



# Some interesting twists in the pulsating sdB star puzzle

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**Abstract.** In investigating the pulsation spectrum of the sdBV star PG 0014+067, we found a peculiar regularity of the pulsation frequencies reminiscent of asymptotic  $p$ -mode pulsation along with rotational splitting. We have found a similar pattern in all other pulsating sdBV stars with multisite coverage. High-order  $p$ -mode pulsations (such as seen in the Sun and roAp stars) should not be present if our models of these stars and their pulsations are any guide to reality. The reasons for this pattern are currently unknown.

**Key words.** Stars: pulsation – Stars: evolution – Stars: subdwarf B

## 1. Introduction

The pulsation frequencies seen in pulsating subdwarf (sdBV) stars show frequencies that are of the same order as the radial fundamental period. However, these stars generally show multiperiodic pulsation, with many modes presenting over a relatively narrow frequency range. The reason for this rich mode spectrum is unclear – proposed explanations range from invoking nonradial modes with degree  $l$  ranging from 0 to 3 or 4, to rotational splittings of  $l = 0 - 2$  modes in stars with rapidly rotating cores. We observed one such star, PG 0014+067 (hereafter PG0014) to explore these possible explanations. What we found is that, while the frequencies might be describable in terms of higher-than-usual values of  $l$  or rapid internal rotation, they show a pattern that looks more like asymptotic  $p$ -mode pulsation. A similar pattern has been found in the other well-studied pulsating sdBV stars.

Unfortunately, the models demand that these stars are showing low-order  $p$ -mode pulsations, which show no such asymptotic patterns.

## 2. PG 0014 Frequencies

The results of the WET run on PG 0014 will appear elsewhere (Vučković et al, in preparation) but Table 1 presents our preliminary frequency list, merged with the frequencies found by Jeffery et al. (2005) using UltraCam on the WHT to do light curves at different colors. In creating this list, we used the WET data as the primary source, and added lower-frequency modes identified by Jeffery et al. (2005). Even though the Jeffery et al. (2005) data are mostly single site, we are confident in their frequency identifications. This confidence comes from their identification of all modes that they have in common with the WET data, without a single instance of selection of a 1 c/d alias in their analysis. In Table 1, the UltraCam peaks are indicated in the Notes column, with

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**Table 1.** Frequencies present in PG0014, along with a “model”

#	Freq. [ $\mu\text{Hz}$ ]	Amp. [mma]	Note	$i$	$j$	Model	Difference
2	5923.4	0.54	fine structure	0	0	5923.2	0.2
3	6193.5	0.44		3	0	5194.1	-0.6
4	6452.9	0.45		7	-1	6454.1	-1.2
5	6632.8	0.65	fine structure	9	-1	6634.6	-1.8
6	6646.5	0.60	UltraCam	8	0	6645.6	0.9
7	6659.9	0.34	UltraCam (w)	7	1	6656.5	-3.4
8	6726.8	0.37	UltraCam (w)	10	-1	6724.9	1.9
9	6826.1	2.38		10	0	6826.1	0.0
10	7088.7	2.98	fine structure	14	-1	7086.1	2.6
11	7187.5	0.66	UltraCam	14	0	7187.3	0.2
12	7289.0	0.65		14	1	7288.5	0.5

the symbol (w) indicating that the mode was seen only in the combined UltraCam data (all colors).

The frequency list of PG 0014 displays some suggestive systematics; concentrating on the second column of Table 1 we see several modes with separations of approximately integral multiples of  $90 \mu\text{Hz}$ . Other separations of nearly  $100 \mu\text{Hz}$  are apparent as well. Our initial inclination was to try to identify the  $100 \mu\text{Hz}$  splitting as a rotational splitting. While this large splitting could be caused by rotation, the implied rotation rate (if solid-body rotation) would be much larger than upper limits based on spectroscopic study of line profiles. On the other hand Kawaler & Hostler (2005) suggest that rapid internal rotation could produce large splittings in a star with a slow surface rotation rate, but their predictions suggest that the splittings of different modes should not show the same value.

### 3. A Phenomenological Model

With two apparent splittings present, we decided to explore an entirely phenomenological parameterization that could then be used to make an empirical fit to the observed frequencies. We chose a form reminiscent of asymptotic  $p$ -mode pulsation with a constant rotation frequency:

total  $p$ -mode pulsation with a constant rotation frequency:

$$f(i, j) = f_o + i \times \delta + j \times \Delta \quad (1)$$

where  $\delta$  represents a small spacing (and  $i$  can range from 0 upwards) and  $\Delta$  represents a large spacing (with  $j$  initially limited to being either -1, 0, or 1). In the equation above,  $f_o$  represents a zero-point for the fit with  $i = j = 0$ .

In the general case of fitting a set of frequencies, we perform a two-dimensional  $\chi^2$  minimization, to find best-fit values for  $\delta$  and  $\Delta$ . Clearly, there is an aliasing problem when the combinations of  $i$ ,  $j$ ,  $\delta$ , and  $\Delta$  produce commensurate spacings, so the  $\chi^2$  surface shows multiple minima. We break that degeneracy (when possible) by choosing the  $(\delta, \Delta)$  pair for which we have at least two modes with the same value of  $i$  but different  $j$ . In practice, we make the further requirement that the fit must include at least two different pairs of modes (i.e. modes with the same value of  $i$ ) that show the same  $\Delta$ ; i.e. the fit must contain modes with  $(i_1, j_1)$ ,  $(i_1, j_2)$ ,  $(i_2, j_3)$ , and  $(i_2, j_4)$ . In a subsequent paper, we will show that enforcing these additional conditions on the fit criteria restrict the aliasing problem and greatly increase the statistical significance of the fit. For PG 0014 this procedure yields values of  $\delta=90.37$ ,  $\Delta=101.22$ , and  $f_o=5923.24$  (all  $\mu\text{Hz}$ ). Using these values in Equation 1, we find the model frequencies

**Table 2.** Frequencies present in PG 1336, along with a “model”

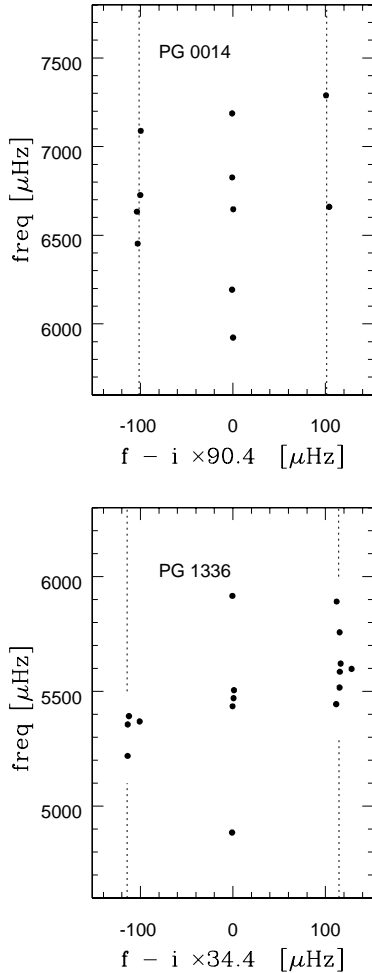
#	Freq. [ $\mu\text{Hz}$ ]	Note	$i$	$j$	Model	Difference
1	4885.1		0	0	4886.7	1.5
2, 7	5219.0, 5444.3		13	-1 1	5218.7, 5447.6	-0.3, 3.3
3, 8, 11	5356.5, 5470.9, 5585.7		17	-1 0 1	5356.0, 5470.5, 5585.0	-0.4, -0.4, -0.7
4, 12	5369.4, 5598.5	no fit	-	-	-	-
5, 9, 13	5392.2, 5505.6, 5621.2		18	-1 0 1	5390.4, 5504.8, 5619.3	-1.8, -0.8, -1.7
6	5435.4	doublet	16	0	5436.2	0.7
10	5516.7		15	1	5516.3	-0.5
14	5757.3	doublet	22	1	5756.7	-0.6
15	5891.5		26	1	5894.1	2.6
16	5916.3		30	0	5917.0	0.7

**Table 3.** Splittings observed in pulsating sdBV stars

Star	$\log g$	$f_o$ [ $\mu\text{Hz}$ ]	$\delta$ [ $\mu\text{Hz}$ ]	$\Delta$ [ $\mu\text{Hz}$ ]	Significance
PG 0014	5.8	5923	90.4	101.2	4.0 $\sigma$
PG 1219	5.85	5812	60.5	-	3.7 $\sigma$
PG 8783	>5.6	7193	58.1	138.9	3.9 $\sigma$
PG 1047	5.9	6310	55.9	186.5	4.2 $\sigma$
PG 1336	5.7	4886	34.4	114.5	6.0 $\sigma$
Feige 48	5.5	2642	13.9	-	-

listed in Table 1. Note that the fit is extremely good - the 11 modes identified in the table are all fit to within 0.05%, with an RMS difference of 0.013  $\delta$ . To estimate the statistical significance of this fit, we performed sets of 1000 trials with randomly chosen frequencies that span the same frequency interval. In each set of trials, the best fit (as judged by the RMS difference between the best model and the random frequency set) was tabulated. This procedure helped us determine the statistical significance of the star’s fit by the model. In the case of PG 0014, the star departs from a random frequency distribution at the 4  $\sigma$  level. Is this asymptotic pulsation? After all, high-order  $p$ -modes show a more-or-less constant frequency spacing; such sequences are seen in helioseismic data and in the rapidly oscillating Ap stars

(i.e. Kurtz et al. (2005)). The sequence of modes split by integral multiples of  $\delta$  cannot be asymptotic  $p$ -mode behavior. Models of PG 0014, and sdBV pulsators in general, indicate that the radial fundamental frequency in the models is usually close (in frequency) to the observed mode frequencies. Asymptotic relations such as Equation (1) are usually valid (at the few-percent level) only for values of  $n \gg l$ , or more generally for large values of  $n$ . Even so, the computed frequency separation for  $p$ -modes in sdBV models yields values of several hundred  $\mu\text{Hz}$  - a factor of 10 or more larger than what PG 0014 shows.



**Fig. 1.** “Echelle” diagrams of the frequencies in PG 0014 (top) and PG 1336 (bottom); the frequencies have been folded on the small spacing  $\delta$  and stacked, showing the uniformity of the frequency spacings within the main band and the two side bands separated from the main band by the large spacing  $\Delta$ .

#### 4. Is PG0014 a freak? No!

To see if PG 0014 is just a strange star, we examined published frequency lists for other sdBV stars that are known to be rich pulsators, and which have extended longitude coverage that ensures correct separation of true frequen-

cies from diurnal aliases. As the Table 3 shows, nearly all sdBV stars with extended-longitude coverage show similar splittings across their temporal spectra.

#### 5. And so, a mystery

While we have no explanation at all for the physics behind this apparent effect, there are several points that may be important.

Feige 48 has the lowest gravity of the bunch; the 5 frequencies from Reed et al. (2004) show a small splitting that is the smallest of the ensemble, but show no evidence of a second, large spacing ( $\Delta$ ). Aside from PG 0014, there appears to be some correlation between  $\log g$  and  $\delta$ . Two stars show spectra that are completely described without the need of a large spacing. And, PG 1336 is a member of a close binary. The large spacing we identify is precisely equal to the orbital frequency. If we can assume that the stars in this system are tidally locked and therefore in synchronous rotation, then in this system at least the large spacing  $\Delta$  can be identified with the stellar rotation rate. Reed (private communication) points out that the two modes for which the spacing model fails are those that display a pulsation geometry aligned not with the orbital plane, but pointing from the primary to the secondary.

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