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Planetesimal dynamics in self-gravitating protoplanetary discs

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Abstract. The interaction between a gaseous protostellar disc and the embedded planetesimals through a drag force generally causes the latter to migrate inwards, the migration rate being very rapid (of the order of 10^3 yrs) for metre-size objects. Here we show how, in the presence of a spiral structure caused by the disc self-gravity, the drag force has the effect of concentrating significantly the planetesimals around the peaks of the gas density. Metre-size particles, in this configuration, can enhance their concentration by more than two orders of magnitude, therefore significantly accelerating their growth and the formation of the cores of giant planets.

Key words. accretion discs – stars: pre-main-sequence – planetary systems: formation planetary systems: protoplanetary discs

1. Introduction

The formation of both terrestrial planets and the cores of gas/ice giant planets is thought to occur through the collisional accumulation of planetesimals. In the case of gas giant planets, a gaseous envelope is accreted once the core has become sufficiently massive. Since this must occur while there is still enough gas in the circumstellar disc, and since observations suggest that most stars lose their gaseous discs within $\sim 10^7$ years, it is generally accepted that gas giant planets have to form within $\sim 10^7$ years. However, the standard core accretion models of giant planet formation (Pollack et al. 1996) suggest formation times that could easily exceed disc lifetimes.

An additional difficulty in the standard core accretion model is the growth of kilometre sized planetesimals from, initially, micron sized dust grains. For standard disc geometries, the pressure gradient in the circumstellar disc tends to be negative and causes the gas to orbit with sub-Keplerian velocities. The dust, which is not affected by the gas pressure gradient, and the gas therefore orbit with different

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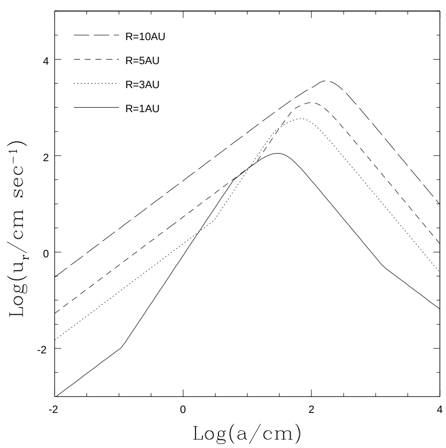

Fig. 1. Radial drift velocity as a function of planetesimal size.

velocities and the resulting drag force causes the dust grains to drift inwards at a rate that depends on their size (Weidenschilling 1977). For small sizes, the dust grains are essentially coupled to the disc gas and the radial drift velocity is consequently small. For large sizes, the grains are decoupled from the gas, move in nearly Keplerian orbits, and again have small radial drift velocities. Particles with intermediate sizes can, however, have large inward radial velocities. Although the exact size range depends on the circumstellar disc properties, the maximum radial velocity may easily exceed 10^3cm/s and is normally thought to occur for objects with sizes between 1cm and 1m (see Fig. 1). In this size range, therefore, these objects may drift inwards before they can become large enough to decouple from the disc gas.

It is quite likely that protostellar discs are self-gravitating in their early stages (Lin & Pringle 1990; Lodato & Bertin 2001). If so, the disc then becomes susceptible to the growth of non-axisymmetric spiral structures which can transport angular momentum very efficiently (Lodato & Rice 2004). In the presence of such spiral structures, the gas pressure gradient, which can be large, changes from positive on one side of the structure to negative on the other, resulting in both super- and sub-Keplerian gas velocities. The gas drag force then causes dust grains to drift both radially inwards and outwards, depending on whether the local gas velocity is super-Keplerian or sub-Keplerian. The net effect is that the dust drifts towards the density maxima, where the pressure gradient is zero (see also Haghighipour & Boss 2003).

In this paper we present the results of three-dimensional, global simulations of self-gravitating accretion discs in which we include both gas and dust, coupled via a drag force (Weidenschilling 1977). The gaseous disc is maintained in a state of marginal gravitational instability, achieved by letting the disc cool down (through a simple parametrisation of the cooling function; Gammie 2001 and Rice et al. 2003). In this way a quasi-steady spiral structure develops in the disc and is maintained during several dynamical time-scales. We are therefore able to follow the process of concentration of the planetesimals in the spiral arms. We have performed several simulations, considering planetesimals of different sizes. We find that, for a given size range, the planetesimals are indeed able to reach high concentrations, in some regions attaining densities comparable to the gas. This could significantly enhance the coagulation of planetesimals into larger bodies, by increasing the planetesimal collision rates and/or by making the planetesimal sub-disc become gravitationally unstable.

2. Generalities and numerical method

We consider a system comprising two interpenetrating discs: a "gas" disc, that is evolved using the standard hydrodynamical equations of motion, and a "planetesimal" disc, that is considered as a collection of test particles evolved under the influence of gravitational and drag forces alone. Both the "gas" disc and the "planetesimal" disc rotate around a central protostar of mass $M_\star$, which we take to be equal to $1M_\odot$.

The gas is supported by pressure, and therefore, in centrifugal balance, it rotates at a sub-Keplerian velocity. We assume that the gas and the planetesimals are coupled through a drag force, as described by Weidenschilling

(1977) (see also more details in Rice et al. 2004).

The three-dimensional gaseous disc used in these simulations is modelled using smoothed particle hydrodynamics (SPH), a Lagrangian hydrodynamics code (see Monaghan 1992). The gas disc consists of 250,000 SPH particles. The central star is modelled as a point mass onto which gas particles may accrete if they approach to within the sink radius (e.g., Bate et al. 1995), here taken to be 0.25 au. Both the central point mass and the gas particles use a tree to determine gravitational forces, and to determine the gas particle neighbours.

The small planetesimals are modelled using an additional type of particle. These particles are, as far as the gas simulation is concerned, massless (i.e., in these simulations we neglect the self-gravity of the planetesimal disc and the back reaction of the drag force on the gas). They experience only gravitational forces (from the central star and from the disc gas) and are coupled to the disc gas via the drag force, as discussed above. In each simulation performed here, we use 125,000 test particles to represent a planetesimal disc which we assume contains particles of a single size.

The gas disc is assumed, in the absence of any cooling mechanism, to have an adiabatic equation of state with an adiabatic index of $\gamma = 5/3$. The gas disc is initially evolved, in the absence of test particles, by imposing a cooling term which is chosen such that the disc ultimately settles into a quasi-steady, self-gravitating state. The test particles are then added and the simulation is evolved for an additional outer rotation period.

3. Results

In this work we use the results of the numerical simulations by Lodato & Rice (2004) of the dynamics of self-gravitating gas discs as an input for our two-component gas-planetesimals disc simulations.

It is well known that the onset of gravitational instabilities in the disc is determined by the value of the parameter $Q = c_s\kappa/\pi G\Sigma$, where κ is the epicyclic frequency and Σ the gas surface density. If Q is smaller than a threshold value of order unity, the disc quickly develops a spiral structure on the dynamical timescale. The presence of the spiral influences strongly the thermal evolution of the disc, in that it provides a source of effective heating. Since Q is proportional to the thermal speed c_s, in the absence of any cooling Q would rapidly become relatively large and the spiral structure would vanish. However, if some cooling is present, a self-regulated state can be achieved where the heating provided by the spiral structure balances the external cooling, leading to a long-lasting spiral.

The simulations by Lodato & Rice (2004) indeed show the effectiveness of the self-regulation process. The disc extends from 0.25 au to 25 au, and is characterized initially by a surface density profile $\Sigma \propto R^{-1}$ and a temperature profile $T \propto R^{-1/2}$. The exact surface density is determined by specifying a total disc mass, and the temperature is determined by specifying that the Toomre Q parameter has an initial value of 2 at the outer edge of the disc. The temperature profile, however, is rapidly modified by the competing heating and cooling processes operating in the disc, as discussed above. At the end of the simulations a self-regulated state is achieved with an almost constant profile of Q, with a value close to unity. The spiral structure obtained in this way is a quasi-steady feature lasting for at least several thermal timescales (i.e., at least until the end of the simulations). The left panel of Fig. 2 shows the final disc structure for a disc whose total mass is $M_{\rm disc} = 0.25 M_{\odot}$. The image shows the logarithm of Σ, with a colour scale covering $1 < \log(\Sigma/{\rm g\ cm^{-2}}) < 4.7$.

In order to cover a wide range of planetesimal sizes we have considered separately planetesimal sizes of 50 cm and 1000 cm. Based on the radial velocity estimates shown in Fig. 1, we expect the 50 cm sized planetesimal to have the largest radial drift. In addition to the 50 and the 1000 cm cases, we have also performed one simulation in which no drag force was included, so as to provide a direct measure of the effect of gas drag on the evolution of the planetesimals.

The planetesimal disc initially extends from $R = 2$ au to $R = 20$ au. The surface den-

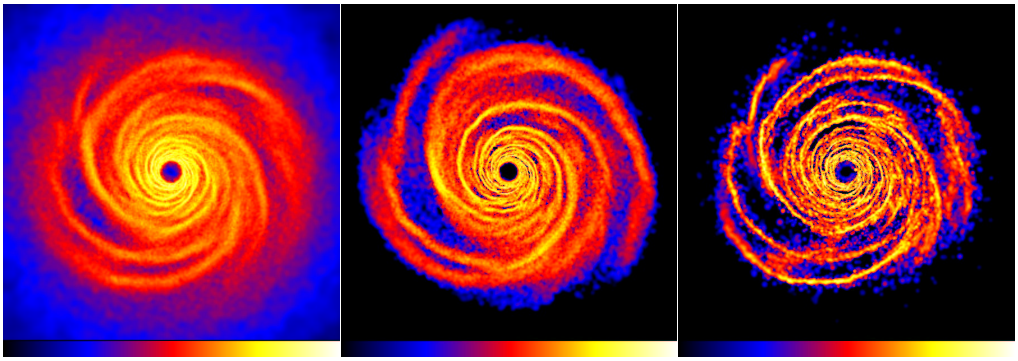

Fig. 2. Surface density structure at the end of the simulation for the gas (left panel), the 1000 cm size planetesimals (middle panel), and the 50 cm size planetesimals (right panel). The panels shows the logarithm of the surface density Σ with the scale covering $1 < \log(\Sigma/\mathrm{g\ cm}^{-2}) < 4.7$. The size of the box is 50 au across.

sity profile of the planetesimals $\Sigma_{\rm p}$ was taken to be proportional to R^{-1}. Since we neglect the planetesimals' self-gravity and the back reaction of the drag force on the gas, the actual value of the planetesimal disc surface density does not influence the results of the simulations (the planetesimal SPH particles are just a "tracer" of the evolution of the solid bodies in the gas disc). However, in order to present illustration values in the analysis of our results, we will assume that the initial ratio of the planetesimal to gas surface densities is 0.01 in all cases.

Initially all the planetesimals are located in the $z = 0$ plane. However, during the simulation, the random motions induced by the gravitational instabilities rapidly stir the planetesimal disc up, so that eventually it acquires a finite thickness $H_{\rm p}$ slightly smaller than the gas disc thickness H.

The middle and right panels of Fig. 2 show the surface density structure of the planetesimal discs, one outer orbital time after the introduction of the planetesimals in the simulations (i.e., after $\sim$ 125 yrs). At this stage, most of the disc has evolved for several dynamical timescales, so that any initial transient features have disappeared. The figure refers to the cases where the planetesimal sizes were 1000 cm (middle panel) and 50 cm (right panel). These plots clearly show how the planetesimal evolution changes with changing planetesimal size. As expected, the 50 cm planetesimals are strongly influenced by the gas drag and display a spiral pattern with very thin spiral arms, indicating that the planetesimals are concentrated at the bottom of the potential. The effect is reduced in the 1000 cm case, where the spiral structure in the planetesimal disc is similar to that of the gas disc, indicating that the planetesimals are pushed into the spirals mainly because of the gravitational field (note that since the planetesimals have no pressure support, we expect the spiral arms to be slightly thinner even if no drag force is introduced).

The concentration of planetesimals is further illustrated in Fig. 3 which shows the distribution of the ratio of the planetesimal volume density $\rho_{\rm p}$ to the gas volume density ρ. The planetesimal volume densities were determined by assuming, as mentioned earlier, that the initial planetesimal to gas surface density ratio was 0.01. This plot shows that the volume density of the 50 cm planetesimals (solid line) may increase to a value similar to the gas density. The volume density ratio for the 1000 cm particles (dotted line), on the other hand, barely exceeds 0.1 and in most regions has a value below 0.05. Given an "unperturbed" density ratio of 0.01, this means that the volume density of the 1000 cm particles is almost never enhanced by more than a factor of 10.

The concentration of planetesimal, due to the combined effect of gas drag and gravity, that we find in our simulations can have a sig-

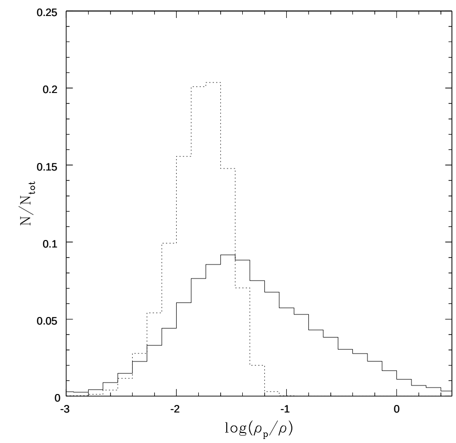

Fig. 3. Distribution of volume density ratios (planetesimal/gas) for the two different sizes considered (solid line: 50 cm; dotted line: 1000 cm) at the end of the simulation.

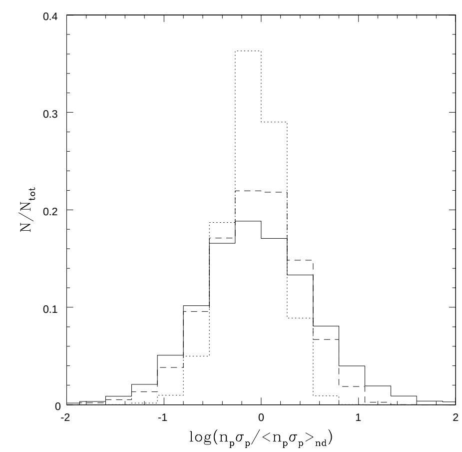

Fig. 4. Distribution of $n_\mathrm{p}\sigma_\mathrm{p}/\langle n_\mathrm{p}\sigma_\mathrm{p}\rangle_\mathrm{nd}$ for the 50 cm case (solid line), for the 1000 cm case (dashed line) and for the no drag simulation (dotted line).

nificant effect on the process of coagulation of planetesimal into larger bodies. This can occur either by increasing the planetesimal collision rate and/or by making the planetesimal sub-disc gravitationally unstable. The collision rate of planetesimals is proportional to $n_\mathrm{p}\sigma_\mathrm{p}$, where $n_\mathrm{p} = \rho_\mathrm{p}/m_\mathrm{p}$ is the planetesimal number density and σ_p is their velocity dispersion. In order to assess the effect of gas drag on the collision rate, we have first computed the azimuthally averaged value of $n_\mathrm{p}\sigma_\mathrm{p}$ from the simulation with no drag force, $\langle n_\mathrm{p}\sigma_\mathrm{p}\rangle_\mathrm{nd}$, as a function of radius. We have then computed, for every planetesimal SPH particle in both the 50 cm case and the 1000 cm case, the ratio $n_\mathrm{p}\sigma_\mathrm{p}/\langle n_\mathrm{p}\sigma_\mathrm{p}\rangle_\mathrm{nd}$, where the average value is computed at the same radial location in the disc. If the gas drag had no effect on the collision rate, the distribution of $n_\mathrm{p}\sigma_\mathrm{p}/\langle n_\mathrm{p}\sigma_\mathrm{p}\rangle_\mathrm{nd}$ would be strongly peaked around unity. Fig. 4 shows the distribution of $n_\mathrm{p}\sigma_\mathrm{p}/\langle n_\mathrm{p}\sigma_\mathrm{p}\rangle_\mathrm{nd}$ that we have obtained in the three cases (no drag force: dashed line; 1000 cm size: dotted line; 50 cm size: solid line). As expected, the distribution for the no-drag simulation is strongly peaked around unity. Fluctuations of $n_\mathrm{p}\sigma_\mathrm{p}$ not related to the gas drag result only in an increase of the collision rate by no more than a factor $\sim$ 6. In contrast, the introduction of the gas drag leads to a broader distribution of collision rates, especially for the 50 cm case where the distribution has a tail extending more than two orders of magnitude above the average value. Depending on how well particles of this size stick together during collisions, this enhanced collision rate could play an important role in the growth of larger planetesimals.

A planetesimal surface density enhancement of a factor $\sim$ 20 may also be sufficient to make the planetesimal sub-disc gravitationally unstable (e.g., Youdin & Shu 2002). This is exactly the range of concentrations that we achieve in 50 cm simulation. However, since we have neglected the planetesimals' self-gravity, we are not able to obtain a gravitational instability in the planetesimal disc in our simulations. In any case, simple estimates of the relative effects of planetesimals self-gravity versus shear show that the density enhancements in the 50 cm simulation may be gravitationally significant and that the planetesimal disc could become gravitationally unstable (see Rice et al. 2004).

4. Conclusions

To summarize, in this work we have shown how the interaction between a self-gravitating gaseous protoplanetary disc and embedded planetesimals plays an important role in accelerating planetesimal growth. A number of possible refinements to the present simple model can be done. These include a proper inclusion of the back-reaction of the planetesimals on the gas and of the planetesimal self-gravity, the inclusion of a range of planetesimal sizes in each simulation, and an exploration of different gas disc properties (total mass, surface density, etc.). However, it seems clear that if protoplanetary discs experience a self-gravitating phase, the resulting disc structures could well play an important role in planetesimal evolution and growth and could ultimately influence the growth of terrestrial planets and the cores of gas/ice giant planets. Also, since a protoplanetary disc is most likely to become gravitationally unstable early in the star formation process, we might expect substantial processing of the dust prior to the optically visible Classical T Tauri phase.

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