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# Fully simultaneous calculations of AGB evolution

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**Abstract.** We present the first fully simultaneous calculations of asymptotic giant branch evolution. We find that third dredge-up occurs at lower core masses and is deeper than in non-simultaneous calculations. At a metallicity of $Z = 0.008$ we form carbon stars at low core masses and hence lower luminosities. This allows us to reproduce the carbon star luminosity function for the Large Magellanic Cloud *without* any ad hoc adjustments to our models.

**Key words.** stars: evolution – stars: AGB – carbon star luminosity function

## 1. Introduction

In the calculation of the evolution of stars of $1-8\,M_\odot$ through the thermally pulsing asymptotic giant branch (TP-AGB) phase of their evolution most evolution codes use only a partially self-consistent approach. The treatment of structure, mixing and nucleosynthesis is not simultaneous, with mixing often being solved in a separate iteration step from the other two. Examples include the codes used by Straniero et al. (1997), Herwig (2000) and Karakas, Lattanzio & Pols (2002). As the phenomenon of third dredge-up (TDU) depends critically on the treatment of convection within a stellar structure calculation it is desirable to combine the calculation of nucleosynthesis, mixing and structure into a single, simultaneous step. The first attempt to do this was made by Pols & Tout (2001) using the STARS evolution code originally developed by Eggleton (1971). In their calculations a $5\,M_\odot$ star was evolved without mass loss through the first 6 thermal pulses and deep TDU occured. The sequence was terminated prematurely owing to numerical instabilities.

Working with the same code Stancliffe, Tout & Pols (2004) developed the concept of a viscous mesh in order to combat these numerical problems. At low timesteps there is an instability associated with the luminosity equation

$$L_{k+1} - L_k = (m'E_1)_{k+\frac{1}{2}} + (m'E_2)_k[\dot{m}_k] \\ -(m'E_2)_{k+1}[-\dot{m}_{k+1}]$$

where $L_k$ is the luminosity at mesh point number $k$. The change in mass with mesh point is denoted $m' = dm/dk$. Square brackets signify a term only included when it is positive.

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The first term containing $E_1$ is related to the usual energy generation terms but evaluated at constant mesh point and the terms containing $E_2$ are an upstream approximation for the advection term owing to the motion of the adaptive mesh. When the timestep becomes small these terms become large and this leads to numerical instability when two large numbers are subtracted to give a small result. On fixing the mesh, the last two terms are removed from the equation and the instability is avoided.

The viscous mesh combines the advantages of adaptive and fixed meshs. The adaptive behaviour of the outer portions of the mesh is retained. Using this technique Stancliffe, Tout & Pols (2004) were able to compute 25 thermal pulses for a $5\,\mathrm{M}_\odot$ star of solar metallicity and 20 thermal pulses for a $3\,\mathrm{M}_\odot$ star of solar metallicity. They found that TDU occured at much smaller core masses than was found in calculations with non-simultaneous codes.

The occurence of TDU at low core masses and deep TDU is important for the formation of carbon stars. These are defined as stars that are M-type and have surface carbon-to-oxygen abundance ratios exceeding unity. Observational evidence suggests that these stars are of low mass, most likely between 1 and $3\,\mathrm{M}_\odot$ (Iben 1981). However, there has been considerable difficulty in producing detailed theoretical models of carbon stars with low enough masses and luminosities. Of the early work on the subject, Boothroyd & Sackmann (1988) were able to produce two carbon star models from initial masses of 1.2 and $2.0\,\mathrm{M}_\odot$ under metal-poor conditions ($Z = 0.001$), while Lattanzio (1989) produced a model initially of $M = 1.5\,\mathrm{M}_\odot$ with $Z = 0.02$. More recently low-mass carbon star models have been produced by Straniero et al. (1997) and, with the aid of convective overshooting, Herwig (2000). However, the core masses found in all these models are too large to explain the carbon star luminosity function (Izzard & Tout 2004).

We briefly review the results of computations with different evolution codes in order to highlight some of the numerical issues that should be borne in mind when considering TP-AGB calculations. Calculations of the evolution of stars between 1 and $6\,\mathrm{M}_\odot$ for $Z = 0.008$ are then presented and these results are used to address the problem of the carbon star luminosity function (CSLF) of the Large Magellanic Cloud (LMC).

## 2. Code Comparison

Figure 1 shows the results of evolution calculations for a $3\,\mathrm{M}_\odot$ star of $Z = 0.02$ made with four different codes: FRANEC (Straniero et al. 1997), MSSSP (Karakas, Lattanzio & Pols 2002), EVOL (Herwig 2000) and STARS (Stancliffe et al. 2004). The STARS model was evolved without mass loss from the pre-main sequence. Details of the other codes can be found in Lugaro et al. (2003). The STARS code gives TDU at much smaller core masses than the other codes (Panel (a) of Figure 1). The TDU and C/O evolution are similar to the EVOL model. This is interesting as the EVOL model includes convective overshooting whilst the STARS model does not. Unlike EVOL, the STARS code produces much lower maximum temperatures in the pulse driven convection zone (PDCZ). The temperatures are also lower than those of MSSSP and FRANEC for much higher TDU efficiencies.

It should be noted that there is a wide range in the details produced by the four codes despite the fact that we are all modelling the same star. All the codes are based on the same equations of stellar structure and nuclear burning. One of the most obvious diffences is in the treatment of the mixing. Some codes, like STARS, include it as a diffusion equation; others adopt a more ad hoc approach. We currently do not have a good understanding of how this affects the results obtained. Other questions involving numerical issues, especially the question of the effect of simultaneous solution on the results obtained also need to be addressed. One way to do this would be to decouple the STARS code but this has so far proved difficult to achieve.

Numerical effects still remain a major source of uncertainty in AGB evolution. Without a thorough analysis of these it will prove difficult to make valuable comparisons between the results obtained using different codes.

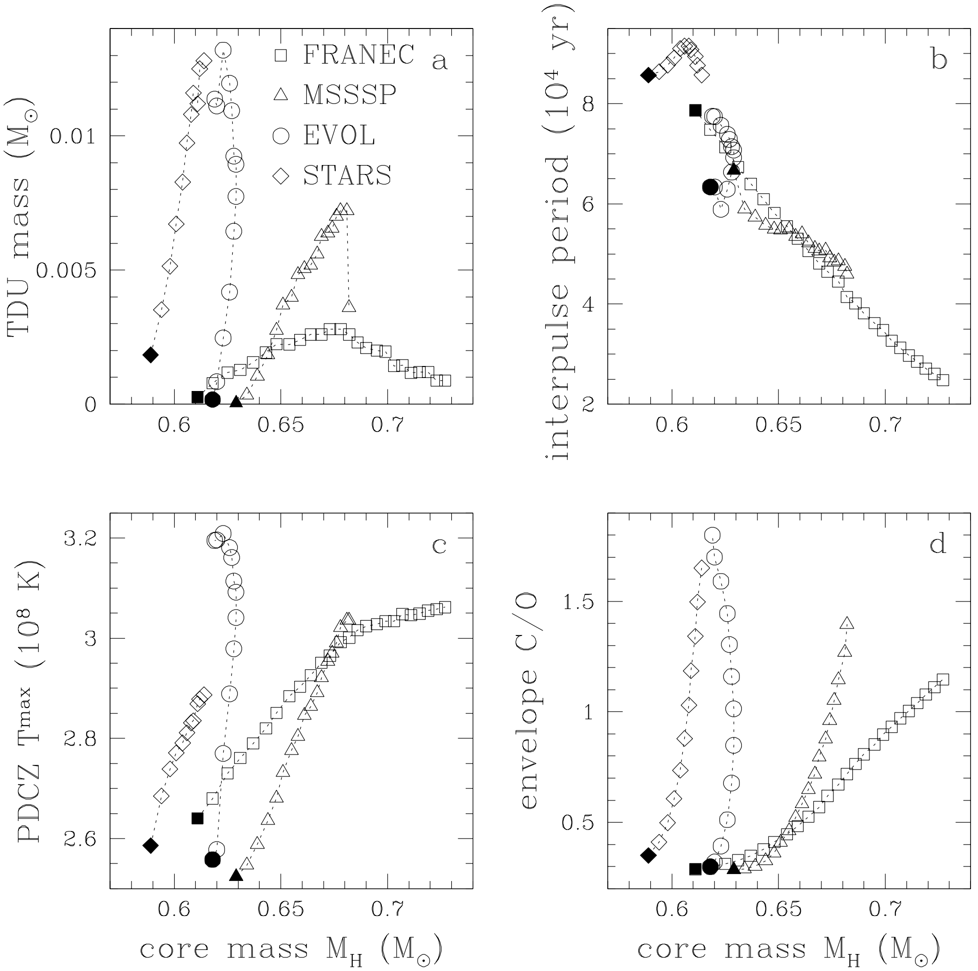

**Fig. 1.** A comparison of models of a 3 $M_\odot$ star of $Z = 0.02$ for four different evolution codes. Panel (a) shows the amount of material pulled into the envelope in a TDU event. Panel (b) shows the interpulse period. Panel (c) shows the maximum temperature reached in the pulse driven convection zone (PDCZ) and Panel (d) shows the envelope carbon-to-oxygen ratio. Adapted from Lugaro et al. (2003).

## 3. Calculations at Z=0.008 with STARS

Models of 1, 1.5, 2, 3, 4, 5 and 6 $M_\odot$ were evolved from the pre-main sequence up to and along the TP-AGB. No convective overshooting or mass loss was considered at any stage of the evolution. We use the formalism of Böhm-Vitense (1958) for convection with $\alpha = 1.925$, based on calibration to a solar model. Models were evolved with 199 mesh points up to core helium burning. At this point the resolution was increased to 499 mesh points in order to facilitate the transition to 999 meshpoints just before the onset of the TP-AGB.

Stars of masses up to around 2.3 $M_\odot$ are expected to undergo a core helium flash. However this phase of evolution is numeri-

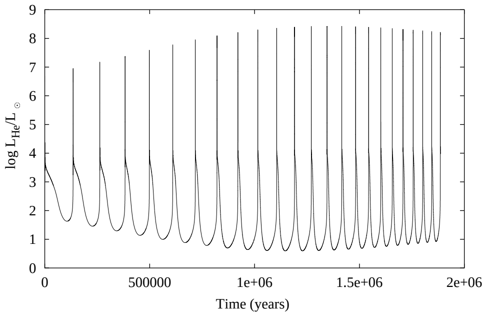

**Fig. 2.** The evolution of the helium luminosity of the 1.5 $M_\odot$ model with time since the first thermal pulse.

**Table 1.** Details of the minimum core mass for TDU $M_c$ and maximum TDU efficiency $\lambda$ for the STARS (S) models. Details for the same models taken from Karakas, Lattanzio & Pols (2002) (M) are provided for comparison. Note the lower core masses for TDU given by the STARS code for stars of 1-2 $M_\odot$.

| Mass | $M_c^S$ | $M_c^M$ | $\lambda^S$ | $\lambda^M$ |
|---|---|---|---|---|
| 1 | 0.573 | 0.657 | 0.169 | 0.002 |
| 1.5 | 0.571 | 0.624 | 0.752 | 0.306 |
| 2 | 0.579 | 0.596 | 0.880 | 0.656 |
| 3 | 0.641 | 0.639 | 0.992 | 0.882 |
| 4 | 0.817 | 0.748 | 1.100 | 0.990 |
| 5 | 0.871 | 0.876 | 1.085 | 0.974 |
| 6 | 0.956 | 0.929 | 1.098 | 0.932 |

cally demanding and the STARS code is currently not suitable for a calculation of the evolution through it. Instead a model of the desired mass is run from the pre-main sequence up to the helium flash. The hydrogen exhausted core mass and the envelope composition are recorded. A 3 $M_\odot$ model is then evolved from the pre-main sequence up to the point where helium ignites in the core. During this evolution helium burning reactions are allowed to produce energy but not consume helium. Once helium has ignited under non-degenerate conditions, mass is stripped from the envelope and the core is allowed to grow until the model has the desired envelope mass and core mass. The envelope composition is then set to that of the pre-flash model. We are therefore assuming that the helium flash proceeds so rapidly that the core mass doesn't change and that there is no change in the envelope composition. These are both standard assumptions.

We find TDU occurs in all these models. Carbon stars are formed in all the models up to 3 $M_\odot$. For 4 $M_\odot$ and above, hot-bottom burning occurs, and because no mass-loss is included, these models do not form carbon stars. Figure 2 shows the evolution of the helium luminosity with time for the 1.5 $M_\odot$ model. The core mass at which TDU occurs is displayed in Table 1 along with the maximum TDU efficiency $\lambda = \Delta M_{\rm DUP}/\Delta M_{\rm c}$, where $\Delta M_{\rm DUP}$ is the mass of material dredged up by TDU and $\Delta M_{\rm c}$ is the amount by which the H-exhausted core grows in the preceeding interpulse. The maximum $\lambda$ value of the 1 $M_\odot$ model quoted in Table 1 is unlikely to be a true representation of the maximum efficiency reached because numerical instabilities prevent us evolving this model to a steady state.

The occurence of TDU at lower core masses for low-mass stars is an important result. It allows us to make carbon stars with lower luminosities than was previously possible.

## 4. The LMC Carbon Star Luminosity Function

There is a long-standing problem with forming carbon stars. Until now, detailed stellar evolution models have proved unable to produce TDU at low enough core masses and hence at low enough luminosities. When Iben formulated the carbon star mystery (Iben 1981) calculations showed that TDU did not occur for core masses below about 0.6 $M_\odot$ (e.g. Sackmann 1980, Wood & Zarro 1981). This is also borne out by more recent calculations but leads to problems in reproducing the observed luminosity functions of the LMC and the SMC. Using the results of Karakas, Lattanzio & Pols (2002), Izzard et al. (2004) determined that the core masses at TDU would have to be lower by 0.07 $M_\odot$.

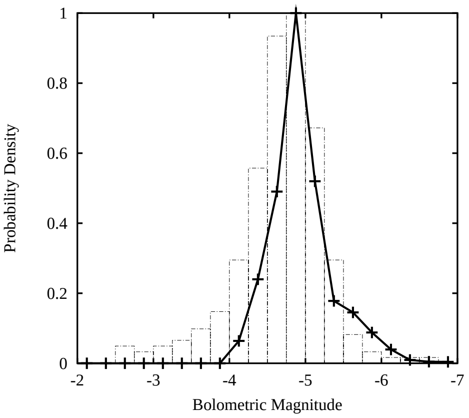

**Fig. 3.** The theoretical fit (solid line) to the LMC CSLF. The histogram is observational data taken from Groenewegen (2002). The CSLF is reasonably well reproduced by the theoretical models.

In order to examine the impact of our new models on the carbon star luminosity function of the LMC we need to generate a population of stars. We utilise the synthetic TP-AGB evolution code of Izzard et al. (2004). The luminosity core-mass relation therein is found to fit our data well and so does not need to be refitted. As in Izzard et al. (2004), the luminosity dip after each thermal pulse (see Iben & Renzini 1983) is modelled by a factor of the form

$$f_{\mathrm{L}} = 1 - 0.5 \times \min\left[1, \exp(-3\frac{\tau}{\tau_{\mathrm{ip}}})\right],$$

where $\tau$ is the time from the beginning of the current pulse and $\tau_{\mathrm{ip}}$ is the interpulse period. It is necessary to include this dip in order to reproduce the low-luminosity side of the CSLF. The core mass at first thermal pulse, core mass at which TDU first occurs and the behaviour of $\lambda$ are all fitted to the detailed models. Three additional models of 1.25, 1.75 and 2.25 $\mathrm{M}_{\odot}$ were created in order to get a more accurate fit for the core mass at which TDU first occurs.

A population of 10,000 stars was evolved for 16 Gyr. A Kroupa, Tout & Gilmore (1993) initial mass function and a constant star formation rate were assumed. Mass loss was included as the Vassiliadis & Wood (1993) type prescription used by Karakas, Lattanzio & Pols (2002). The superwind phase is turned on when the Mira pulsation period of the star reaches 500 d. We have fitted the synthetic code to results from detailed models without mass loss. Because the Vassiliadis & Wood formalism causes significant mass loss only in the later pulses and our models form carbon stars rapidly, the impact of this approximation on our results is limited.

The results of the population synthesis are shown in Figure 3. The model is normalised such that it matches the peak of the observations. The model fits the observations of the LMC CSLF very well except for a slight underabundance of carbon stars between $\mathrm{M}_{\mathrm{bol}} = -4$ and $\mathrm{M}_{\mathrm{bol}} = -4.75$. Note that we do not fit the very low-luminosity carbon stars. These are likely to be extrinsic, rather than intrinsic, carbon stars (see Izzard & Tout 2004). It should be noted that our models consist only of a single metallicity and do not account for the finite size of the clouds. Both these effects would broaden the luminosity function slightly.

## 5. Conclusions

We have higlighted the difference in results produced by some of the main codes currently being used to tackle TP-AGB evolution. It is important that we reach a firm understanding of where results differ due to different input physics (e.g. the inclusion of convective overshooting) and where they arise from different numerical treatments. For example, we need to determine the effects of using a fully simultaneous solution.

We have presented the first fully simultaneous calculations of TP-AGB evolution at $Z = 0.008$. We find deep TDU to occur at much lower core masses for low mass stars compared to the results of other codes. We are able to form carbon stars at lower luminosity than previously thought. The detailed models have been used to provide fits for a population synthesis code *without* any ad hoc adjustments and we are able to reproduce the CSLF of the LMC. We thus consider the case closed on the carbon star mystery in the LMC.

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