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Turbulence in models of a star other than Sun: matching η Bootis observed p-modes

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Abstract. Standard stellar models for η Boo fail to reproduce the newly observed low frequency p-modes from space and the high frequency p-modes observed from the ground, simultaneously. This discrepancy can be removed by including turbulence in the modeling of the outer layers of η Boo. We include turbulence by applying the effects of turbulent pressure and turbulent kinetic energy — extracted from a hydrodynamical 3D convection simulation for the Sun — at the correct depth in the 1D models of η Boo.

1. Introduction

Detailed high-precision modeling of the outer layers of stars has so far been restricted to the Sun because only helioseismology (e.g., Christensen-Dalsgaard et al., 1996) has provided us with extensive p-mode spectra, which subsequently led to the need to refine the modeling of the outer stellar layers in order to reproduce the observed frequencies. Standard solar models fail to reproduce the high order p-modes because of the insufficiencies present in the modeling of the outer convection zone, usually approximated with standard mixing length theory (MLT), which turns out to be a poor approximation of the complicated physics present in the outer layers of the Sun: the physics of shallow turbulent convection.

The largest uncertainties in the models are confined within a layer just below the photosphere: the superadiabatic layer (SAL) of convection. Since the peak of the SAL is located at $r \sim 0.995\,R_\odot$ (e.g., Monteiro et al., 1996) and inversion techniques only probe to solar radii of $r < 0.96\,R_\odot$ (Antia & Basu, 1994), it is clear that the exact structure of the model cannot be uniquely derived from observations. Thus, the correct structure can only be determined by theory which must result in the reproduction of the observed frequencies.

We believe that the correct structure of the outer layers can only be firmly established by full 3D hydrodynamical simulations of shallow convection which have recently been performed by Stein & Nordlund (1998); Robinson et al. (2004). Based on these simulations, a better match to the observed p-mode frequencies can be achieved. (Rosenthal et al., 1999) used Stein & Nordlund (1998)'s 3D simulation and patched it to the envelope solution which yields a good match to observations. In another approach Li et al. (2002) used the turbulent pressure and turbulent kinetic energy extracted from horizontal- and time-averages of a so-

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Send offprint requests to: Christian W. Straka, e-mail: `straka@astro.yale.edu` lar 3D simulation from Robinson et al. (2003) to modify the outer layers of the 1D models. These authors were also successful in reproducing the observed *p*-mode frequencies.

With the space satellite MOST[1] (Walker et al., 2003), high quality *p*-mode frequencies have now become available for the star η Boo. With the observed *p*-mode frequencies from ground (Kjeldsen et al., 2003) we show that the combined data set points strongly towards the need for refinements of the modeling of the outer layers of η Boo compared to standard models with MLT.

2. Stellar Modeling of η Boo

2.1. Starting Model

We start from a model for η Boo that has been selected with the *quantified dense grid method* (QDG) developed by Guenther & Brown (2004), a method which selects models based on their *p*-mode frequency spectra alone. Guenther et al. (2005)'s best model fit for η Boo was determined by comparing 8 observed MOST $l = 0$ low frequency *p*-modes to theoretical modes extracted from a huge set of models along their evolutionary path. The set of models covers a large parameter space with different initial hydrogen content $X = (0.69, 0.71)$, metallicity $Z = (0.02, 0.03, 0.04)$ and stellar masses between $1.4\,M_\odot$ and $1.9\,M_\odot$ with a fine grid resolution of $0.005\,M_\odot$.

A small χ^2 number (Guenther & Brown, 2004) indicates a good agreement between model and observations which was achieved for the best-fit model with a $\chi^2 < 1.4$. This model consists of a mass of $1.71 \pm 0.05\,M_\odot$, $(X, Z) = (0.71, 0.04)$, a mixing length of 1.8 and no element diffusion at an age of 2.40 ± 0.03 Gyrs. Within the searched parameter-space it is the model that best fits the 8 observed MOST p-modes. Significantly, the model also happens to be consistent with the observed luminosity and effective temperature (Di Mauro et al., 2003), although, these parameters were not used to constrain the best-fit model.

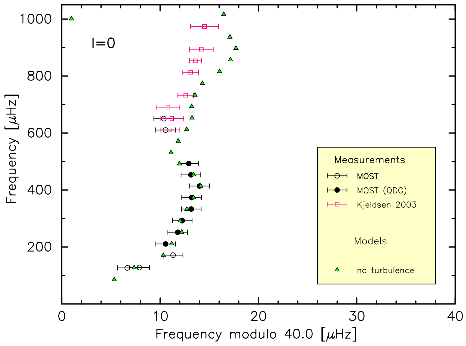

Fig. 1. Echelle diagram showing the standard model (triangles), searched for giving a good fit to 8 MOST modes together with the ground based $l = 0$ measurements.

While this best-fit model gives a good match for the observed MOST low *p*-mode frequencies it fails to reproduce the $l = 0$ high frequency *p*-modes which were observationally determined by Kjeldsen et al. (2003). The model frequencies appear to be at slightly higher folded frequencies with a difference that increases from $1.5\,\mu$Hz at $600\,\mu$Hz to $4\,\mu$Hz at $900\,\mu$Hz (Figure 1). These differences between model and observations are significant within observational uncertainty above $700\,\mu$Hz.

2.2. Inclusion of Turbulence

Li et al. (2002) have demonstrated for the Sun that a better match between observed and model *p*-mode frequencies can be achieved when the 1D stellar models are modified by including the combined effects of turbulent pressure and turbulent kinetic energy extracted from horizontally and time averaged quantities of a full 3D simulation of the outer layers of the Sun Robinson et al. (2003).

If we want to do the same for η Boo, we need to extract the data from a 3D simulation of this star. We are currently working on a 3D simulation for η Boo. Nevertheless, we believe

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[1] MOST (Microvariability & Oscillations of STars) is a Canadian Space Agency mission, jointly operated by Dynacon Inc., the University of Toronto Institute for Aerospace Studies and the University of British Columbia, with the assistance of the University of Vienna.

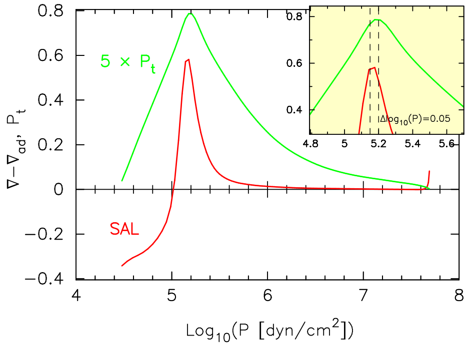

Fig. 2. Averaged quantities taken from a 3D turbulent convection simulation for the Sun. The peak of the SAL coincides with the peak of turbulent pressure.

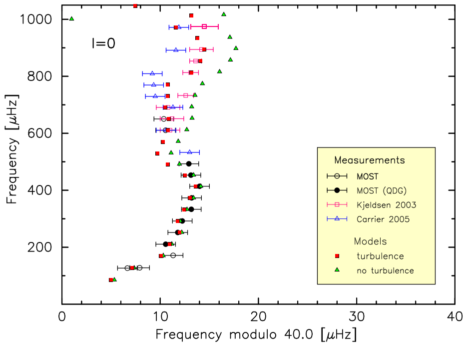

Fig. 3. Echelle diagram showing the non-adiabatic p-mode frequencies derived from a best-fit theoretical model without turbulence (triangles) in comparison to a model with turbulence (squares) on top of the ground and space based observational measurements.

that the *solar* 3D simulation data provides us with a reasonable first estimate if we apply this data sensibly to η Boo. In this work, we apply the solar 3D simulation averages for the turbulent pressure and turbulent kinetic at the correct depth within the outer layers of η Boo.

A characteristic found in the 3D simulations of evolutionary models of the Sun, taken at a sampling of four different ages along the evolutionary track of models for the Sun, is that the SAL peak closely coincides with the turbulent pressure peak (Figure 2). The offset between the peaks measured in pressure difference is always smaller than $\Delta\log_{10}(P\,[\mathrm{dyn/cm^2}]) = 0.1$. In order to apply the solar turbulence data to η Boo we match the turbulent pressure peak from the 3D simulation data with the SAL of the 1D model with a small offset of $\Delta\log_{10}(P\,[\mathrm{dyn/cm^2}]) = 0.0758$, a mean value derived from the four solar models.

2.3. Model Calibration

When we change the outer layers of the η Boo models by including the effects of turbulence we must insure that the previous good fit to the 8 measured MOST frequencies is preserved. Since the lower frequency p-modes anchor the model effectively in mass and age (Guenther et al., 2005) and also in radius we must calibrate the models to the same luminosity and effective temperature by varying the initial helium content and mixing length parameter.

This procedure is completely analogous to the calibration of a solar 1D model with turbulence included (Li et al., 2002). However, for the Sun, the effective temperature and luminosity is known to high precision independent of the p-mode analysis. In the case of η Boo we need to rely on the determined luminosity and effective temperature by matching the low frequency spectrum alone.

3. Results

It is evident from Figure 1 that the model for η Boo which reproduces the $l = 0$ low frequencies observed by MOST yields high frequency p-modes that deviate significantly from the observed ground based high frequency p-modes by Kjeldsen et al. (2003). We now show in Figure 3 that our calibrated η Boo model that includes the depth-adjusted effects of turbulence as derived from a 3D convection simulation for the Sun reproduces both the low frequency p-modes observed by MOST *and* in addition six out of the eight ground based $l = 0$ frequency data points by Kjeldsen, five within their 1σ uncertainty and one within 2σ. Also note that one frequency not matching the data is still a match within 3σ.

The region of $600 - 650\,\mu$Hz where the models coincide with the two MOST modes that have been independently confirmed by the ground based measured modes of both Kjeldsen et al. (2003) and Carrier et al. (2005) add credibility to our modeling. As already noted in Guenther et al. (2005), MOST had measured two modes below $200\,\mu$Hz that also fit into the $l = 0$ sequence of our models.

To provide a more quantitative measure we calculate the χ^2 numbers for the combined data sets (MOST plus Kjeldsen et al.) of all $l = 0$ modes in the range $200 - 900\,\mu$Hz. For the model without turbulence we get $\chi^2 = 18$ compared to $\chi^2 = 2.5$ for the model that includes turbulence.

Our model with turbulence fits the Kjeldsen data much better than the data from Carrier et al. (2005). The combined set of MOST plus Carrier data gives a $\chi^2 = 18$. Since the Carrier data appears consistently at lower folded frequencies, the standard model without turbulence is very far off with a $\chi^2 = 131$. Hence the model with turbulence is still much closer to the Carrier data than a model without turbulence.

4. Conclusions

This paper demonstrates that the measured p-mode frequencies of η Boo from space (by MOST) and the ground (by Kjeldsen) can be *jointly* matched with our theoretical models by including the effects of turbulence in the outer stellar layers.

Specifically, we show that the inclusion of turbulence can account for the difference of $1.5 - 4\mu$ Hz in the echelle diagram between the ground based $l = 0$ data points and the models without turbulence. The quantitative agreement for the model including turbulence is excellent for the combined data set (MOST plus Kjeldsen) of the $l = 0$ p-mode frequencies between $200 - 900\,\mu$Hz with a $\chi^2 = 2.5$.

Since the turbulent pressure and turbulent kinetic energy is taken from a 3D simulation for the Sun we must still confirm that a 3D simulation for η Boo will yield similar values for these quantities. Since we shift the solar data to apply them at the correct depth our results are qualitatively robust. We are now working to produce a 3D simulation simulation for η Boo to add unambiguous evidence to the quantitative correctness of these results.

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