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Calculating Stellar Models in 3D: Djehuty and the Core Helium Flash

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Abstract. We present some of the first calculations obtained with the *Djehuty* 3D stellar structure code. This is a fully explicit hydrodynamics code, and hence not suited to evolutionary calculations except on the shortest timescales. With this in mind, we applied the code to the peak of the core Helium flash to see how the model differs form the standard results obtained in 1D. We determine that, to the time when calculations were stopped, the 1D hydrostatic and 3D hydrodynamic codes agree remarkably well. Perhaps more importantly, we show that such calculations are possible and that the *Djehuty* code can be applied to problems of astrophysical interest.

Key words. Stars: evolution – Stars: core flash – Stars: hydrodynamics

1. Introduction

The core Helium flash is an exciting event in the life of a star, marking the beginning of the end, in many ways. Gone are the days of lazy H burning, even the relatively fast pace of shell H burning on the giant branch will soon seem sedate in comparison to later phases. The core flash marks the ignition of helium under degenerate conditions, and hence it occurs for stars in the range of about $1M_{\odot}$ to $2.5M_{\odot}$. The ignition causes a thermal runaway in highly degenerate matter where there is a dramatically reduced feedback between the temperature and the density (expansion) through the equation of state. Stars on the horizontal branch are identified as post-flash objects so we know empirically that stars survive this potentially destructive event. But what happens deep in the interior? Does it have observable consequences? Could it affect the later evolution?

Most calculations to date have been made using standard stellar evolution codes, and hence they have made standard stellar structure assumptions: spherically symmetric, hydrostatic, and usually instantaneous mixing in convective regions, which are determined using the Schwarzschild (or perhaps Ledoux) criterion. How relevant are these to a structure that is evolving on a timescale measured in perhaps minutes or at most hours?

The pioneering work of Dupree and coworkers was the first to remove most of these assumptions. This work, summarized in Dupree (1996), consisted of both 2D axially symmetric as well as 3D calculations (of necessarily low resolution, at the time). One of his results, that mixing caused heating to rapidly extend into the very centre of the star, is found in our 3D results as well. He warned that the details depended critically on assumptions for eddy viscosity, which we believe are not a concern for the 3D results we present in this paper, which is a summary of Dearborn et al. (2006).

2. The "Djehuty Project"

The *Djehuty* Project at LLNL is an effort to model stars in 3D with as much physics as is computationally possible. All of the physics from standard 1D codes is included, such as opacities, nuclear reaction rates, equation of state etc, but to this is added an explicit hydrodynamical formulation that makes no assumptions about mass motions and hence is ideally suited to investigate convective instabilities in stars and consequent mixing. It is hoped to soon model binary stars, and tackle such problems as common envelope evolution, realistically, for the first time. These advances are only possible with the advent of massively parallel computing technologies.

2.1. The mesh

The *Djehuty* code is comprised of seven meshes as show in Fig 1. The central mesh is an $N \times N \times N$ cube and each face is connected to an $N \times N \times L$ cuboid. The cuboid gradually distorts the mesh in the radial (L) direction from cubic at the bottom to become the face of a sphere at the edge. This structure allows good resolution in the centre and forces no symmetry on the material under consideration. The mesh is adjusted according to the ALE (Arbitrary Lagrangian Eulerian) method as discussed by Anderson et al. (2004). Scalar quantities are defined in cell centres and vector quantities at the nodes.

2.2. Input Physics and Starting Model

The *Djehuty* code uses a suite of 21 nuclear species, enough to cover the major H, He and C burning phases. It also includes a small nuclear statistical equilibrium approximation, which is not relevant to our study. The equation of state is that due to Eggleton et al. (1973) with updates from Pols et al. (1995). It reproduces the OPAL equation of state (Rogers & Iglesias 1992) to better than 1%. OPAL opacities are used for high temperatures and Alexander & Ferguson (1994) for low temperatures. Conductive opacities are from Hubbard & Lampe (1969) and Itoh et al. (1993). Radiation and matter are treated separately in the code, each with its own temperature. The radiation is treated with a flux-limited diffusion equation.

Our initial model is taken from a one dimensional calculation performed with a standard hydrostatic code using essentially the same input physics.

3. The Core Flash in 1D

The core flash is a relatively rapid event, and hence suited to an explicit hydro code. Further, we know that the stars survive, so in a crude sense we know the final result. There are still many open questions, mostly concerned with the development of the convective shell that forms once the helium reactions ignite. Does this reach the convective envelope? One dimensional models would indicate that it does not, and this seems to fit the observed abundances of horizontal branch stars. Nevertheless there are some exceptions, and it is to these cases that we hope to later return. Our aim here is to see how close does the convection predicted by the 1D code, using Schwarzschild criterion and mixing-length theory, match the results from a 3D code where matter moves according to the local forces.

The sequence of events, as calculated in 1D, is well known. Increasing density causes increased neutrino energy losses from the core, resulting in a relative cooling of the very centre. The position of the temperature maximum moves off-centre. It is at this maximum that helium eventually begins to burn. The energy deposited by the triple-alpha reactions is unable to drive sufficient expansion in the degenerate matter, so the burning rate increases even further, leading to a runaway which is eventually quenched through the small (but non-zero) expansion despite the highly (but not infinitely) degenerate equation of state. The convection driven by the flash extends from the temperature peak almost to the hydrogen shell, just below the convective envelope. The luminos-

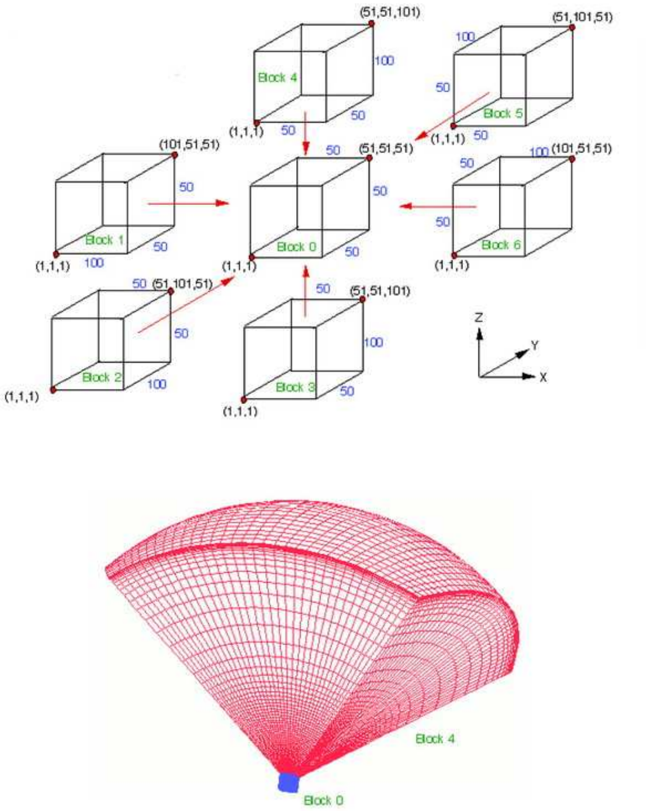

Fig. 1. Logical structure of a *Djehuty* mesh. Here $N = 35$ and $L = 70$.

ity may exceed $10^9 L_\odot$ before dropping. This main pulse is followed by 3 or 4 smaller pulses, after which the star settles onto the ZAHB with a convective helium-burning core (and a hydrogen-burning shell). Models show that about 3% of the He in the core is burned to C during the core flash.

4. The Core Flash in 3D

Our hydrodynamic code has its time-steps limited by the Courant condition, and the very high sound speed in the degenerate matter hurts us quite a lot. Hence we started with 1D models very close to the peak of the flash. These models were mapped onto the 3D mesh of approximately 1 million zones.

We must ensure adequate resolution of the regions of interest. The inner 50% of the mass is contained in about 10^{-12} of the volume of the star! Yet most of the volume is unaffected by the events deep in the interior. Hence we will place our outer boundary in a region far from the centre, so that it does not affect the calculations there, but also well in from the surface of the star, because nothing of interest will happen there. We place our outer radius in the radiative zone below the convective envelope and outside the hydrogen shell. Our calculation encloses a volume of about 30 times the volume of the helium core, and we include a range of over 5 orders of magnitude in density. The mesh and stellar structure in a $1M_\odot$ model with $\log(L_{He}) = 4$ is shown in Fig 2.

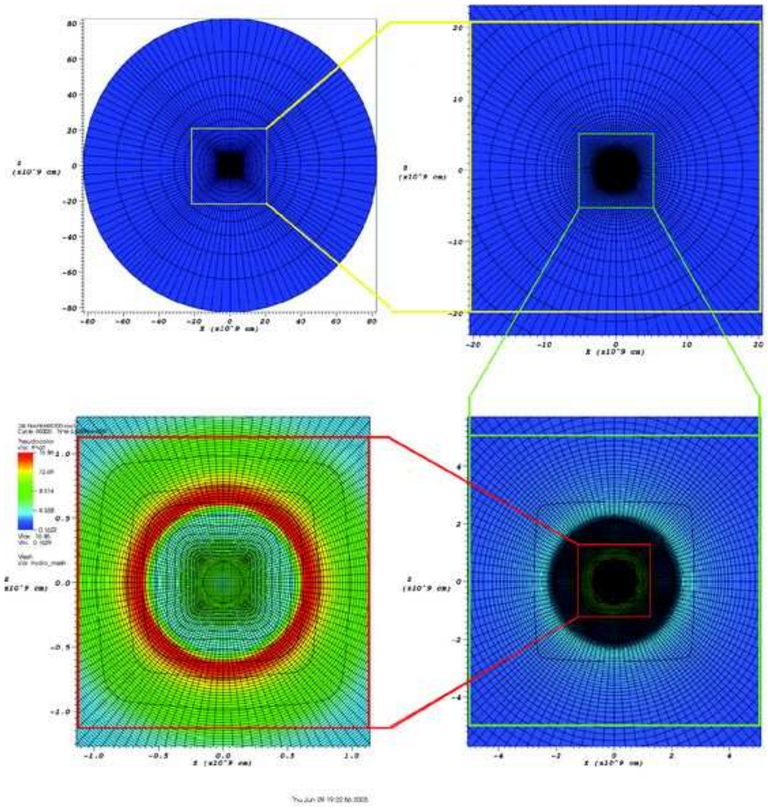

Fig. 2. A slice through the centre of the model. The background colours show the temperature (blue is colder than red) and hence the H shell is very clearly seen in the bottom left frame. Small arrows show the velocity field. Some squarish contours are artifacts of the plotting routines.

Mapping the star to 3D introduces some noise and deviations from spherical symmetry. The velocity field, in particular, is of concern. Although we may indeed have an estimate of the magnitude of the velocity from the mixing-length theory, we do not have a vector field to use. Instead, we start with zero velocities throughout. This will cause large accelerations initially, as material starts to move under its buoyancy. Until it is moving, it will feel a prolonged exposure to the high temperatures and we also expect a temporary blip in the luminosity. Further, as helio-seismology has shown us, stars are extremely good oscillators and waves can travel for long periods within a star. We attempt to damp these motions, as well as the initial excursions, by using the following equation of motion:

$$\frac{D\boldsymbol{v}}{Dt} = \boldsymbol{a} \text{ if } |\boldsymbol{v}| \leq \mathrm{V_t}$$

but

$$\frac{D\boldsymbol{v}}{Dt} = \boldsymbol{a} - \boldsymbol{v}\left(\frac{|\boldsymbol{v}| - V_t}{\delta t|\boldsymbol{v}|}\right) \text{ if } |\boldsymbol{v}| \geq \mathrm{V_t}$$

where δt is the time-step and V_t is a critical "treacle" speed. When the models settled down they tended to have a slightly lower luminosity than in the 1D case, by up to 50%. Part of this may be caused by the expansion associated with the initial luminosity peak, which is again due to a zero velocity field being imposed initially. Overall, we consider that the agreement between the equilibrium models in 1D and 3D is very good.

We next investigated the development of convection in a model with a helium luminosity of $3\times10^9 L_\odot$. The 1D model reaches its peak helium burning rate some 31 hours later. In the preceding few days of star time, the 1D model has shown the hydrogen-exhausted core growing at 2.5 ms^{-1}, while the helium convective shell encompasses $0.43 M_\odot$ from $0.0085 R_\odot$ to $0.0195 R_\odot$. The 3D model, with no mass motions initially, reaches a peak of $5\times10^9 L_\odot$ briefly, before it quickly drops to an equilibrium value of about $2\times10^9 L_\odot$. Our 3D simulation covers only about an hour of star time, but during this period we can observe the development of convection in the helium burning zone. This can be seen in Fig. 3. Here we show contours of the ^{18}O isotope, which is produced by α-capture on ^{14}N. The ^{18}O isotope is not included explicitly in the 1D code so its distribution is an excellent marker of burning and mixing in the 3D code. The figure shows velocity vectors on a slice through the centre of the star. The black circle shows the limit of convection in the 1D model. In the outer regions we see a persistent azimuthal ringing in the velocity field, but this produces no mixing. In the neighbourhood of the ^{18}O contour the velocity field is dramatically different. Here we see the development of plumes which rise and fall, transporting energy from the burning region.

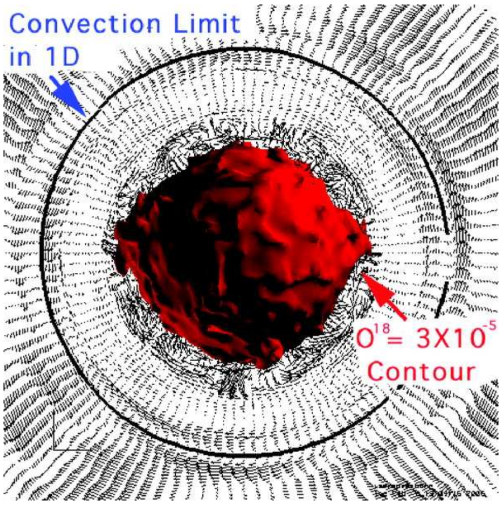

Fig. 3. Velocity vectors in a slice through the centre. The extent of convection in the 1D code is shown as is an outer contour of ^{18}O, essentially the outer convective region, from the 3D code.

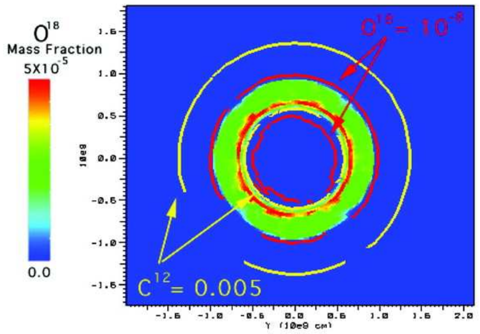

Fig. 4. Contours of ^{18}O and ^{12}C on a slice through the centre of the star. Mixing inward in the 3D model is pushing ^{18}O toward the core, and inside the inner edge of convection as found in the 1D case. The convection in the 3D model has not yet extended all the way to the border determined by the 1D code.

An alternative view of the development of the convective region is shown in Fig. 4. Here the yellow lines mark the contours where the abundance of ^{12}C is 0.005, thus marking the inner and outer regions of the helium burning region in the 1D code. The convection is again indicated by the positions of the ^{18}O contours, which clearly show the extension of the convection *inward* as found earlier by Dupree and co-workers. In the hour of star time calculated in 3D, the convection has developed over about half of the region predicted by the 1D models. It is computationally very expensive to continue the 3D calculation to see the maximum extent of the convective region, and compare with the 1D calculation. We intend to do this but have not done so yet. Nevertheless, we can estimate the speed with which the convective region is growing. We find this zone to be increasing as $r \propto \sqrt{t}$ and we predict that the 3D model will reach the 1D convective extent after 5 hours of star time. It is also worth noting that the 3D model shows the helium core growing at a rate of about 18 ms^{-1} which is in good agreement with the 1D estimate of 13 ms^{-1}. Because these speeds are well below the sound speed we expect that hydrostatic modelling should capture most of the features of the core flash.

5. The Core Flash in a Rotating Star

We performed more tests of our 3D core flash models, as discussed in Dearborn et al. (2006), but here we present a brief summary of some new calculations including rotation.

Beginning with a mature non-rotating model near the peak luminosity, we created new models by imposing solid body rotation (through the core region) with periods of 4.8, 1.6, and 0.87 hours. Outside of the core region, it was necessary to cap the rotation velocity to avoid centrifugal accelerations in excess of gravity. While these rotation periods cover much of the range observed in white dwarfs, the rotation speed imposed in the helium burning shell is well below the convective plume speeds in all cases. Further, the ratio of gravitational to centrifugal acceleration never exceeds 10^{-4} in the Helium convective region.

Given the modest size of the perturbation imposed by these rotation speeds, it is not surprising that the differences between the rotating and non-rotating models are small. Over comparison periods of 1000 to 2000 seconds, the energy production rate in the rotating models appears to increase by 1/4 to 1/2 percent. This increase is difficult to measure, as the RMS variation in the energy production rate is about 4%, but appears to be significant in 2 of the 3 rotation periods tested.

We also examined the radius of the convection region in the slowest rotating and non-rotating models. After 2000 seconds, they had both increased by about 450 Km, and were effectively indistinguishable. We see what appears to be redistribution of angular momentum in the convective region, but the high convective velocities and significant acoustic wave energy (even in the stable regions) requires a statistical approach. A preliminary assessment shows significant deviations from the imposed rotation profile in regions where there is vertical displacement of material. This was expected in the convective helium burning region, but was also seen outside of the hydrogen burning shell. The cause for that motion will be discussed in a subsequent publication on a new mechanism for giant branch mixing.

6. Conclusions

Simply put, our calculations indicate that the 1D hydrostatic models of the core helium flash are surprisingly close to the results of our first explorations in 3D. Further calculations and comparisons will be the subject of future papers, including one on the rotating models discussed very briefly here.

Perhaps the most important conclusion, however, is that the study of 3D hydrodynamic stellar models is now possible, with *Djehuty*.

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References

Anderson, R. W., Elliott, N. S., and Pember, R. B., 2004, J. Comput. Phys., 199, 598.

Alexander, D. R. and Ferguson, J. W., 1994, ApJ, 437, 879

Dearborn, D. S., Lattanzio, J. C., and Eggleton, P. P., 2006, ApJ, 639, 405

Dupree, R. G. 1996, ApJ, 471, 377

Eggleton, P. P., Faulkner, J. and Flannery, B. P., 1973, A&A, 23, 325

Hubbard, W. B., and Lampe, M., 1969, ApJS, 18, 297

Itoh, N., Mitake, S., Iyetomi, H. and Ichimura, S., 1983, ApJ, 273, 774

Pols, O. R., Tout, C. A., Eggleton, P. P., and Han, Z., 1995, MNRAS, 274, 964

Rogers, F. J., and Iglesias, C. A., 1992, ApJ, 401, 361