



# Interferometric Observations of Cepheids

## $p$ -factor and center to limb darkening measurements.

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**Abstract.** Cepheids distances are usually inferred from the Period-Luminosity relationship, calibrated using the semi-empirical Baade-Wesselink (BW) method. Using this method, the distance is known to a multiplicative factor, called the projection factor. Presently, this factor is computed using numerical models - it has hitherto never been measured directly. Based on our new interferometric measurements obtained with the CHARA Array and the already published parallax, we present a geometrical measurement of the projection factor of a Cepheid,  $\delta$  Cep. The value we determined,  $p = 1.27 \pm 0.06$ , confirms the generally adopted value of  $p = 1.36$  within 1.5 sigmas. Our value is in line with recent theoretical predictions of Nardetto et al. (2004). Moreover, center-to-limb variation (CLV) remains a possible slight source of bias for the interferometric BW method. In order to address this problem, we are in the process of measuring the CLV of Polaris.

**Key words.** Techniques: interferometric – Stars: variables: Cepheids – Stars: individual:  $\delta$  Cep – Stars: individual: Polaris  $\gamma$ — Cosmology: distance scale

## 1. Introduction

The most commonly used alternative to measure the distance to a pulsating star is the Baade-Wesselink (BW) method. It utilizes the pulsational velocity  $V_{\text{puls}}$  of the surface of the star and its angular size. Integrating the pulsational velocity curve provides an estimation of the linear radius variation over the pulsation. Comparing the *linear* and *angular* amplitudes of the Cepheid pulsation gives directly its distance. The most recent implementation (Kervella et al., 2004) of the BW method

makes use of long-baseline interferometry to measure directly the angular size of the star.

Unfortunately, spectroscopy measures the apparent radial velocity  $V_{\text{rad}}$ , i.e. the Doppler shift of absorption lines in the stellar atmosphere, projected along the line of sight and integrated over the stellar disk. This is where  $p$ , a projection factor, has to be introduced, which is defined as  $p = V_{\text{puls}}/V_{\text{rad}}$ . There are in fact many contributors to the  $p$ -factor. The main ones are the sphericity of the star (purely geometrical) and its limb darkening (due to the stellar atmosphere structure). A careful theoretical calculation of  $p$  requires modeling dy-

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namically the formation of the absorption line in the pulsating atmosphere of the Cepheid (Parsons, 1972; Sabbey et al., 1995; Nardetto et al., 2004).

Until now, the  $p$ -factor was estimated from numerical models. Any uncertainty on the value of  $p$  will create the same relative uncertainty on the distance estimation, and subsequently to the P-L relation calibration. In other words, the Cepheid distance scale relies implicitly on numerical models of these stars. But how good are the models? To answer this question, one should confront their predictions to measurable quantities. Until now, this comparison was impossible due to the difficulty to constrain the angular diameter and the distance from observations.

## 2. The $p$ -factor of $\delta$ Cep

Interferometric observations were undertaken in 2004 at the CHARA Array (ten Brummelaar et al., 2005), in the infrared K' band ( $1.95 \mu\text{m} \leq \lambda \leq 2.3 \mu\text{m}$ ) with the Fiber Linked Unit for Optical Recombination (Coudé du Foresto et al., 2003) (FLUOR) using 250 and 313 m baselines. The pulsation phase was computed using the following period and reference epoch (Moffett & Barnes, 1985):  $P = 5.366316 \text{ d}$ ,  $T_0 = 2\,453\,674.144$  (Julian date), the 0-phase being defined at maximum light in the V band.

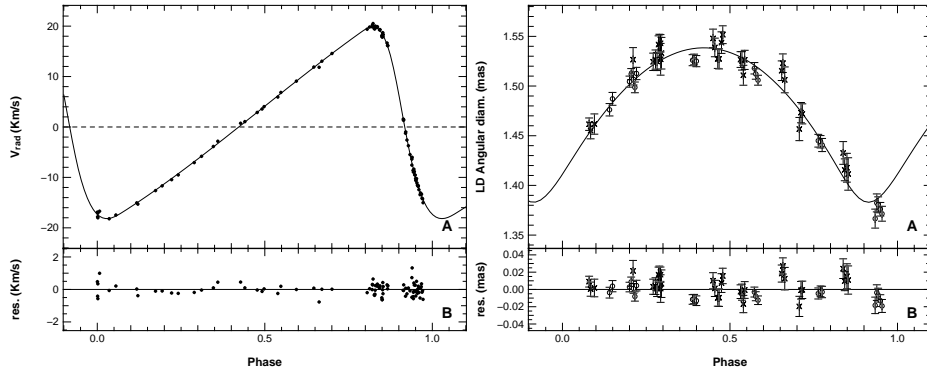
Among the various sets of measurements of the radial velocity  $V_{\text{rad.}}(t)$  available for  $\delta$  Cep, we chose measurements from Bersier et al. (1994) and Barnes et al. (2005). These works offer the best phase coverage, especially near the extrema, in order to accurately estimate the associated photospheric amplitude. In order not to introduce any bias due to a possible mismatch in the radial velocity zero-point between the two data sets, we decided to reduce them separately and then combine the resulting  $p$ -factor. An integration over time is required to obtain the photospheric displacement. This process is noisy for unequally spaced data points: the radial velocity profile was smoothly interpolated using a periodic cubic spline function.

Fitting the inferred photospheric displacement and observed angular diameter variations, we adjust three parameters: the mean angular diameter  $\bar{\theta}$ , a free phase shift  $\phi_0$  and the projection factor  $p$  (see Fig. 1). The mean angular diameter is found to be  $1.475 \pm 0.004 \text{ mas}$  (milliarcsecond) for both radial velocity data sets. Assuming a distance of  $274 \pm 11 \text{ pc}$  (Benedict et al., 2002), this leads to a linear radius of  $43.3 \pm 1.7$  solar radii. The fitted phase shift is very small in both cases (of the order of 0.01). We used the same parameters (Moffett & Barnes, 1985) to compute the phase from both observation sets and considering that they were obtained more than ten years apart, this phase shift corresponds to an uncertainty in the period of approximately five seconds. We thus consider the phase shift to be reasonably the result of uncertainty in the ephemeris.

The two different radial velocity data sets lead to a consolidated value of  $p = 1.27 \pm 0.06$ , once again assuming a distance of  $274 \pm 11 \text{ pc}$ . The final reduced  $\chi^2$  is 1.5. The error bars account for three independent contributions: uncertainties in the radial velocities, the angular diameters and the distance. The first was estimated using a bootstrap approach, while the others were estimated analytically (taking into account calibration correlation for interferometric errors): for  $p$ , the detailed error is  $p = 1.273 \pm 0.007_{\text{Vrad.}} \pm 0.020_{\text{interf.}} \pm 0.050_{\text{dist.}}$ . The error is dominated by the distance contribution.

## 3. the CLV of Polaris

Observations were undertaken using the same instrumental setup, using smaller baselines in 2003, 2004 and 2005. The pulsation of Polaris is not detectable at our level of precision: according to latest radial velocity survey, the pulsation is of the order of 0.4% in diameter (Moskalik & Gorynya, 2005) while our sensitivity is of the order of 1%. Polaris is also an astrometric and spectroscopic binary. According to precedent studies (Wielen et al., 2000; Evans et al., 2002), the contrast in the K band ( $\Delta K \approx 6.5$ ) is way to large to be detected by our instrument ( $\Delta K \approx 3$ ). Thus, our data taken at differ-



**Fig. 1.** Left: Radial Velocity smoothed using splines. A. Radial velocity data points from Bersier et al. (1994), as a function of pulsation phase (0-phase defined as the maximum of light) and spline fit (line). B. Residuals. Right:  $p$ -factor determination. A. Our angular diameter measurements (points) at medium medium baseline (250 m, crosses), and large baseline (313 m, circles). The continuous line is the integration of the 4-knots periodic cubic spline fitted to the radial velocities. Integration parameters:  $\theta = 1.475$  mas,  $p = 1.269$  and  $d = 274$  pc. B. Residuals of the fit

ent phases and baselines can be mixed in order to study the CLV of the star.

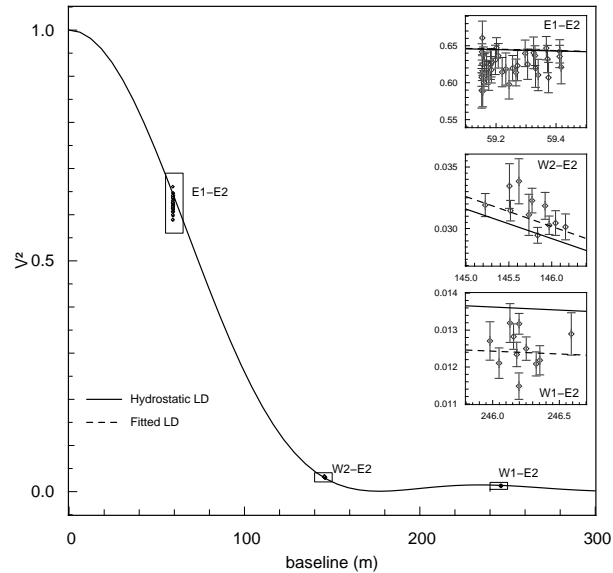
Claret (2000) tabulated CLV coefficients from hydrostatic ATLAS models. If we use following parameter  $T_{\text{eff}}=6000\text{K}$ ,  $\log g = 2.5$  and solar metallicity, we get from the database the following LD coefficients for the K band:  $a_1 = 0.6404$ ,  $a_2 = -0.1182$ ,  $a_3 = -0.2786$ ,  $a_4 = 0.1802$ . Using these, the only adjusted parameter in the fit to our interferometric data is the angular diameter of the star  $\theta = 3.152 \pm 0.003$  mas and the corresponding reduced  $\chi^2 = 4.5$ .

Because at highest spatial frequencies, in the second lobe of the visibility curve, the visibility is overestimated by the hydrostatic model (Fig. 2), this means that the CLV may be stronger. A stronger limb darkening would lower the second lobe. In order to investigate a possible departure of the CLV from the hydrostatic profil we chose a single parameter CLV law, the power law:  $I(\mu) = \mu^\alpha$  (Michelson & Pease, 1921; Hestroffer, 1997). The hydrostatic model corresponds to  $\alpha = 0.16$ : this model reproduces the hydrostatic visibility profile (from Claret's law and 4 parameters) at the  $10^{-3}$  (relative) level. A larger value of  $\alpha$  means a stronger CLV, while  $\alpha = 0$  corresponds to the uniform disk profile.

The best fit, leads to  $\theta = 3.189 \pm 0.005$  mas and  $\alpha = 0.26 \pm 0.01$ ; the reduced  $\chi^2$  is then 2.5. Based on the  $\chi^2$ , the fit is significantly better (previously 4.5). The CLV is stronger and the corresponding diameter is thus larger, as expected. However, one should notice that this model still fails to fit the mid-first lobe (see E1-E2 panel on Fig. 2, dash line and solid line overlap). The measured  $V^2$  are lower than anything predicted by any limb darkened law for the lowest baselines: in order to change the first lobe in shape, one has to invoke something larger than Polaris itself to disturb the lower spatial frequencies, while a change in CLD only affects the second lobe (highest spatial frequencies). Thus, we think that this strong CLD is not a realistic model.

#### 4. Conclusion

1. The  $p$ -factor of  $\delta$  Cep has been directly measured, using the interferometric Baade-Wesselink method, assuming a distance of  $d = 274 \pm 11$  pc (Benedict et al., 2002). We found a value of  $p = 1.27 \pm 0.06$ ;
2. Conversely, assuming a perfectly known  $p$ -factor, the formal error on the distance would have been 2%;
3. The center-to-limb variation of Polaris has been studied: neither an hydrostatic nor ad-



**Fig. 2.** Interferometric squared visibilities ( $V^2$ ) measurements of Polaris as a function of baseline. The continuous line is the expected hydrostatic profile while the dash line is an adjusted profile. Even if the latter one leads to a smaller reduced  $\chi^2$  and better agreement in the far first lobe and in the second lobe (baselines W2-E2 and W1-E2), there is still a significant departure at the smallest baseline (E1-E2).

justed limb darkened disk would fit the data. A fainter and larger component, a few times larger than the star itself, seems to disturb our measurements.

These two first points are detailed in Mérand et al. (2005).

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