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On the formation of line-driven winds near compact objects

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Abstract. We consider a general physical mechanism which could contribute to the formation of line-driven winds at the vicinity of strong gravitational field sources. It is demonstrated that if gravitational redshifting is taken into account, the radiation force becomes a function of the local velocity gradient (as in the standard line-driven wind theory) and the gradient of g_{00} (or gravitational potential). It is shown that the proposed mechanism could have an important contribution to the formation of line-driven outflows from compact objects.

Key words. radiation mechanisms: general – stars: mass loss – stars: winds, outflows – galaxies: active

1. Introduction

In the paper of Dorodnitsyn (2003) (hereafter **D1**) it was proposed a mechanism when line-driven acceleration occurs in the vicinity of compact object so that the gravitational redshifting can play an important role. The generalization of these studies in the frame of General Relativity (GR) is the problem that we address in this paper. A mechanism that we study is quite general and can be considered to work in any case when there is enough radiation to accelerate plasma and radiation driving occurs in strong gravitational field. Particularly we discuss winds in active galactic nuclei as they manifests most important properties of accretion disk + wind systems keeping in mind however that our treatment allows to consider their low mass counterparts.

In the standard line-driven wind theory a given parcel of gas sees the matter that is upstream redshifted because of the difference in velocities (assuming that a wind is accelerating gradually). This helps a line to shift from the shadow produced by the underlying matter and to expose itself to the unattenuated continuum. It was shown in **D1**, that together with Sobolev effect Sobolev (1960) the gravitational redshifting of the photon's frequency should be taken into account when calculating the radiation force. In case of strong gravitational field the gradient of the gravitational potential works in the same fashion as the velocity gradient does when only Sobolev effect is taken into account, so that the radiation force becomes $g_l \sim (dv/dr + \frac{1}{c} d\phi/dr)$. As it was shown in **D1** now the gravitational field works in exposing the wind to unattenuated radiation of the central source. Thus we call such a flow "Gravitationally Exposed Flow" (GEF).

To compare GEF regime with the standard line-driven wind (SLDW) we choose the simplest possible model: plasma is moving with non-relativistic velocities, spherical-symmetry, stationarity, wind is assumed isothermal, and

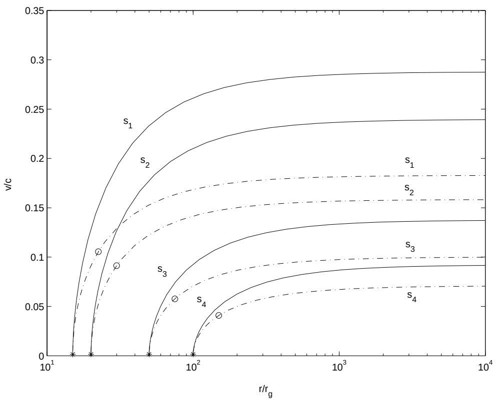

Fig. 1. Solutions of the equation of motion. Solid line - "Gravitationally Exposed Flow" (GEF) solution, dashed line - standard line-driven wind (SLDW) regime. Stars indicate GEF critical points, circles - SDLW critical points. cf. Figure 1. of Dorodnitsyn (2003) Labels s_1 - s_4 mark solutions with different locations of the critical point.

no ionization is treated and the radiation field is assumed to originate form a point source.

2. Optical depth and the radiation force in Sobolev approximation

A photon emitted at a given radius will suffer a continuous both gravitational and Doppler redshifting and may become resonant with a line transition at some point downstream. Thus a ray of a frequency ν_d, emitted by the matter that for simplicity is assumed to be at rest at radius r_d, at a given point r has a frequency ν_{lab}, as measured by the observer $\hat{O}$ and that is obtained from relation:

$$\nu_d \sqrt{h_d} = \nu_{lab} \sqrt{h(r)} = \nu^\infty, \qquad (1)$$

where ν^∞ is the frequency of the ray at infinity. We restrict ourself to the *radially streaming photons* only and assume that they are emitted from a point source. In such a case the Sobolev optical depth may be calculated without solving the radiation transfer equation. Optical depth between r_d and a given point r can be written:

$$\tau_l = \int_{r_d}^{r} \chi_{l,\,lab}\, \hat{dl} = \int_{r_d}^{r} \chi_{l,\,lab} \frac{dr}{(1-\frac{2M}{r})^{1/2}}, \qquad (2)$$

where $\hat{dl}$ is the proper-length element, and $\chi_{l,\,lab}\,(\mathrm{cm}^{-1})$ is the absorbtion coefficient as measured by $\hat{O}$. However, it is more appropriate to measure absorbtion (as well as the emission) in the local frame co-moving with the fluid. In such a case, opacity is transformed according to the reation: $\chi_{l,\,lab} = \chi_{l,com}\, \tilde{\nu}/\nu_{lab}$, where in the co-moving frame the frequency of the ray is $\tilde{\nu} = \gamma\nu_{lab}(1-\beta)$, $\beta = v/c$, and $\chi_{l,com}$ is the absorbtion coefficient measured by the co-moving observer. In the co-moving frame, the line-center opacity is determined by the following relation:

$$\chi_l^0 = \frac{\pi e^2}{mc} g f \frac{N_L/g_L - N_U/g_U}{\Delta\nu_D}, \qquad (3)$$

where $\Delta\nu_D = \nu_0 v_{th}/c$ is the Doppler width, and ν_0 is the line frequency, f is the oscillator strength of the transition, g is the statistical weight of the state, N_U, N_L and g_U, g_L are respective populatios and statistical weights of the corresponding levels of the line transition.

Then, one have to compare $\tilde{\nu}$ with the frequency of the line ν_0, to see whether it is in the range of the line profile $\varphi(\tilde{\nu}-\nu_0)$. That is in fact a standard procedure that is followed in order to calculate the Sobolev optical depth, but with an additional step - the gravitational redshifting. Thus, both Doppler and gravitational redshifting of the photons frequency are taken into account.
Introducing a new frequency variable:

$$y \equiv \tilde{\nu} - \nu_0 = \gamma(1-\beta)\frac{\nu^\infty}{\sqrt{h}} - \nu_0, \qquad (4)$$

We change the integration variable in (2) from r to y:

$$\tau_l = \int_{y(r_d)}^{y(r)} \frac{\tilde{\nu}}{\nu_{lab}} \frac{\varphi(y)\chi_l^0 \Delta\nu_D}{\nu^\infty \left| \frac{d\eta}{dr} - \mathrm{w}\frac{\eta}{\sqrt{h}} \right|} dy, \qquad (5)$$

where $\eta \equiv \gamma(1-\beta)$. The relation (4) was used in order to calculate dx/dr. The last term in the denominator of (5) is due to the "gradient of the gravitational field": $\mathrm{w} = \frac{d}{dr}\sqrt{h}$. Note that $\mathrm{w}\cdot c^2 \equiv \frac{GM}{r^2\sqrt{h}}$ - equals the acceleration of the free-falling particle that was initially at rest in the Schwarzschild metric. In case of $w = 0$ we obtain the result of Hutsemekers & Surdej (1990): $\tau_l = \frac{\chi_l^0 v_{th}(1-\beta)}{\gamma dv/dr}$. Assuming that the line profile is a δ- function, or, equivalently, that the region of interaction is infinitely narrow, we find the optical depth in Sobolev approximation: $\tau_l^\star = \frac{\chi_l^0 \Delta\nu_D(1-\beta)}{\nu_0\left|\sqrt{h}\gamma\frac{d\beta}{dr}+\mathrm{w}\gamma^{-1}\right|}$, where it was taken into account that $\eta' = -\gamma^3(1-\beta)\beta'$. In our treatment we retain only terms $O(v/c)$ (in the equation of motion) and thus resultant Sobolev optical depth can be written in the form:

$$\tau_l = \frac{\kappa_l^0 \rho_0 v_{th}}{\left|\sqrt{h}\frac{dv}{dr} + c\mathrm{w}\right|}, \tag{6}$$

where κ_l^0 ($\mathrm{cm}^2\cdot\mathrm{g}^{-1}$) is the mass absorbtion coefficient: $\kappa_l^0 = \chi_l^0/\rho_0$, measured in the rest-frame of the fluid, and $\gamma = 1$.

In the weak field limit the optical depth (6) will transform to equation (11) of **D1**. If there is no gravitational redshifting taken into account then the Sobolev optical depth is obtained: $\tau_{sob} = \frac{\kappa_l^0\rho_0 v_{th}}{|dv/dr|} = \chi_l^0\, l_{sob}$, and the Sobolev length scale $l_{sob} = v_{th}/\,(dv/dr)$ determines a typical length on which a line is shifted on about its thermal width. In general case from (6) we conclude that

$$\tau_l = \chi_l^0\, l_{GEF}, \tag{7}$$

where $l_{GEF} = \frac{v_{th}}{\sqrt{h}\frac{dv}{dr} + c\mathrm{w}}$.

The radiation force form a single line exerted by the material as measured in its rest frame reads:

$$g_i \simeq \frac{F_\nu^0(\nu_l)\chi_l^0/\rho_0\Delta\nu_D}{c}\frac{1-e^{-\tau_l}}{\tau_l}, \tag{8}$$

where $F_\nu^0(\nu_l)$(erg $\cdot\mathrm{cm}^{-2}\cdot\mathrm{Hz}^{-1}\cdot\mathrm{s}^{-1}$) is the radiation flux at the line frequency in the rest-frame of the fluid. Note that τ_l in (8) should be calculated with taking into account the redshifting and is given by (6). The $e^{-\tau_l}$ term reflects the fact that the incident flux at r is reduced in comparison with the initial flux F_ν. $1-e^{-\tau_l}$ gives the "penetration probability" for a ray to reach a given point.

In our treatment we neglect special relativistic terms all equations. We may expect that final results will be at least qualitatively correct for the flows as fast as $\sim 0.2 \div 0.3c$.

According to the well accepted notation let introduce the optical depth parameter:

$$t = \frac{\sigma_e\rho_0 v_{th}}{\left|\sqrt{h}\frac{dv}{dr} + c\mathrm{w}\right|}, \tag{9}$$

where σ_e is the electron scattering opacity per unit mass, and t is connected to τ_l via relation: $\tau_l = \xi t$, where $\xi = \kappa_l^0/\sigma_e$.

The role of the parameter ξ is very important because it allows to separate the line optical depth into two parts: the first (ξ) that depends on statistical equilibrium, and the second (t) which depends only on the redshifting law (4). It will alow us to use the standard parameterization law for the force multiplier when calculating the radiation force. Summing (8) over the ensemble of of optically thin and optically thick lines we obtain total radiation acceleration:

$$g_l = \sum_i g_i = \frac{F\sigma_e}{c}M(t), \tag{10}$$

where F is the total flux and the "force multiplier" M(t) equals:

$$M(t) = \sum_{\tau_l<1}\frac{F_\nu}{F}\,\xi\,\Delta\nu_D + \sum_{\tau_l>1}\frac{F_\nu}{F}\frac{\Delta\nu_D}{t}. \tag{11}$$

CAK found that M(t) can be fitted by the power law: $M(t) = kt^{-\alpha}$. Thus the radiation pressure force can be cast in the form:

$$g_l = \frac{F\sigma_e}{c}k\left(\frac{\sigma_e\rho_0 v_{th}}{\sqrt{h}\frac{dv}{dr} + c\mathrm{w}}\right)^{-\alpha}. \tag{12}$$

3. Gravitationally Exposed Flow

The equation of motion for stationary, spherically-symmetric, isothermal wind reads Sobolev (1960):

$$\frac{b}{h}\left(vh\frac{dv}{dr}+\frac{GM}{r^2}\right)+\frac{1}{\rho_0}\frac{dP}{dr}-\frac{\sigma_e}{\sqrt{h}}\frac{L}{4\pi r^2 c}$$
$$-\frac{\sigma_e}{\sqrt{h}}\frac{L}{4\pi r^2 c}k\left(\frac{4\pi}{\sigma v_{th}\dot{M}}\right)^{\alpha}\times$$
$$\times\left\{\sqrt{h}\,vr^2\left[\sqrt{h}\frac{dv}{dr}+c\mathrm{w}\right]\right\}^{\alpha}=0, \qquad (13)$$

Here we retain only terms of the order $O(v/c)$. We adopt the equation of state for the ideal gas: $P=\rho_0\mathcal{R}T$, $E_i=3/2\mathcal{R}T$, where $\mathcal{R}=k/m_p$ is the gas constant. For a given position of the critical point r_c we can calculate the value of the velocity and velocity gradient in the critical point. Adjusting the position of the critical point r_c we integrate the equation of motion inward, looking for the solution that satisfies the inner boundary condition. The qualitative picture that has been obtained in **D1** is confirmed throughout our calculations. We detect a considerable gain in terminal velocity both in comparison with CAK case and between fully relativistic calculations presented here, and semi-classical treatment of **D1**.

4. Conclusions

Finally we summarize most important results which have been obtained in the current studies (Dorodnitsyn & Novikov 2004):

1. A wind driven by the radiation pressure on spectral lines was considered in the frame of General Relativity. Following Dorodnitsyn (2003), we argue that it is important to take into account the gravitational redshifting of the photon's frequency, when calculating the radiation force.
2. A generalization of the Sobolev approximation in GR was developed and the general relativistic equation of motion with the radiation pressure force on spectral lines was derived.
3. The results of the numerical integration of the equation of motion demonstrate that taking into account gravitational redshifting can result in a wind that is considerably more fast than previously assumed on the ground of the CAK theory.

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