



# Solar irradiance, luminosity and photospheric effective temperature

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**Abstract.** Models of the observed solar irradiance variations related to solar surface magnetism respond to about 95 % of reconstruction; the remaining may originate from non-magnetic effects. Sofia & Li (2000) have suggested that the activity cycle modulation of irradiance is primarily produced by a luminosity variation (both in terms of changes in radius,  $dR$ , and effective surface temperature,  $dT$ ). Taking into account the ellipticity of the solar shape and from the computed best fit of modelled to observed irradiance, we show here that  $dT$  must be no more than 1.2 K if combined with a radius variation amplitude  $dR = 10$  mas. Moreover, we show that the amplitude of solar irradiance modulation is very sensitive to small photospheric temperature variation. We underline a phase-shift (correlated or anticorrelated radius and luminosity variations) in the  $[dR, dT]$ -parameter plane ( $dT$  less than 0.05 K). From gravitational energy considerations, we obtain an upper limit on the amplitude of cyclic solar radius variations between 2.2 and 2.5 km. To understand the origin of the irradiance modulation, the asphericity-luminosity parameter  $w$  is computed:  $w \approx -3.7 \cdot 10^{-3}$ . Such a small (negative) value indicates that solar surface phenomena play a major role in producing the solar luminosity.

## 1. Photospheric temperature variation

Studies of temperature variation and related parameters for solar-like stars have shown effective temperature to be the likely dominant variable (Gray et al. 1996). Concerning the Sun, the variability of this parameter with the solar cycle is still a matter of study, but a change in  $T$  must have an effect on the irradiance and could explain part of irradiance mod-

ulation during the solar cycle. Detection of solar temperature variation with the required precision is very hard to set up. Gray & Livingston (1997), using the ratios of spectral line depths as indicators of stellar effective temperature, showed that the observed variation of photospheric temperature is in phase with the solar cycle. The amplitude  $dT = 1.5 \pm 0.2$  K found by these authors can account for nearly the entire variations of total solar irradiance during the solar cycle. However, spectroscopic measurement of variation in the solar temperature has proved to be elusive: the  $dT$  estimate of Gray

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& Livingston (1997) depends on a calibration coefficient. On theoretical grounds, Caccin et al. (2002) obtained  $dT \approx 3$  K, which is twice the experimental value (they noted that the gravitational acceleration  $g$  was not the same for all the stars observed and they found a  $g$ -dependence within the spectral line depths).

Recently, monitoring the spectrum of the quiet atmosphere at the center of the solar disk during thirty years at Kitt Peak, Livingston and Wallace (2003) and Livingston et al. (2005) have shown a “nearly immutable basal photosphere temperature” during the solar cycle within the observational accuracy, i.e.  $dT \approx 0$  with error bars of  $\sim 0.3$  K. In this study, we emphasize the sensitivity of irradiance changes to surface effective temperature variation.

## 2. Reconstruction of solar irradiance

When reconstructing solar irradiance variations, the choice of a suitable model depends on the time-scales considered. For example, surface magnetic features, i.e. spots, faculae and magnetic network, mostly produce variations of total solar irradiance on periods from hours to months. On such grounds, Krivova et al. (2003) presented a model in which solar surface magnetism is responsible for all total irradiance changes. Such a “surfacic” model explains approximately 95 % of the variations, but the 5 % left must be justified. On longer time-scales, Sofia & Li (2000) proposed that the observed total solar irradiance variations (in phase with the activity cycle) are due to changes in solar luminosity,  $dL$ , with structural reajustments of the solar interior induced by changing magnetic fields. This thesis is supported by other authors such as Kuhn (1996, 2003), Kuhn et al. (1998) or Kuhn & Armstrong (2004), who showed that faculae, which are supposed to increase the irradiance, can be dark in the disk center when measured in continuum wavelengths. Following Li et al. (2003), we state that observed irradiance variations may be due to: a) internal solar structure changes, b) solar photospheric temperature changes, c) solar radius (more precisely shape) changes.

We propose here a simple model taking into account the two last proposals, i.e. effective temperature,  $dT$ , and outer shape area,  $dA$ , variations (without any magnetic parameter). In order to determine the relevant solar layers at which irradiance variations may take place (Gough 2001), one needs to compute the parameter  $w$  (Sofia & Endal 1980), called *asphericity-luminosity* (Lefebvre & Rozelot 2004):  $w = (dR/R)/(dL/L)$ .

## 3. Radius variations and luminosity changes

Our model can be found in Fazel et al. (2005), based upon a luminosity variation of the form:  $L_{model} = L_0 + \sin(2\pi t/P + \phi)dL$  where parameters have their usual meanings. Two models of the solar shape, that is an ellipsoidal surface and a distorted shape developed on a basis of Legendre polynomials, were investigated, but results were similar. Hence, we present the case of an ellipsoidal shape only, for which luminosity temporal variations will be reproduced by  $dR$  in the range [10, 200] mas ( $dR = 0$  being the case of a sphere of radius  $R_\odot$ ).

### 3.1. Solar outer shape

The outer shape of the Sun is defined by the surface radius  $R$ , as a function of colatitude ( $\theta$ ), taking into account the non-uniform velocity rate and the non-homogeneous distribution of the mass inside the Sun. Latitudinal variations of the solar radius (Lefebvre & Rozelot 2003) can be expressed as:  $R(\theta) = R_{sp} \left[ 1 + \sum_{n, even} c_n P_n(\theta) \right]$  where  $R_{sp}$  is the radius of the best sphere fitting both  $R_{pol}$  and  $R_{eq}$  (note that  $R_{sp}$  is different from the semi-diameter of the Sun,  $R_\odot$ );  $c_n$  the shape coefficients (“asphericities”) and  $P_n(\theta)$  the Legendre polynomials of degree  $n$  ( $n$  being even due to axial symmetry). We need to compute the solar surface area,  $A$ , from  $R(\theta)$ , which is:  $A = 4\pi \int_0^{\pi/2} R(\theta) \left[ 1 + \left( \frac{dR(\theta)}{d\theta} \right)^2 \right]^{\frac{1}{2}} d\theta$ .

The best available values of the shape coefficients are  $c_2 \in [-2 \times 10^{-6}, -1 \times 10^{-5}]$  and  $c_4 \in [6 \times 10^{-7}, 1 \times 10^{-6}]$  (Rozelot et al. 2004). For

convenience, we express these results in fractional parts of the best sphere area which corresponds to the radius  $R_{sp}$ . Computations were carried up to  $n = 4$ , leading to  $dA(c_2, c_4)/A_{sp} \in [1.82 \times 10^{-6}, 6.37 \times 10^{-6}]$ , where the minimum corresponds to the lower bound of  $c_2$  and  $c_4$ , while the maximum corresponds to their upper bound. Those values can be compared to the ones deduced from an ellipsoid. The area of an ellipsoid of  $R_{eq} = a = 6.959918 \times 10^8$  m,  $R_{pol} = b = 6.959844 \times 10^8$  m and  $c = \sqrt{a^2 - b^2}$  is:

$$A_{ell} = 2\pi \left[ a^2 + (ab^2/c) \ln \frac{a+c}{b} \right]. \quad (1)$$

Assuming  $dR_{sp} = da = db (= dc)$  varies in the interval  $[0, 200]$  mas (Rozelot & Lefebvre, 2003) we find  $dA/A_{ell} \in [3.08 \times 10^{-6}, 6.16 \times 10^{-5}]$ .

### 3.2. Solar effective temperature variation

We computed our model for different pairs of  $dT$ , i.e.  $[0, 5]$  K and  $dR$ , i.e.  $[0, 200]$  mas. The corresponding deviations from the first component in the Singular Spectrum Analysis,  $RC1$ , were computed over two solar cycles and the best fit is  $dT = 1.2$  K at  $dR = 10$  mas. This estimate of the effective temperature variation, is close to that obtained by Gray and Livingston (1997), Caccin et al. (2002), Li et al. (2003) as discussed in the first section. We further noticed that computed irradiance was very sensitive to surface effective temperature. For small values of  $dT$ , i.e.  $[0, 1.5]$  K, a phase transition curve in the  $[dR, dT]$ -plane given by:  $[dT_{critical} = 5.10^{-8} dR^2 + 4.10^{-4} dR + 5.10^{-4}]$  where  $dT_{critical}$  belongs now to  $[0, 0.085]$ . In the  $[dR, dT]$ -plane, points under the phase transition curve correspond to anticorrelated solar radius variations with respect to irradiance ones, and conversely for points above the curve. Observations show a correlation between irradiance variations and the solar activity cycle. Hence, we see how precise measurements of surface effective temperature variation over a solar cycle are crucial to deduce, observationally, if solar radius variations are correlated (or anti) with the solar cycle, i.e. above (or below) the phase transition curve.

## 4. Effects of the gravitational energy variations on $R$ and $L$

In order to investigate the consequences of gravitational energy changes on solar radius variations, Callebaut et al. (2002) used a self-consistent approach, and they computed  $\Delta R/R$  and  $\Delta L/L$  associated with the energy responsible for the expansion of the upper layer of the convection zone. We use here the same formalism, but we consider an ellipsoidal surface (Eq. 1). Let  $\alpha$  be the fractional radius ( $0 < \alpha < 1$ ): if the layer above  $\alpha R$  expands, the expansion is zero at  $\alpha R$  and is  $\Delta R$  at  $R$ . The increase in height at a radial distance  $r$  in the layer interval  $(\alpha R, R)$ , with  $R = R_{sp} = \sqrt[3]{R_{eq}^2 R_{pol}}$ , is given

by  $h(r) = \frac{(r-\alpha R)^n \Delta R}{R^n (1-\alpha)^n}$ , where  $r$  is the usual radial coordinate and  $n = 1, 2, 3, \dots$  is the order of development. The relative increase in thickness for an infinitesimally thin layer at  $r = R_{sp}$  is obtained from  $h(r)$ , and the relative change in temperature can be expressed in terms of the relative change in radius. Applying this to an ellipsoid and assuming  $dR_{eq} = dR_{pol} = dR_{sp}$ :

$$\frac{\Delta L}{L} = - \left[ \frac{4n(\gamma - 1)}{1 - \alpha} + \left( 2 + \frac{3c}{ab^2} + \frac{(b - a^2)c}{ab(a^2 - b^2)} + \frac{b(\frac{1}{a+c} - 1)}{(a+c) \ln(\frac{a+c}{b})} \right) \right] \frac{\Delta R_{sp}}{R_{sp}} \quad (2)$$

where  $n = 1$ ,  $\gamma = 5/3$ , and  $\alpha \approx 0.96$ . Our theoretical approach, through Eq. 2, implies that a decrease of  $R_{sp}$  corresponds to an increase of  $L$  in the solar cycle, that is to say solar radius variations are anticorrelated with solar luminosity variations.

The usual adopted value  $\Delta L/L = 0.0011$  (Dewitte et al. 2005, using TSI composite data from 1987 to 2001, mean value  $L_0 = 1366.495$  W/m<sup>2</sup>) leads to  $\Delta R_{sp}/R_{sp} = 3.5 \times 10^{-6}$  (or  $\Delta R_{sp} = 2.5$  km). Fröhlich (2005), reanalyzing composite TSI data from 1978 to 2004 (mean value  $L_0 = 1365.993$  W/m<sup>2</sup>), found  $\Delta L/L = 0.00073$ , which yields  $\Delta R_{sp}/R_{sp} = 3.1 \times 10^{-6}$  (or  $\Delta R_{sp} = 2.2$  km). Our absolute estimate of  $\Delta R_{sp}/R_{sp}$  agrees with that of Antia (2003), who used  $f$ -mode frequencies data sets from MDI (from May 1996 to August 2002) to estimate the solar seismic radius, 2.09 km (with an accuracy of about 0.6 km).

**Table 1.** The asphericity-luminosity parameter  $w$ .

Sofia & Endal (1980)	$-7.5 \cdot 10^{-2}$	
Dearborn & Blake (1980)	$5.0 \cdot 10^{-3}$	
Spruit (1992)	$2.0 \cdot 10^{-3}$	
Gough (2001)	$2.0 \cdot 10^{-3}$	(1)
	$1.0 \cdot 10^{-1}$	(2)
Lefebvre & Rozelot (2004)	$-7.5 \cdot 10^{-2}$	

## 5. Asphericity-luminosity parameter

The asphericity-luminosity parameter  $w$  can now be computed from the above results based on Dewitte et al. (2005) and Fröhlich (2005) data sets:  $w \approx -3.7 \cdot 10^{-3}$ . The sign and the value of  $w$  are relevant as they characterize solar luminosity production. A minus sign means that radius variations are anticorrelated with luminosity ones. For a given  $dL/L$ , small values of  $dR/R$  lead to a small  $w$  and that means  $L$  is produced in the upper-most layers (Gough 2001); conversely, the luminosity production mechanism would be located in deeper solar layers: see respectively note 1 and 2 in Table 1 (where it seems that some authors quoted here have provided absolute values):

## 6. Conclusions

From the point of view of observations, we obtained constraints on surface effective temperature and related radius variations, even if magnetic field were neglected. The best fit over nearly two solar cycle gives  $dT = 1.2 \text{ K}$  at  $dR = 10 \text{ mas}$ . However, we underlined a phase-shift in the  $[dR, dT]$ -parameter plane between the solar radius and irradiance variations, certainly due to the ellipticity of the solar shape. We show that solar irradiance variations modeling is very sensitive to small  $dT$ , as suggested by Livingston et al. (2005); such a finding support our conclusion. Noting that observed irradiance variations are in phase with solar cycle, the faint radius variations with time would be theoretically, according to the amplitude of the temperature modulation: anticorrelated (for  $dT \approx 0.005 \pm 0.005 \text{ K}$  (or correlated if  $dT$  greater than  $0.01 \text{ K}$ ).

From the point of view of theory, we applied the Callebaut et al. (2002) method to an

ellipsoidal Sun. We could conclude to solar radius anticorrelated with solar luminosity variations. We further obtained an upper limit on the amplitude of  $dR$ , 2.2-2.5 km. Our estimate agrees with those based on helioseismology. We then deduced the corresponding value of the asphericity-luminosity parameter,  $w = -3.7 \cdot 10^{-3}$ . Its sign reflects the  $(dR, dL)$  anticorrelation and its value comforts an upper layer mechanism as the source of luminosity variations. We conclude that we need more precise measurements of  $(dT, dR)$  during the solar cycle (and on smaller time-scales) to confront the observational with the theoretical approach on solar radius versus irradiance variations.

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