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A necessary extension of the flux transport model

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Abstract. Customary two-dimensional flux-transport models for the evolution of the magnetic field at the solar surface do not account for the radial structure and the volume diffusion of the magnetic field Schrijver et al. (2002). When considering the long-term evolution of magnetic flux, this omission can lead to an unrealistic long-term memory of the system and to the suppression of polar field reversals. In order to avoid such effects, we propose an extension of the flux-transport model by a linear decay term derived consistently on the basis of the eigenmodes of the diffusion operator in a spherical shell. The value of the volume diffusivity η associated with this term can be estimated to be in the range 50–100 km^2s^{-1} by considering the reversals of the polar fields in comparison of flux-transport simulations with observations. We show that the decay term prohibits a secular drift of the polar field in the case of cycles of varying strength, like those exhibited by the historical sunspot record.

1. Introduction

The large scale evolution of the magnetic flux distribution at the photosphere can be described as a result of the emergence of bipolar magnetic regions and the transport of the corresponding radial magnetic flux by the horizontal flows due to convection, differential rotation and meridional circulation (e.g., Leighton 1964; DeVore et al. 1984; Wang et al. 1989b; van Ballegooijen et al. 1998; Schrijver 2001; Mackay 2002; Baumann et al. 2004). When applying this flux transport model to multiple solar cycles, the secular evolution resulting from the simulations is not in agreement with the observations: activity cycles of varying strength lead to a drift of the polar fields and even to the disappearance of polar reversals (Schrijver et al. 2002; Wang et al. 2002).

This disagreement of flux transport models with the observations arises from ignoring the vectorial nature and the radial structure of the magnetic field and, particularly, from omitting the part of the diffusion operator depending on the radial coordinate (see also Dikpati and Choudhuri 1994). This leads to an unwanted long-term memory of the surface flux since the decay time of a dipolar surface field increases strongly in the presence of a poleward meridional flow. We therefore suggest to extend the flux transport model by a modal version of the decay term proposed by Schrijver et al. (2002). We derive this term by considering the decay of the eigenmodes of the (volume) diffusion operator in a spherical shell. This leads to a consistent decay rate for each eigenmode, which can simply be incorporated in a code based upon expansion of the magnetic field into surface harmonics. The volume diffusion coefficient remains as a free parameter, which can be estimated by comparison of simulation results with observational constraints (like polar field reversals).

2. Decay modes of a poloidal field in a spherical shell

The time evolution of a magnetic field is described by the pure diffusion equation if there are no systematic flows present,

$$\frac{\partial \mathbf{B}}{\partial t} = -\eta\, \nabla \times (\nabla \times \mathbf{B})\,, \tag{1}$$

where η is the (constant) magnetic diffusivity. The full magnetic field vector can be written as the sum of a poloidal and a toroidal part. In the following it is sufficient to consider the decay of a poloidal magnetic field in a spherical shell (representing the solar convection zone) since the flux transport model assumes a purely radial magnetic field at the photosphere. The evolution of this field is described by Eq. (1), where **B** now represents a poloidal field. In spherical geometry the poloidal magnetic field can be represented by a scalar function, $S\,(r, \theta, \phi, t)$, as

$$\mathbf{B} = -\nabla \times (\mathbf{r} \times \nabla S) = -\mathbf{r} \vartriangle S + \nabla \frac{\partial}{\partial r}(rS)\,, \tag{2}$$

where $\vartriangle$ is the Laplace operator in spherical coordinates and **r** is the radius vector (Bullard and Gellman 1954; Krause and Rädler 1980). Inserting this field representation into Eq. (1) and choosing appropriate boundary conditions (e.g. radial magnetic field at the surface and no field lines entering the ideally conducting core at the bottom of the convection zone), a general solution for S can be written as a decompostion into orthogonal decay modes (Elsasser 1946),

$$S\,(r, \theta, \phi, t) = \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} \sum_{m=-l}^{l} R_{ln}(r)\; Y_{lm}(\theta, \phi)\; T_{ln}(t)\,. \tag{3}$$

Here, we omit the monopole term (l = 0). Y_{lm} are the spherical surface harmonics. Separation of variables leads to an exponential time dependence

$$T_{ln}(t) = \exp(-\eta\, k_{ln}^2 t)\,, \tag{4}$$

where $1/\eta\, k_{ln}^2$ is the decay time of the mode characterized by the wave numbers l and n. The dipole mode ($l = 1$) with $n = 0$ has the longest decay time of about 5 years. The decay of the higher radial modes is much more rapid. It is therefore justified to consider only the most slowly decaying modes with $n = 0$ in the decay term for the surface transport model.

3. Extension of the surface flux transport model

We extend the surface flux transport model (DeVore et al. 1985; Wang et al. 1989a) by a decay term describing the volume diffusion of the poloidal magnetic field in the convection zone.

$$\begin{aligned}\frac{\partial B_r}{\partial t} \;&=\; -\omega(\theta)\frac{\partial B_r}{\partial \phi} - \frac{1}{R_\odot \sin\theta}\frac{\partial}{\partial \theta}\left(v(\theta) B_r \sin\theta\right) \\ &+\; \frac{\eta_{\mathrm{h}}}{R_\odot^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial B_r}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 B_r}{\partial\phi^2}\right] \\ &+\; Q(\theta, \phi, t) - D(\eta)\,,\end{aligned} \tag{5}$$

where $\omega(\theta)$ is the angular velocity of the photospheric plasma, $v(\theta)$ is the meridional flow velocity on the solar surface, $Q(\theta, \phi, t)$ is a source term describing the emergence of new magnetic flux, and η_{h} is the turbulent magnetic diffusivity associated with the nonstationary supergranular motions on the surface. We specify the decay term $D(\eta)$ on the basis of the decay modes in a spherical shell as determined in the previous section. To this end, we expand the instantaneous radial surface magnetic field into spherical harmonics,

$$B_r\,(R_\odot, \theta, \phi, t) = \sum_{l=1}^{\infty} \sum_{m=-l}^{m=+l} c_{lm}(t)\; Y_{lm}\,(\theta, \phi)\;. \tag{6}$$

The volume diffusion leads to exponential decay of each of these modes with the corresponding decay times, $\tau_{ln} = 1/(\eta\, k_{ln}^2)$, depending on the radial structure of the magnetic field. Since the latter is unknown in the framework of the flux transport model, we only consider the mode $n = 0$. This is justified by the fact that all higher modes ($n \geq 1$) decay much more rapidly, so that they do not affect the long-term,

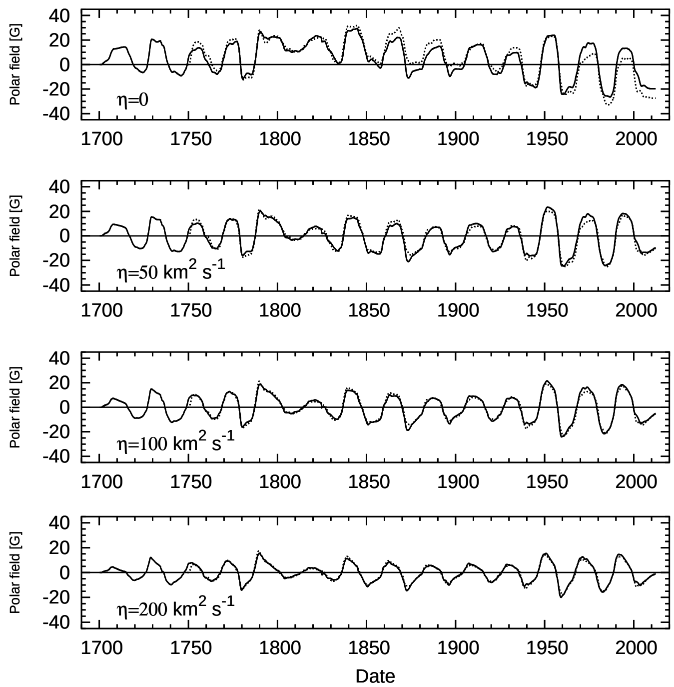

Fig. 1. Evolution of the north polar field (average above 75° latitude) for a sequence of synthetic cycles with the flux emergence rate taken proportional to the sunspot numbers since 1700 (full line) and 1750 (dotted line), respectively, for different values of the volume diffusivity, η. The top panel (η=0) shows the strong drift of the polar field resulting from the secular increase of solar activity during the last century (see also Schrijver 2001). Finite values of η reduce the secular effect and lead to more symmetric variations, as observed; the volume diffusion term also largely removes the dependence of the results on the initial conditions (difference between full and dotted lines).

large-scale behaviour of the surface field that the flux transport models aim to describe. We therefore write $D(\eta)$ as

$$D(\eta) = \sum_{l=1}^{\infty} \sum_{m=-l}^{m=+l} \frac{c_{lm}(t)}{\tau_{l0}} \, Y_{lm}(\theta, \phi) \ . \tag{7}$$

Note that the decay times do not depend on the azimuthal wave number, m, of the mode. The modal decay term given by Eq. (7) is particularly simple to implement in flux transport codes based upon an expansion into surface harmonics (e.g., van Ballegooijen et al. 1998; Mackay 2002; Baumann et al. 2004).

The decay term, $D(\eta)$, depends on the turbulent volume diffusivity, η, which will generally differ from the diffusivity $\eta_{\rm h}$ related to the flux transport by the horizontal surface motions of supergranulation.

4. Application of the extended flux transport model

One of the main problems with the original formulation of the flux transport model is the too long memory of the system. In the case of cycles with varying strength this can lead to a secular drift of the polar fields and a suppression of polar field reversals. In order to illustrate this, we consider the secular variation of solar activity in the historical record of sunspot numbers (see also Schrijver 2001). Using the record of sunspot numbers since 1700, we have simulated a series of solar cycles with the flux transport code. We have used the 'standard' parameters (butterfly diagram of emerging bipolar regions, tilt angles, polarity rules, transport parameters, etc.) of Baumann et al. (2004), except for taking the emergence rate of the bipolar regions proportional to the sunspot numbers, thereby also using cycle lengths in agreement with the observed record. The evolution of the polar fields according to simulations using the unmodified and the extended flux transport model for various values of η is shown in Fig. 1. In order to evaluate the effect of the arbitrary initial condition (zero surface field) we also show a run starting from 1750, represented by the dotted line. In the case $\eta = 0$ (no volume diffusion), the long memory of the system leads to a drift of the polar fields in the 20$^{\text{th}}$ century due to the secular increase of solar activity, so that the oscillations become very asymmetric, in striking contrast to the observed evolution of the polar fields. Finite values for η of the order 100 km^2s^{-1} lead to more symmetric oscillations and a suppression of the unrealistic drift. The amplitude of the simulated polar field in the last cycles for a value of $\eta = 100\ \text{km}^2\text{s}^{-1}$ is roughly consistent with the published observational data, which indicate amplitudes for the field strength of 10–20 G (e.g., Arge et al. 2002; Dikpati and Choudhuri 1994; Durrant et al. 2004). Comparing the two runs with different starting times (full line and dotted line), we find that the long memory of the system in the case $\eta = 0$ leads to a significant difference between these two runs. For $\eta \neq 0$, on the other hand, there is almost no dependence on the initial condition after a few cycles.

5. Conclusion

We have shown that a modified (modal) version of the decay term first introduced into the flux transport model by Schrijver et al. (2002) can be derived consistently from the volume diffusion process that is neglected in the standard flux transport models. Including this term removes the unrealistic long memory of the system and thus prohibits a secular drift or a random walk of the polar fields in the case of activity cycles with variable amplitude. The value of the (turbulent) magnetic volume diffusivity, η, can be estimated by considering the reversal times of the polar field relative to the preceding activity minimum and by comparing with direct observations or values inferred from proxy data. We find that values of η in the range 50–100 km^2s^{-1} are consistent with the observational constraints. This is also the order of magnitude suggested by simple estimates based on mixing-length models of the convection zone. With this consistent extension and improvement of the model, flux transport simulations of the large-scale magnetic field on the solar surface over many activity cycles can be carried out.

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