrared lines where the Zeeman split is increased relative to the thermal line broadening. The Zeeman observations of the Fe XIII 1075 nm line, however, still required almost an hour of integration time.

For forbidden coronal lines with a life time much larger than the Larmor period of the excited atom a first order interpretation of the observed spectropolarimetric signals can be given as follows. The Stokes V signal is proportional to the line-of-sight magnetic field component $B_{\|}$ at the location where the line is emitted, the Stokes Q and U polarisation signals indicate the orientation $\mathbf{B}_{\perp}/B_{\perp}$ normal to the line of sight.

This picture of the line formation, however, is a strong simplification. The physics of the emission of these forbidden lines has been studied in great detail in recent years, e.g., by House (1977), Sahal-Brechot (1977) and Querfeld (1982) and our understanding has been largely refined with respect to the above crude interpretation.

The above crude picture, however, also a neglects the fact that the coronal observations are line-of-sight integrals through an optically thin medium and do not represent directly its local properties. The goal of our study is to obtain a more sophisticated interpretation of the observations also in this respect. In particular, we want to investigate the whether these observations suffice to determine a global model of the coronal magnetic field. Due to a lack of space we can here only briefly discuss the approach we have adopted (chapter 2) and present initial results of some of our test calculations (chapter 3). Necessary future work work is outlined in the final section (chapter 4). More details can be found in Kramar (2005) and Kramar et al. (2006).

2. The inversion problem

The inversion of line-of-sight observations is commonly referred to as tomography. In many fields this is a well established technique which solves for the distribution of an isotropic scalar quantity in a limited region of space from line-of-sight integrals of the scalar along a discrete number of directions. The inversion is ill-posed but has no null space which means that all structures of the density distribution can in principle retrieved, the quality of the reconstruction depends on the resolution and signal to noise ratio of the individual images taken and on the angular sampling of the image view directions.

Tomography has also been applied to solar physics. Davila (1994) proposed to use coronagraph observations to construct the coronal plasma density. According calculations have been performed by Zidowitz (1999) and Frazin & Janzen (2002). For the application of tomography to space observations some additional difficulties have to be coped with which are absent in laboratory applications. The observations are usually made with a coronagraph with the Sun being occulted. This causes a data loss at the center of each image which increases the condition of the inversion problem. Observations are traditionally made from ground-based telescopes or from space craft close to the ecliptic more or less continuously

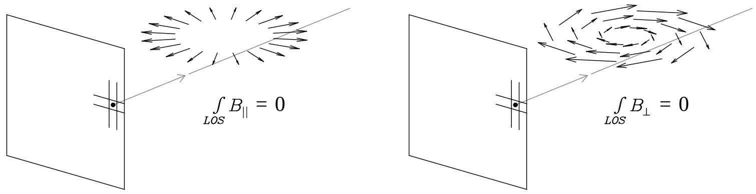

Fig. 1. Illustration of the inability to see div**B** and $\mathbf{e}_{LOS}\times$curl**B** from certain line of sight integrals

over half a rotation period of the Sun. The tilt of the Sun's rotation axis with respect to the ecliptic and the instationarity of coronal structures during the time of the observation may seriously affect the quality of the reconstruction.

The inversion problem we are faced with bears a further very fundamental complication. We want to reconstruct a field rather than a scalar quantity and as observations we have line of sight integrals of the field components (or functions thereof) along and perpendicular to the view direction. This extension to scalar tomography has been investigated in recent years (e.g. Howard 1996) and has been termed vector tomography. If we adopt for a moment the very simplified interpretation of the spectropolarimetric observations mentioned in the previous chapter, for the Hanle effect we for the moment even assume that we can measure the strength of the field in the plane of the sky, we would observe quantities like

$$\begin{pmatrix} D_V \\ D_{(Q,U)} \end{pmatrix} = \int_{\mathrm{LOS}} \begin{pmatrix} \epsilon_V \\ \epsilon_{(Q,U)} \end{pmatrix} d\ell \sim \int_{\mathrm{LOS}} \begin{pmatrix} B_{\|} \\ \mathbf{B}_{\perp} \end{pmatrix} d\ell \qquad (1)$$

It is a well known result from vector tomography, that the top integral is completely insensitive to the any divergent part of the field B and the bottom integral does not respond to the components of the curl of **B** normal to the line of sight (see Fig. 1). Hence we have to deal with an extended null spaces in the field we want to reconstruct from the above measurements.

How far the problem of a finite null space also applies to real Zeeman and Hanle effect observations is not obvious because their emissivities are more complicated than was assumed in (1). We therefore have developed an inversion procedure to numerically test the condition of the field inversion. In our test computations we use the emissivity expressions (House 1977; Sahal-Brechot 1977; Querfeld 1982)

$$\begin{pmatrix} \epsilon_I \\ \epsilon_V \\ \epsilon_Q \\ \epsilon_U \end{pmatrix} \propto \begin{pmatrix} 2\Sigma + \Delta(3\cos^2\theta - 1) \\ 2\Sigma\bar{g}\omega_L\cos(\theta) \\ 3\Delta\sin^2(\theta)\cos(2\alpha) \\ 3\Delta\sin^2(\theta)\sin(2\alpha) \end{pmatrix} \qquad (2)$$

Here, Σ and Δ are proportional to the ion density and to the sum and difference, respectively, of the upper sublevel population of the emitting electron transition, $\bar{g}$ is the effective Landé factor of the transition. The wavelength dependence of the different Stokes lines is neglected here assuming that representative moments of the line signal were taken. The magnetic field enters into (2) through the field intensity in the Larmor period ω_L and the field angles θ with respect to the line of sight and α, its orientation normal to the line of sight.

Our inversion procedure is based on a least square iteration between the polarimetric observations and a forward modelling of the data calculated from the respective line of sight integrals of (2) assuming a model field **B**. The field is then successively improved until the difference reaches the instrument's noise level. According to the above arguments we have to expect that the forward modelling may have null spaces and our minimisation may not yield unambiguous results. To stabilise our inversion, we therefore add a second term to our expression $L(\mathbf{B})$ to be minimised

$$L(\mathbf{B}) = \sum_{\mathrm{pixels}} | \begin{pmatrix} D_V \\ D_Q \\ D_U \end{pmatrix}^{\mathrm{obs}} - \begin{pmatrix} D_V \\ D_Q \\ D_U \end{pmatrix}^{\mathrm{sim}} (\mathbf{B}) |^2$$

$$+ \int_{\text{corona}} |\text{div}\mathbf{B}|^2 \, dv \qquad (3)$$

This second term is an obvious constraint to the magnetic field and it has far reaching consequences. The calculation of the divergence at the inner coronal boundary requires the normal component of **B** on the Sun's surface as boundary condition. Hence our data set has to be extended to include not only the coronal observations but also the photospheric surface magnetograms. We consider this extension an advantage because this additional information is merged in a natural way with our inversion procedure. If, e.g., we had no coronal observations at all, the procedure would only minimise the divergence term in (3). If in addition we could insure that the total field energy $\int |\mathbf{B}|^2 dv$ is also minimised, the solution will be the potential field which complies with the measured surface magnetogram. The energy minimisation, however, is implicitly taken care of if a proper minimisation algorithm (e.g., conjugent gradients) is chosen along with a minimum energy initial field for the iteration. Hence we obtain the potential field approximation of the coronal magnetic field for free and every individual coronal observation included in (3) drives our solution towards a more realistic, current carrying corona.

The problem we have to investigate can be restated much more precisely now: is the first term in (3), i.e. the coronal spectropolarimetric data, sufficient to resolve the magnetic field perturbations of all current systems to be expected in the corona ? Reversely, are there current systems which the coronal observations do not respond to and which will consequently not appear in our solution. We note in passing that we could easily constrain the magnetic field even further by adding a third term in (3) proportional to the integral of the squared $\mathbf{j} \times \mathbf{B}$ force in the corona. This would additionally stabilise our inversion and it is in fact exactly the approach some popular surface field extrapolation schemes make use of (see S. Régnier's contribution in this issue), except that there the term of the coronal observations in (3) is not accounted for.

3. Test of the inversion approach

In this section we report on first initial test calculations using the approach described above. The test consisted of the retrieval of given coronal magnetic field configurations from simulated observations. The coronal density was assumed to fall off radially according to a classical power law (Newkirk 1970) and assumed was known for the field inversion because it can, at least approximately, be obtained from a scalar tomography inversion of the line intensity. In Fig. 2 we show the first model we tested. It consists of a dipole field with an isolated current loop across the equator confined on a meridional plane. The simulated data set was generated form this model field as if observed by a space craft on an ecliptic plane tilted by 10 degrees with respect to the z axis. The data set thus comprised images of the Stokes signals taken from 36 equidistantly distributed viewing directions with 5% noise added to the calculated signals intensities.

The inversion cannot avoid to reconstruct the field in the whole domain of the corona. This should not be a problem where the field is a potential field and we therefore emphasise the perturbed field region when presenting the inversion results. Fig. 3 shows these field perturbations in the equatorial plane in the immediate neighbourhood of the point where the current loop intersects the plane. Note that the background dipole field is normal to this plane so that it does not show in Fig. 3. For test purposes we have here only made use of either the Zeeman (centre panel in Fig. 3) or the Hanle (right panel) observations for the reconstruction. For comparison, the left panel displays the perturbation of the original field. Obviously, the Zeeman observations yield a much better reconstruction than the Hanle data. The reason seems to be that for this orientation of the current, the Hanle data responds very little while the Zeeman effect alone provides sufficient information to resolve this kind of field perturbation.

As an alternative model, we investigated a dipole field with a current loop in the equatorial plane. Again in Fig. 5 we show the original and the reconstructed magnetic field in the

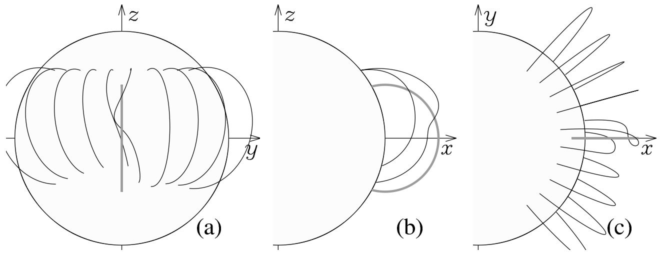

Fig. 2. Coronal magnetic field model 1 with a current loop in the meridional x, z plane. Along with the current loop we also show the distorted field lines.

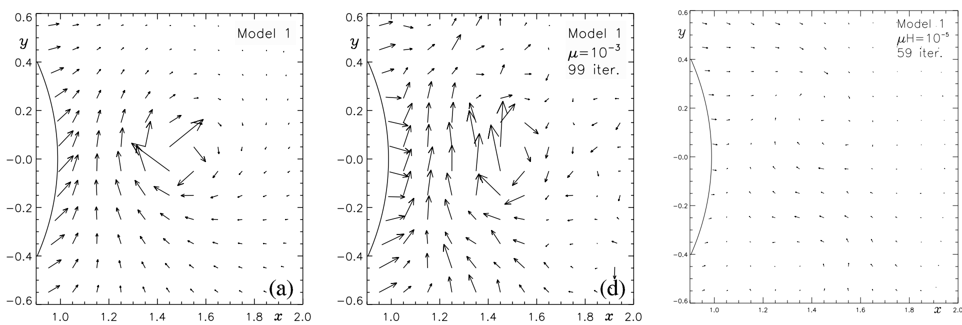

Fig. 3. Field perturbations in the equatorial plane close to where the current loop of model in Fig. 2 intersects the plane. From left to right we show the original field, the reconstruction taking only account of the Zeeman observations and of only the Hanle observations in (3).

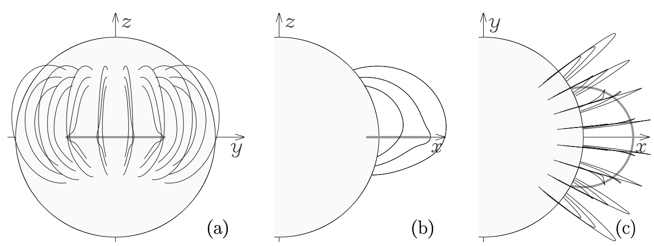

Fig. 4. Coronal magnetic field model 2 with a current loop in the equatorial x, y plane. We also show the distorted field lines.

x, y-plane normal to the current loop and concentrated to the neighbourhood of the current intersection. Note that now the background dipole field is superposed and the effect of the current cannot so clearly be discerned. It can be concluded though that now the Zeeman observations miss the field perturbation entirely and only allow to reconstruct the background dipole field. The Hanle observations on the other hand give a decent response and its information included in the inversion reproduces to some extent the effect of the current.

4. Summary and outlook

We have presented first results which aim to use coronal spectropolarimetric observation to reconstruct the magnetic field of the corona. From our findings we can state that Hanle or

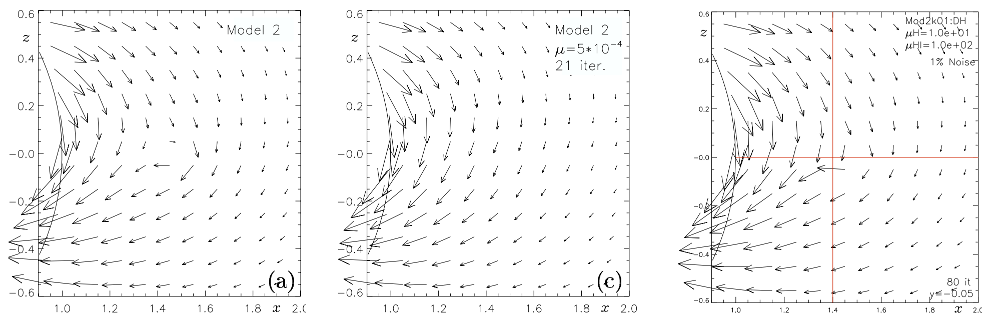

Fig. 5. Field perturbations in the meridional plane centred between the loop's foot points and close to where the current loop of model in Fig. 2 intersects the plane. From left to right we show as in Fig. 3 the original field, the reconstruction taking only account of the Zeeman observations and of only the Hanle observations in (3).

Zeeman observation alone are not sufficient for the reconstruction. Instead the Stokes V, Q and U components and in addition surface magnetograms are necessarily required to be sensitive to the most obvious coronal current systems. We are confident that this data set is also sufficient to yield a realistic coronal magnetic field model. This, however, has to be verified in future experiments. If it turns out that the inversion is too unstable to yield satisfactory field models, we still have the option, as mentioned above, to include a force-free constraint in our minimising function (3).

A future critical test is the dependency of the inversion output on the noise level of the observations. This test besides its insight into the condition of the inversion problem can also provide us with an estimate of the maximum tolerable signal to noise ratio. This way we may also optimise the tradeoff between image noise and angular resolution since the Zeeman observations require long integration times. We are confident that this information could be quite helpful to improve real observations. We are just at the beginning with our investigations of this new method and there is quite some work ahead. In future we intent to also include the Stokes I component in our data set and treat the particle density as additional variable so that an a-priori density model is no more required. In the more distant future we may also consider to take account of the full spectral profile of the Stokes measurements and in return relax a-priori assumptions about the coronal temperature distrubution.

Acknowledgements. The work by M.K. was supported by the International Max Planck Research School on Physical Processes in the Solar System and Beyond. This research school is located at the Max-Planck Institute for Solar System Research and at the Universities Göttingen and Braunschweig.

References

Arnaud, J. 1982 , Astron. Astrophys, 112, 350
Charvin, P. 1965, Ann. d'Astrophys., 28, 877
Davila, J. M. 1994, Astrophys. J., 423, 871
Frazin, R. A., Janzen, P. 2002, Astrophys. J., 314, 782
Howard, J. 1996, Plasma Phys. Control. Fusion, 38, 489
Kramar, M. 2005, Thesis, University of Göttingen
Kramar, M., Inhester, B., Solanki, K. S. 2006, Astron. Astrophys., 456, 665
Querfeld, C. W. 1982, Astrophys. J., 255, 764
House, L. L. 1977, Astrophys. J., 214, 632
Lin, H., Penn, M. J., Tomczyk, S. 2004, Astrophys. J., 541, 83
Lin, H., Kuhn, J. R. Coulter, R. 2004, Astrophys. J., 613, 177
Newkirk, L. L. 1970, Solar Phys., 15, 15
Sahal-Brechot, S. 1977, Astrophys. J., 213, 887
Sakurai, T. 1998, Space Sci. Rev., 51, 11
Zidowitz, S. 1999, J. Geophys. Res., 104, 9727