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The effect of rotation on Petersen Diagrams

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Abstract. The well-known Petersen diagrams are a useful technique to constrain the mass and metallicity of models for double-mode radial pulsators. However, when moderately rotating stellar models are considered this method may fails. A preliminary study of the effect of rotation on the first overtone to fundamental period ratios is discussed for slow to moderate rotational velocities. The impact on the mass and metallicity determination is examined.

Key words. Stars: variables: Sct – Stars: rotation – Stars: variables: RR Lyr – Stars: oscillations – Stars: fundamental parameters – Stars: variables: Cepheids

1. Introduction

Stars studied by means of Petersen Diagrams, are double-mode radial pulsators like double-mode Cepheids, RR Lyrae or high-amplitude δ Scuti stars (HADS). For these objects, it is particularly useful to analyse the behaviour of the ratio between the fundamental radial mode and the first harmonic. Such ratios, firstly studied by Petersen (1973, 1978), are generally used as a mass and metallicity indicators, to test mass-luminosity and/or radius-luminosity relations, and even to dertermine the distance modulus to the SMC (Kovács 2000). The recent analysis of data from large-scale projects like OGLE (Optical Gravitational Lensing Experiment Szymanski 2005; Udalski et al. 1997), NSVS (Northern Sky Variability Survey Woźniak et al. 2004), ASAS (All Sky Automated Survey Pojmanski 2002, 2003) or MACHO (Alcock et al. 2000) has permitted to better study the properties of such double-mode pulsators.

Only a few theoretical works considering rotation are found in the literature. Such works are rather incomplete and mainly focused on very rapid rotators (Perez Hernandez et al. 1995; Pamyatnykh 2003; Suárez et al. 2005a). In the present work, the consequences of neglecting the effect of rotation on radial period ratios is examined. To do so, up-to-date techniques taking properly into account the rotation in the modelling (in equilibrium models and in oscillation frequencies) are used. Similarly as done in Pamyatnykh (2003) and Suárez et al. (2005a) near degeneracy is also considered.

2. The $\Pi_{1/0}\,(\Omega)$ ratios

The well-known dependence on the metallicity of $\Pi_{1/0}$ is depicted in a classic PD for

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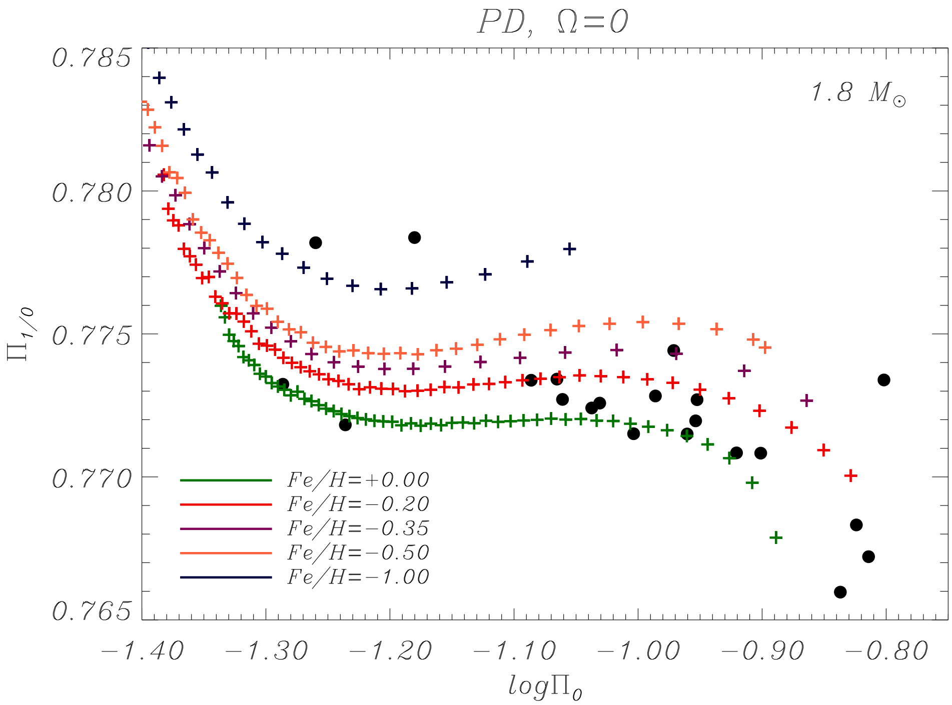

Fig. 1. Classic PD (Π_0 in d^{-1}) containing tracks of 1.8 $M_\odot$ evolutionary models computed with different initial metal content [Fe/H], from solar fraction (green) to -1.00 (blue). Filled circles represent the observed period ratios of the double-mode high-amplitude δ Scuti stars reported in Poretti et al. (2005).

1.8 $M_\odot$ evolutionary tracks in Fig. 1. Similar tracks for different mass and metallicity ranges can be found in Petersen (1973), Petersen & Christensen-Dalsgaard (1996) and Petersen & Christensen-Dalsgaard (1999). Such dependence makes the period ratios increase when decreasing the stellar initial metal content, which is commonly used to discriminate, in the context of radial pulsators, Pop. I from Pop. II stars. Typical $\Pi_{1/0}$ values found for main sequence Pop. I stars are in the range of $\Pi_{1/0} = [0.772, 0.776]$. As a reference, the observed period ratios of the double-mode high-amplitude δ Scuti stars known up to now are also included (values obtained from Poretti et al. 2005, and references therein).

However, it is well known that rotation modifies the structure of stars and thereby the cavity where modes propagate. Therefore, in a general case, the period ratios are explicitly dependent on the rotation velocity. Furthermore, since observationally only projected rotational velocities $v\sin i$ are provided by observations, it is possible to consider the period ratio $\Pi_{1/0}(\Omega)$ as a function of:

$$\Pi_{1/0}(\Omega) = \Pi_{1/0}(M, R, Z, \Omega(i)) .$$

The impact of considering $\Pi_{1/0}(\Omega)$ rather than $\Pi_{1/0}$ on metallicity determinations is here analysed. To do so, several evolutionary tracks have been computed for different metallicities and initial rotational velocities, fixing the mass of models to 1.8, $M_\odot$ which typically corresponds to a δ Scuti star.

Period ratios are then calculated from adiabatic oscillations, which have been computed from the previously computed *rotating* models.

In Fig. 2 such *rotational PD* (hereafter RPD) are displayed for tracks computed for $\Omega_\mathrm{i} = 50\,\mathrm{km\,s^{-1}}$. The main effect of rotation is similar to decrease the metallicity in classic

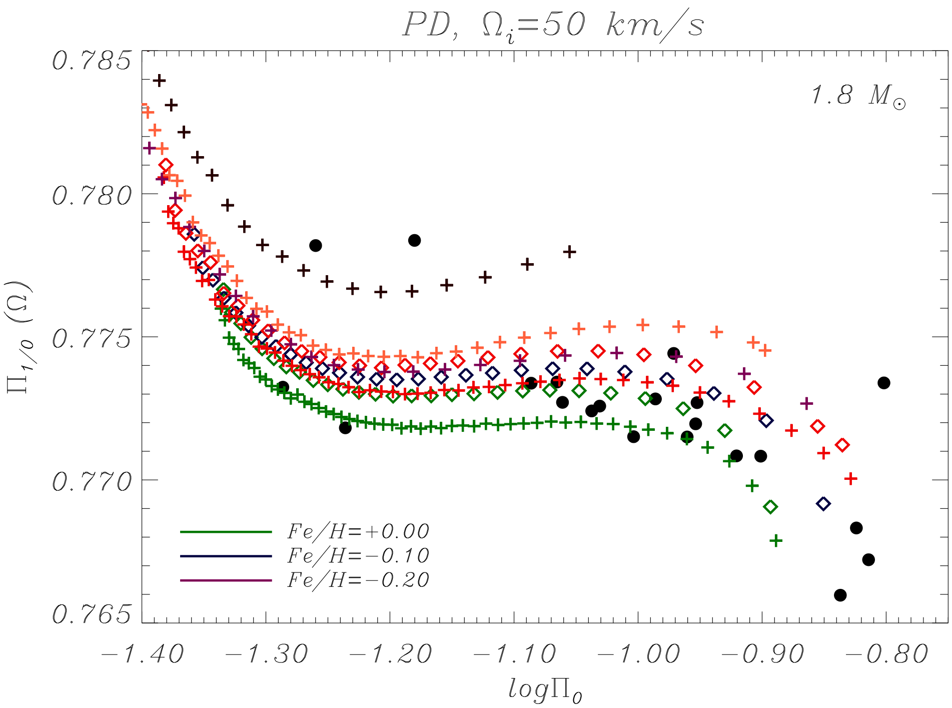

Fig. 2. RPD illustrating the comparison between rotating and non-rotating tracks (classic PD). Tracks for an initial rotational velocity of 50 km s^{-1} have been considered. For convenience, the following symbols are used: crosses, representing non-rotating models; and diamonds, those evolved with $\Omega_{\rm i} = 50\,{\rm km\,s^{-1}}$. As in Fig. 1 filled circles represent the observed period ratios of the double-mode high-amplitude δ Scuti stars reported in Poretti et al. (2005).

PD. This can be understood in terms of variations of the density distribution in stellar model interiors due to rotation effects (Suárez et al., work in preparation). In addition, the shift to larger period ratios is dependent on the metallicity. In particular, the higher the rotational velocity the closer the models are for different metallicity in RPD. This means that the effect of rotation on period ratios is systematically larger for increasing metallicity values (up to the solar value, in the present study).

The previous differences in period ratios can also be analysed in terms of metallicity. For shortness, tracks will be specified, from now on, with the subscript corresponding to the rotational velocity considered. For instance, the track computed with $\Omega_{\rm i}$ = 50 km s^{-1} and [Fe/H] = −0.1 will be called as $[-0.1]_{50}$. The analysis of Fig. 2 reveals that $[0.00]_{50}$ tracks are located closely to $[-0.20]_0$ ones. Similarly, $[-0.10]_{50}$ tracks may be confused with $[-0.20, -0.35]_0$ ones, and finally, $[-0.20]_{50}$ tracks are close to $[-0.50]_0$ ones. As can be seen, the mix-up is important for Pop. I stars when considering 1.8 $M_\odot$ rotating models evolved with $\Omega_{\rm i}$ = 25, 50. Such effect is equivalent to modify the mass of models, i.e. larger masses produces larger period ratios (for a given metallicity), and vice–versa. This occurs systematically for any *rotating track*. The present discussion for 1.8 $M_\odot$ models can be extended to any other mass. For a complete analysis and discussion for different masses and metallicities, see Suárez et al. (2005b).

3. Conclusions

As claimed in our recent paper Suárez et al. (2005b), the effect of rotation must be taken into account when analysing double-mode pulsators with Petersen Diagrams.

Taking into account the effect of rotation on both equilibrium models and on the adiabatic oscillation computation, the new FI/FO period ratios *ratiorot* are calculated. From these ratios, we have constructed the corresponding *rotational* Petersen Diagrams (RPD). The analysis of these RPD reveals that the difference in period ratios increases with the rotational velocity for a given metallicity. It remains around 10^{-3} for rotational velocities up to $50\,\mathrm{km\,s^{-1}}$. Such difference have been found enough to produce a significant confusing scenario when analysing RPD in terms of metallicity variations. In particular, for $1.8\,\mathrm{M}_{\odot}$ stellar models, differences in metallicity up to $\delta[\mathrm{Fe/H}] \sim 0.30$ can be found when considering models evolved with initial rotational velocities of $50\,\mathrm{km\,s^{-1}}$. Furthermore, such confusion may still increase when including other stellar masses, rotational velocities and metallicities.

The analysis of period ratios in the framework of a perturbative theory, as well as the inclusion of the effect of differential rotation in the manner described in Suárez et al. (2005c) should provide constraints which help us to improve our modelling.

In the context of HADS, a revision of recent results reported by Poretti et al. (2005) may be envisaged, as well as the extension to evolved models like double-mode Cepheids stars, in particular to analyse the effect of rotation on FI/FO period ratios.

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