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Acoustic oscillations in centrifugally flattened polytropic star

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Abstract. We present numerical calculations of axisymmetric acoustic modes in polytropic models of star deformed by the centrifugal force. In the range of flatness and frequencies considered, we found that, as flatness increases, (i) differences with perturbative methods becomes rapidly significant (ii) the structure of the spectrum is strongly modified (iii) the mode amplitude at the surface tends to concentrate near the equator.

Key words. Stars: pulsations – rotation

1. Introduction

The effect of rotation on stellar oscillations has been mostly studied using perturbative methods (Saio, 1981; Soufi et al., 1998). Although fully justified for slowly rotating stars like the sun, this approach might not be accurate for rapidly rotating stars or when the oscillation frequency is close to the rotation rate (Dintrans & Rieutord, 2000). Moreover, the limit of validity of perturbative methods is unknown. The same problem holds for usual identification techniques, like the asymptotic large and small frequency separations, the rotational splitting or the mode's visibility. New methods able to compute eigenmodes of rapidly rotating stars are therefore needed.

We developed such a method and as a first step we only considered the effect of the centrifugal force through its impact on the equilibrium state of the star; we thus neglected the Coriolis force. In this paper we present computations of low frequency axisymmetric p-modes. The problem has been further simplified by using uniformly rotating polytropes as equilibrium models and assuming adiabatic perturbations as well as the Cowling approximation. The rotation rate has been varied from $\Omega = 0$ up to $\Omega/\sqrt{GM/R_e^3} = 0.59$ which corresponds to a typical δ Scuti star with an equatorial velocity of 150 kms^{-1}.

2. The formalism

Except in the special cases of spherically symmetric media and uniform density spheroids, the eigenvalue problem of acoustic resonances in arbitrary axially symmetric cavities is not separable in the radial and meridional variables. A two-dimensional eigenvalue problem has then to be solved. The mathematical formalism and the numerical method that we used are briefly described below.

For accuracy reasons, the numerical grid is chosen to match the stellar surface and to be

Send offprint requests to: F. Lignières spherical at the center. The coordinate system (ζ, θ, ϕ) defined from usual spherical coordinates (r, θ, ϕ) by

$$\begin{cases} r = (1-\epsilon)\zeta + A(\zeta)(S(\theta') - 1 + \epsilon) \\ \theta = \theta' \\ \phi = \phi' \end{cases} \qquad (1)$$

fulfill these requirements as $\zeta = \zeta_0$ describes the surface $r = S(\theta)$ and $A(\zeta)$ goes to zero more rapidly than ζ (Bonazzola et al., 1998). The parameter ϵ is the flatness of the surface. The eigenvalue problem is written with these coordinates and, when necessary, vectors are projected onto the associated natural basis. The numerical discretization is done using spectral methods, spherical harmonics for the angular coordinates and Chebyshev polynomials for the pseudo-radial coordinate ζ. Finally, the resulting algebraic eigenvalue problem is solved with either a QZ algorithm or an Arnoldi-Chebyshev algorithm as in Rieutord & Valdettaro (1997).

Because of the symmetries with respect to the rotation axis and the equator, one obtains a separated eigenvalue problem for each absolute value of the azimuthal number $| m |$ and each parity with respect to the equator.

The method has been tested for axisymmetric ellipsoids of uniform density because another method using separability exists in this case (Lignières et al., 2001). The computation of axisymmetric p-modes in a uniformly rotating polytrope presented below has been tested by solving the same problem but using a different form of the starting equations.

3. Parameter range and mode identification

Self-gravitating uniformly rotating polytropes of index $n = 3$ and specific heat ratio $\gamma = 5/3$ have been computed for rotation rates varying from $\Omega = 0$ up to $\Omega/\sqrt{GM/R_e^3} = 0.59$. In this range, the flatness of the star's surface defined as $\epsilon = 1 - R_p/R_e$ increases from 0 to 0.15, R_e and R_p being the equatorial and polar radii, respectively. For each model, low frequency axisymmetric p-modes have been computed.

In the absence of rotation, separability enables the identification of modes by three

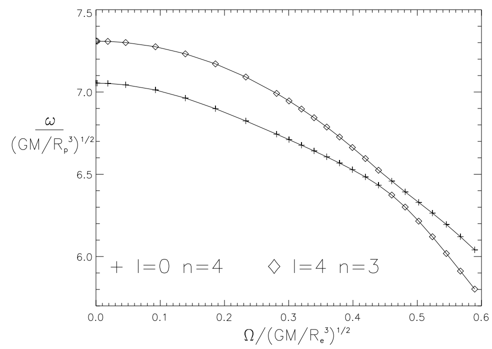

Fig. 1. Evolution of two eigenfrequencies as a function of the rotation rate. In the absence of rotation the modes are identified by their degree and order, respectively $\ell = 0,\ n = 4$ and $\ell = 4,\ n = 3$ and we kept this labeling by progressively increasing the rotation rate. An avoided crossing occurs near $\Omega/\sqrt{GM/R_e^3} = 0.45$. Although the modes have a mixed character at that point, they recover their original properties after the crossing which enabled us to unambiguously label the modes.

"quantum" numbers n, ℓ, m which characterize respectively their radial, latitudinal and azimuthal structure. In the rotating case, we compute a spectrum of modes having the same $|m|$ and the same parity with respect to the equator without a priori information about their radial and latitudinal structure.

To obtain a meaningful classification of these modes, we assumed that any mode can be associated unambiguously with a non-rotating mode and thus can be identified by the three quantum number of that non-rotating mode. In practice, we started at zero rotation with the $\ell = 0-7,\ n = 1-10,\ m = 0$ modes and tried to follow them by progressively increasing the rotation. This process presents no difficulty except when eigenfrequencies try to cross. Indeed modes having the same $| m |$ and the same parity with respect to the equator cannot have the same frequency and, as illustrated on Fig. 1, an avoided crossing takes place. Near the closest approach of the avoided crossing, the two modes acquire a mixed character. Then, it is difficult to distinguish between the two modes at that point. However, after the avoided crossing, the modes recover their original properties and can therefore be identified again.

Up to the fastest rotation considered, we were always able to follow unambiguously the $\ell = 0 - 7,\ n = 1 - 10,\ m = 0$ modes through the sequence of avoided crossings.

4. Comparison with perturbative methods

According to perturbative analysis, centrifugal effects on pulsations appear at second order in Ω (Saio, 1981). Thus, if ω_0 denotes the non-rotating eigenfrequency, $(\omega(\Omega) - \omega_0)/\Omega^2$ should have a finite limit as Ω vanishes. Approximate frequencies valid up to second order in Ω have been obtained by computing this limit from our data. Then, to assess the range of validity of the perturbative approach, we compared these frequencies to the exact frequencies. Figure 2 displays the relative differences for the $\ell = 0-2,\ n = 1-10$ modes. It shows that the error made in using perturbative method becomes rapidly important as rotation increases. Indeed, a maximum relative error of 1 percent is reached near $\Omega/\sqrt{GM/R_e^3} = 0.2$ which is already significant considering the accuracy of present and future frequency determinations. Figure 2 also shows that departures

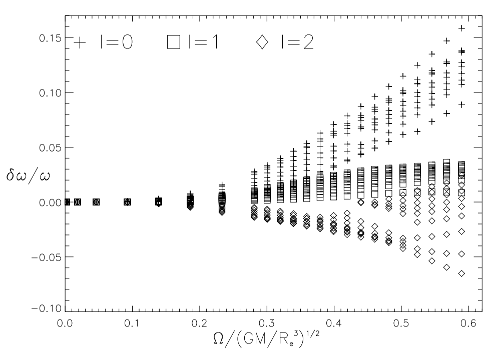

Fig. 2. The relative difference between the exact frequency and its second order perturbative approximation (second order in terms of the small parameter $\Omega/\sqrt{GM/R_e^3}$) namely, $\delta\omega/\omega$ where $\delta\omega = \omega - (\omega_0 + \omega_1\Omega^2/(GM/R_e^3))$ is displayed as a function of the rotation rate for all the $\ell = 0 - 7,\ n = 1 - 8$ or $10,\ m = 0$ computed frequencies.

from perturbative calculations depend on the type of mode. For a given parity with respect to the equator, discrepancies are stronger for low degree and high order modes.

5. The structure of the frequency spectrum

Does rotation modify the usual structure of the acoustic frequency spectrum ? Figure 3 presents a comparison of the spectrum of all $\ell = 0-7, n = 1-10, m = 0$ eigenfrequencies for two different rotation rates, (a) $\Omega = 0$ and (b) $\Omega/\sqrt{GM/R_e^3} = 0.59$. The degree ℓ is given by the height of the vertical bar. Comparison of both spectra shows that the centrifugal flattening translates the spectrum towards smaller frequencies as well as contracts it. Both effects are simply due to the volume growth of the oscillation cavity induced by the centrifugal force. Another more subtle effect is that rotation changes the usual frequency ordering. While eigenfrequencies of given order n increase monotonically with ℓ in the absence of rotation, this is no longer true for high rotation rates as can be seen on Fig. 3. Indeed, a stronger effect of the centrifugal distortion on modes symmetric with respect to the equator, results in a progressive shift of the spectrum of symmetric modes with respect to the spectrum of anti-symmetric modes. Again, this effect is stronger for low degree and high order modes.

It is indeed not surprising that the symmetrical deformation produced by the centrifugal force affects differentially modes which are respectively symmetric and anti-symmetric with respect to the equator. Moreover, for a given parity, modes of large latitudinal length scales (low ℓ) and concentrated near the surface (high n) should be more affected since the centrifugal force induces a large scale distortion which is stronger near the surface.

As revealed by asymptotic studies ($\omega \rightarrow +\infty$), the structure of the acoustic frequency spectrum in the absence of rotation is characterized by a constant large frequency separation $\omega_{n,l} - \omega_{n-1,l}$ and a vanishingly small frequency separation $\omega_{n,l} - \omega_{n-1,l+2}$. Despite the limited range of computed frequencies, we do observe a clear tendency towards this asymp-

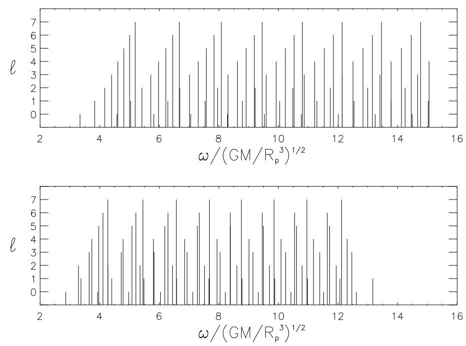

Fig. 3. Frequency spectrum of $\ell = 0 - 7$, $n = 1 - 8$ or 10, $m = 0$ modes for (a) $\Omega = 0$ and (b) $\Omega/\sqrt{GM/R_e^3} = 0.59$. The degree number ℓ associated with the frequency is shown by the height of the vertical bar.

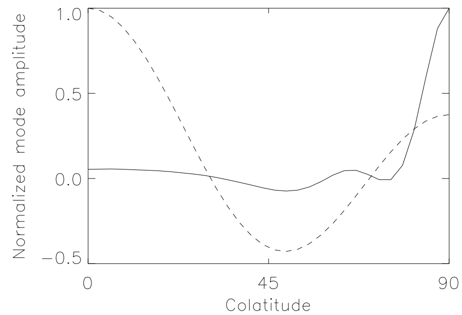

Fig. 4. Surface distribution of the amplitude of a $\ell = 4$, $n = 4$ mode for $\Omega = 0$ (dashed line) and $\Omega/\sqrt{GM/R_e^3} = 0.59$ (continuous line). Centrifugal distortion induces a strong concentration of the mode amplitude towards the equator.

totic state. Increasing the rotation, we still find that the large separation tends to be constant. However, the small separation has no longer a tendency to vanish for high orders *n*. This absence of the usual small frequency separation together with the shift between the symmetric and anti-symmetric spectra strongly modify the structure of the frequency spectrum in the presence of a non-negligible centrifugal force.

6. Surface distribution

The surface distribution of the mode amplitude is no longer described by a single spherical harmonic. The new pattern induced by the centrifugal effect is of particular interest as it could have an impact on the mode visibility. Figure 4 compares the surface distribution of a given mode for two different rotation rates. It clearly shows a concentration of the mode amplitude towards the equator. This striking effect seems to be general for all the modes computed. It was already mentioned by Clement (1998).

Interestingly, this result is compatible with observations of δ Scuti pulsations showing an increase of the pulsation amplitudes with *vsini* (Suárez et al., 2002). However, realistic calculations of the mode visibility including in particular the gravity darkening effect are needed to draw observational consequences.

7. Conclusion

The complete treatment of the centrifugal distortion reveals significant departures with perturbative methods. It also shows new effects like the absence of the so-called "small frequency separation", strong differences between modes respectively symmetric and anti-symmetric with respect to the equator, and the profound modification of the mode visibility. The next step is to include the Coriolis force (see Reese et al. (2006) for the first results).

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