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RR Lyrae stars in M3

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Abstract. Predicted relations connecting pulsational (period and amplitude of pulsation) and structural (mass, luminosity and effective temperature) parameters, as based on a wide set of nonlinear convective pulsating models of RR Lyrae stars with $Z = 0.001$, $Y = 0.24$ and mass and luminosity suitable for the "old" (age $t > 8$ Gyr) variables observed in globular clusters, are used together with sound constraints on the mass of pulsators, as inferred from up-to-date evolutionary models of horizontal branch stars, in order to provide a self-consistent "pulsational" framework for the analysis of observed variables in M3. It is found that current uncertainties on the intrinsic luminosity of up-to-date horizontal branch models, as due to the input physics used in the computations by the different authors, have a quite low influence (~ 0.02-0.03 mag) on the pulsational distance modulus. On the contrary, the effect of the different bolometric corrections adopted to convert bolometric luminosity into absolute magnitude is of the order of ~ 0.05-0.06 mag.

Key words. CM diagram – He burning stars – RR Lyrae stars – Distances

1. Introduction

RR Lyrae variables are of great relevance in several fields of modern astrophysics In particular, they are widely used, via the calibration of their absolute visual magnitude M_V(RR) in terms of the Iron-to-Hydrogen content [Fe/H], as standard candles for distance determination in the Local Group. On this basis, they provide an independent test of the Cepheid distance scale for nearby galaxies (Magellanic Clouds, M31) as well as a calibration of secondary distance indicators (e.g., the globular cluster luminosity function) in external galaxies, thus yielding relevant clues about the value of the Hubble constant. On the other hand, since the distance yields the absolute magnitude of the globular cluster main sequence turn-off, and then information on the age of these ancient stellar systems, one easily understands the relevance of an accurate RR Lyrae distance scale for cosmological studies.

In spite of the huge amount of work made in the last years to establish the absolute magnitude of Horizontal Branch (HB) stars, and particularly of RR Lyrae variables, a general consensus has not been achieved yet, both on the zero-point and the slope of the M_V(RR)-[Fe/H] relation.

In particular, the "theoretical route" based on HB models suggests absolute magnitudes which are somehow brighter than empirical estimates, at fixed [Fe/H]. However, evolutionary predictions themselves, as provided by the various researchers, still suggest significant discrepant (both in the zero-point and the slope) M_V(RR)-[Fe/H] relations (see VandenBerg et

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al. 2000, hereafter VDB; Caputo et al. 2000 and references therein).

On this issue, it should be noted that the constraints inferred by HB evolutionary models deal with the bolometric luminosity L and the overall metallicity Z. Thus, in the comparison among evolutionary predicted M_V(RR)-[Fe/H] relations two factors are of critical importance: a) the conversion of bolometric luminosity and effective temperature into magnitude and color, i.e. the adopted bolometric correction and temperature-color transformation, and b) the scaling between the theoretical Z and the measured [Fe/H] value, i.e. the adopted ratio between α-elements and Iron. These factors are obviously still more important when theoretical predictions are compared with observed quantities. In addition, we wish also to mention that HB models refer to "static" stars, whereas the RR Lyrae observed magnitudes are "mean" quantities averaged over the pulsation cycle, which not necessarily are exactly alike static values.

On this ground, we planned to construct an exhaustive and homogeneous theoretical framework, both on the pulsational and the evolutionary side, aiming at a sound analysis of observed RR Lyrae stars at various metal content (see Castellani, Caputo & Castellani 2003, Paper I, for the observational scenario of RR Lyrae variables in Galactic globular clusters). Extensive sets of nonlinear convective pulsation models (Di Criscienzo, Marconi & Caputo 2003, Paper III, in preparation) and HB evolutionary tracks (Cassisi, Castellani, Caputo & Castellani 2003, Paper IV, in preparation) are computed using the same input physics. Then, adopting similar bolometric corrections and color-temperature transformations, self-consistent predictions concerning various stellar parameters are derived. The general procedure is discussed in Marconi, Caputo, Di Criscenzo & Castellani (2003, Paper II), where the results at Z=0.001 and Y=0.24 are compared with RR Lyrae stars in M3, which is the most RR Lyrae rich cluster in the Galaxy and among the best studied ones. The main results of such an investigation are here reported.

2. Theoretical pulsational scenario

For the pulsation models with $Z = 0.001$, two masses (0.65 $M_\odot$ and 0.75 $M_\odot$) and three luminosity levels ($\log L/L_\odot = 1.51, 1.61, 1.72$) have been adopted in order to explore the role of the stellar mass and luminosity. For each given set of entry parameters, the computations are performed by decreasing the effective temperature T_e by steps of 100-200 K. In this way, the blue edge (FOBE) and red edge (FRE) of the pulsation region are derived.

For each fundamental (F) and first-overtone (FO) model, the computations provide the pulsation period and the bolometric light curve. hen, using bolometric corrections and color-temperature transformations (as in previous papers, we adopt Castelli, Gratton & Kurucz 1997a, b, hereafter CGK), the static magnitude M_i, i.e. the value the star would have were it not pulsating, mean intensity-weighted $< M_i >$ and magnitude-weighted (M_i) magnitudes, and amplitudes A_i in the various photometric bands are derived.

As for the principal results[1], we get

a) the Period-Magnitude-Color (PMC) relation:

$$\log P^F = -0.61 - 0.34 M_V - 0.66 \log M + 1.31[M_B - M_V] \quad (1)$$

b) the Near-infrared Period-Magnitude (PM_K) relation:

$$M_K^F = -0.18 - 2.20 \log P^F - 1.45 \log M - 0.66 \log L \quad (2)$$

c) the Period-Amplitude (PA) fundamental relation (adopting a mixing-length parameter l/H_p=1.5, see later):

$$\log P^F = 0.13 - 0.38 M_V - 0.30 \log M - 0.19 A_V \quad (3)$$

d) the Edges of the instability strip in the HR diagram, at l/H_p=1.5:

$$\log T_e^{FOBE} = 3.970 - 0.06 \log L + 0.09 \log M \quad (4)$$

$$\log T_e^{FRE} = 3.957 - 0.10 \log L + 0.07 \log M \quad (5)$$

[1] We give here only the results for F pulsators.

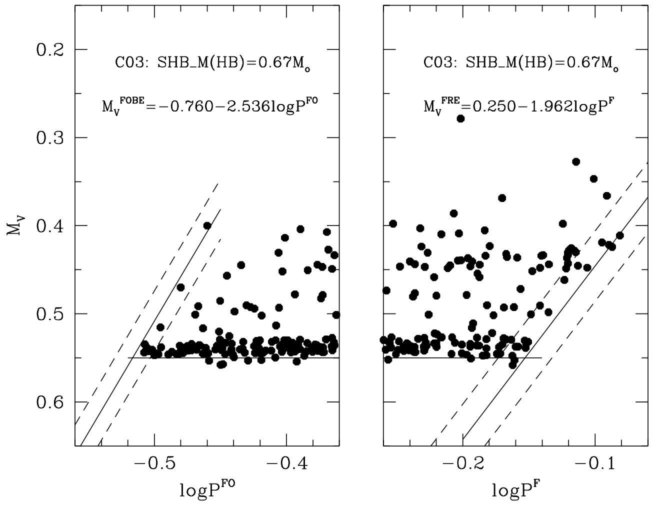

Fig. 1. Absolute visual magnitude of C03 synthetic HB pulsators as a function of period. Left and right panel refers to first-overtone and fundamental pulsators, respectively. The solid lines (see the labelled relations) are the predicted edges of the instability strip, while the dashed lines depict their intrinsic dispersion.

Before proceeding, it is worth mentioning that a significant source of uncertainty on the pulsation limits is due to the efficiency of convection in the star external layers. As the depth of convection increases from high to low effective temperature, and the effect of convection is to quench pulsation, we expect that varying the value of the mixing length parameter l/H_p will modify the red edge of the instability strip by a larger amount with respect to the blue edge. Model computations show that when increasing l/H_p from 1.5 to 2.0 the FOBE temperature decreases by 100 K, whereas those at FBE and FRE increase by 100 K and 300 K, respectively.

As a consequence, adopting l/H_p=2.0 yields that the zero-point in eq. (4) and eq. (5) varies by −0.006 and +0.020, respectively. Moreover, the Period-Amplitude relation of F pulsators becomes:

$$\log P^F = 0.02 - 0.38 M_V - 0.35 \log M - 0.14 A_V \quad (6)$$

All the relations presented so far cannot be straightaway applied to observed RR Lyrae variables because they hold for static values, whereas empirical measurements refer to quantities averaged over the pulsation cycle.

The synthetic visual magnitudes for F and FO pulsators show that mean magnitudes (M_i) of fundamental pulsators, as derived averaging the magnitude over the cycle, are fainter than the corresponding static values with a discrepancy increasing as the pulsation amplitude increases. A similar discrepancy, though to a less extent, is present also for the intensity-averaged magnitudes $< M_i >$. Since for a given effective temperature the amplitude decreases from blue to visual to infrared, the difference between static and mean values for K magnitudes is almost negligible, whereas in the optical bands they differ from the corresponding static values up to $\sim$ 0.18 mag (B), $\sim$ 0.14 mag (V), and $\sim$ 0.08 mag (I), at A_V = 1.6 mag. Concerning first-overtone pulsators, they show low-amplitude and quite symmetric light curves, thus the discrepancy is less pronounced. It is interesting to note that the behavior of synthetic magnitudes agrees with observed differences $(V)- < V >$ and $(K)- < K >$, as reported in previous empirical studies on RR Lyrae stars (e.g. Fernley 1993).

As for the mean colors, one has that the $(M_B - M_V)$ color is redder, whereas $< M_B > - < M_V >$ is bluer, than the static color. Also in this case, the evidence for a positive difference between predicted (M_B-M_V) and $< M_B > - < M_V >$, with an amount that increases with A_V, agrees with quite well settled observed trends (see, e.g. Liu & Janes 1990; Storm, Carney & Beck 1991; Carney et al. 1992, Caputo et al. 1999).

As a whole, mean magnitudes and colors can be corrected by the amplitude effect to get static values (see also Bono et al. 1995). However, static magnitudes can be directly estimated when both the mean quantities are measured. In particular, either for fundamental and first-overtone pulsators, we derive:

$$M_V = -0.35(M_V) + 1.35 < M_V > \quad (7)$$

and

$$\begin{aligned} M_B - M_V \; = \; & 0.49\,[(M_B - M_V)] \\ & +0.50\,[< M_B > - < M_V >] \end{aligned} \quad (8)$$

with an intrinsic uncertainty of ±0.01 mag.

3. The evolutionary connection

Due to somehow different input physics, current updated HB models still provide slightly different luminosity values. As a matter of example, let us compare Cassisi et al. (2003, hereafter C03) models with $Z = 0.001$ with VDB results. The two sets of models adopt quite similar values for the mixing length parameter ($l/H_p = 2.0$ and 1.9, respectively), no Helium and heavy element diffusion, and can be taken as representative of the range in *absolute magnitude* predicted by recent HB evolutionary calculations with updated input physics. Moreover, the VDB models have the same *overall* metallicity $Z = 0.001$, although referring to different chemical mixtures ([α/Fe] = 0, [Fe/H] = −1.31 and [α/Fe] = 0.3, [Fe/H] = −1.54).

From the comparison of zero-age-horizontal-branch (ZAHB) models at fixed effective temperature, one derives that

1. C03 models are brighter by $\delta \log L \sim 0.01$ dex;
2. C03 models are more massive by $\sim 0.02\, M_\odot$;
3. increasing [α/Fe] up to 0.3, which is the mean value estimated for globular cluster stars with [Fe/H] < -0.6 (see Carney 1996), leaves quite unvaried the value of mass and luminosity. In other words, the mass and luminosity of RR Lyrae pulsators are expected to depend on the overall metallicity Z, quite independently of the internal ratio between α and heavy elements (see also Salaris, Chieffi & Straniero 1993).

Using HB models and the predicted edges of the pulsation region [eq. (4) and eq. (5)], synthetic horizontal branches (SHB) can be constructed, yielding the predicted numbers of blue (B), variable (V) and red (R) stars, as well as the predicted mass of pulsators. In particular, based on C03 computations, one has that the average mass of the variable stars is quite constant, $< M(RR) >= 0.67 \pm 0.02\, M_\odot$, with the (B-R)/(B+V+R) ratio in the range of $\sim -0.45 \rightarrow +0.55$. Furthermore, the predicted edges of the distribution of synthetic HB pulsators in the M_V-$\log P$ plane is derived.

Figure 1 shows the case at $M(\mathrm{HB}) = 0.67\, M_\odot$ (HB index $\sim$ 0.10, as observed in M3), but quite similar results are obtained with $M(\mathrm{HB}) = 0.65$ and $0.69\, M_\odot$. As already mentioned, CGK bolometric corrections are adopted. The left panel in the figure with the short-period side of synthetic first overtone pulsators: the solid line is the resulting FOBE of the synthetic distribution (see the labelled relation), while the dashed lines depict its intrinsic dispersion of ±0.034 mag. In the right panel, which refers to the long-period side of fundamental pulsators, the solid and dashed lines are the predicted FRE (see labelled relation) and its intrinsic uncertainty by ±0.04 mag, respectively. The horizontal line drawn in the two panels depicts the lower envelope magnitude $M_{V,le} = 0.55$ mag of the synthetic pulsator distribution.

Before proceeding, it is important to mention that if the main differences between C03 and VDB models are taken into account (see points 1 and 2 at the beginning of this section), then either FOBE and FRE magnitudes should be increased by $\sim$ 0.03 mag, at fixed period, while the absolute magnitude of the lower en-

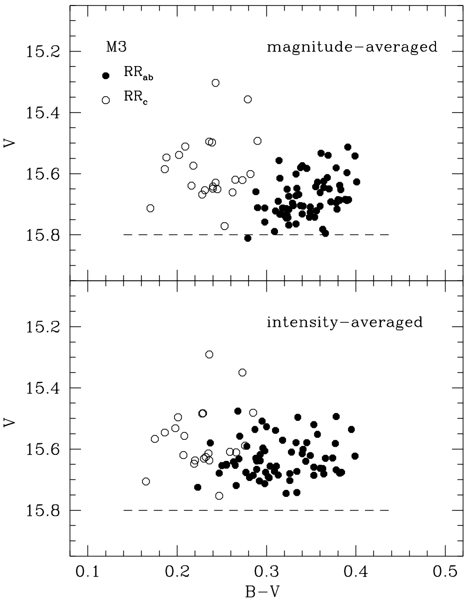

Fig. 2. Color-magnitude diagram of RR Lyrae stars in M3. The two panels refer to magnitude-weighted and intensity weighted values. Different symbols refer to different pulsation modes. The dashed line is drawn at $V = 15.8$ mag.

velope is $M_{V,le} = 0.58$ mag, *provided that CGK bolometric corrections are adopted.*

4. The RR Lyrae population in M3

Figure 2 shows M3 variables (data by Corwin & Carney 2001) plotted in the color-magnitude diagram, according to magnitude-averaged (upper panel) and intensity-averaged (lower panel) quantities. As a matter of reference, we draw a line at $V = 15.8$ mag. It is quite evident that the *ab*-type magnitude plotted in the upper panel is on average fainter than in the lower panel. Moreover, the magnitude-averaged color of the bluest *ab*-type variables is evidently redder than the intensity-averaged one. As a whole, it is found that the behavior of the measured difference between the empirical mean quantities follows the predictions faithfully. On this ground, eq. (7) and eq. (8) can be used to get static magnitudes and colors, with an error of ±0.01 mag, allowing us to compare

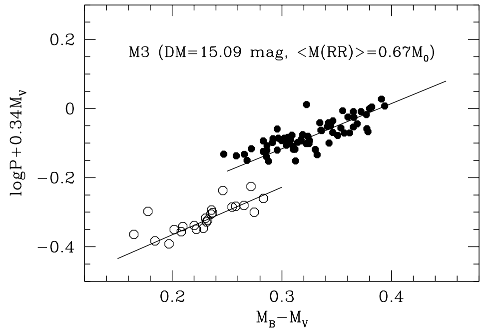

Fig. 3. Reduced period versus color for RR Lyrae stars in M3 (symbols as in Fig. 2).

the predictions presented in the previous sections with observed data. Moreover, Longmore at al. (1990) near-infrared photometry is used. In the following, we will adopt $E(B-V) = 0$.

a) Period-Magnitude-Color relation

According to eq. (1), one may derive the distance modulus for each given value of the mass. Adopting $< M(RR) >= 0.67 \pm 0.02\, M_\odot$ as suggested by SHB results, the resulting distance modulus is $DM(PMC) = 15.09 \pm 0.08$ mag (see Fig. 3).

b) Near-infrared Period-Magnitude relation

Figure 4 shows that the distribution of measured K magnitudes as a function of periods agrees quite well with the predicted slope at constant mass and luminosity (solid line). Since the predicted PM_K relation is slightly dependent on the intrinsic luminosity [see eq. (2)], we adopt $\log L = 1.75 \pm 0.15$ as a quite safe luminosity range for RR Lyrae with $Z = 0.001$. Then, taking again $< M(RR) >= 0.67 \pm 0.02\, M_\odot$, we derive $DM(PM_K) = 15.08 \pm 0.06$ which is very close the PMC result. Note that these two methods are independent of the adopted mixing-length parameter.

c) Period-Magnitude-Amplitude relation

As a further independent method to estimate the distance modulus, let us consider the period-magnitude-amplitude (PMA) of fundamental pulsators, bearing in mind that the result is now depending on the mixing-length parameter. Adopting again $< M(RR) >= 0.67 \pm 0.02\, M_\odot$, the predicted PMA relation with $l/H_p = 1.5$ and $l/H_p = 2.0$ yields

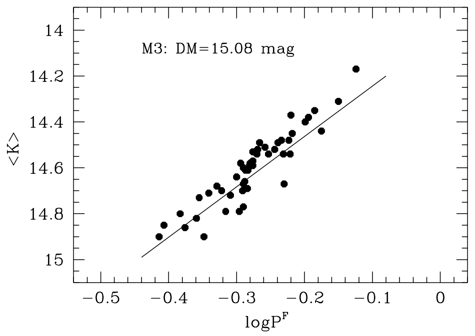

Fig. 4. Near-infrared magnitudes of RR Lyrae stars in M3 as a function of period. The solid line is the predicted slope at constant mass and luminosity.

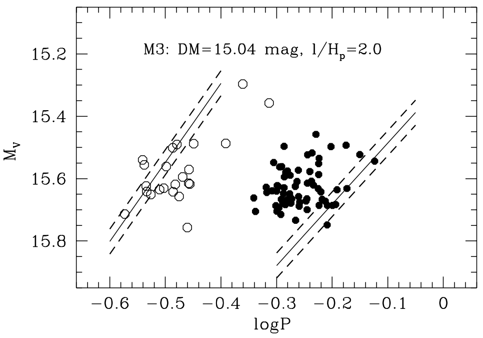

Fig. 6. RR Lyrae stars in M3 in the V-log$P$ plane in comparison with the predicted edges of the instability strip at l/H_p = 2.0, as corrected with the labelled distance modulus.

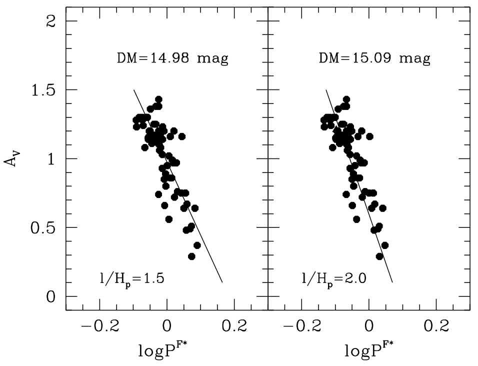

Fig. 5. Period-amplitude diagram of RR$_{ab}$ in M3 in comparison with the predicted relations (solid lines) under different assumptions on the mixing-length parameter. The labelled distance moduli are the resulting values adopting an average mass $< M(RR) >= 0.67 \pm 0.02\ M_\odot$.

$DM(PMA)$ = 14.98 ± 0.08 mag and 15.09 ± 0.07 mag, respectively (see Fig. 5).

d) Edges of the instability strip

Figure 6 finally illustrates the comparison between the observed distribution of RR Lyrae stars in the V-log P plane and the predicted edges of the instability strip, as inferred by SHB simulations with l/H_p = 2.0 (see Fig. 1). A nice agreement between predictions and observations is attained adopting DM=15.04 mag, even though just a couple of c-type variables turns out be in the hot stable region. As far as the case l/H_p = 1.5 is concerned, one would derive DM = 15.05 mag from the observed distribution of c-type variables and 14.90 mag from that of ab-type variables.

In summary, the assumption l/H_p = 1.5 yields rather discordant results, while a quite close agreement among the various approaches is reached with l/H_p = 2.0. However, an even better consistency is reached adopting that the mixing-length parameter increases from FOBE (l/H_p = 1.5) to FRE (l/H_p = 2.0). As a result, the *pulsational* distance modulus of M3, as given by the weighted mean over the above results, is DM = 15.07 ± 0.05 mag, at least adopting RR Lyrae masses from C03 HB models and CGK bolometric corrections.

In order to compare this result with the classical *evolutionary route*, let us go back to Fig. 1, and to the lower envelope of the absolute magnitudes of synthetic pulsators based on C03 HB models, $M_{V,le}$ = 0.55 mag. Figure 7 shows that the actual lower envelope for the M3 variables is V = 15.7 mag (solid line), thus providing an *evolutionary* distance modulus DM = 15.15 mag which is 0.08 ± 0.05 mag longer than the *pulsational* value. It is important to note that such a small but equally disturbing discrepancy *does not depend on the adopted HB models, nor on the adopted bolometric correction*. In fact, were the VDB models used, but maintaining CGK bolometric corrections, then $M_{V,le}$ = 0.58 mag,

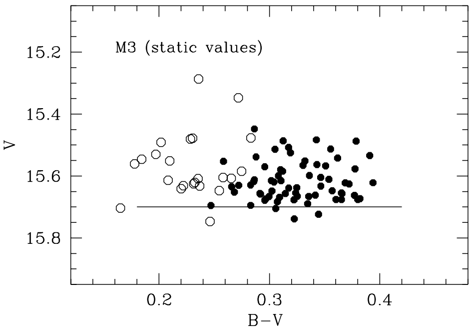

Fig. 7. Color-magnitude diagram of RR Lyrae stars in M3. The solid line is the observed magnitude of the lower envelope.

leading to an *evolutionary* distance modulus DM = 15.12 mag. On the other hand, the *pulsational* distance modulus of M3 becomes $DM = 15.04 \pm 0.05$ mag, which is again 0.08 ± 0.05 mag shorter with respect to the evolutionary predictions (i.e. DM = 15.15 mag). As for the effects of different bolometric corrections, they modify the absolute magnitudes, and then the derived distance modulus, but leaves unvaried the relative values.

On this issue, it is worth mentioning that there is a significant difference between CGK values and those adopted in VDB models, in the sense that the former give absolute magnitudes brighter by $\sim$ 0.05-0.06 mag, at fixed luminosity and effective temperature. In summary, the distance modulus of M3, as based on pulsational models and mass constrained by stellar evolution theory, varies from 15.00 mag (VDB computations and bolometric corrections) to 15.04 mag (VDB computations and CGK bolometric corrections) to 15.07 mag (C03 computations and CGKa,b bolometric corrections), all with a total error of ±0.05 mag. It follows that updated HB computations do yield consistent results, *provided that similar bolometric corrections are used*. Conversely, current uncertainty on the BC scale require some firm solutions.

Independently of the adopted BC correction, the pulsational approach would suggest that recent HB models are over-luminous by 0.08 ± 0.05 mag. However, a clear way to reduce the HB luminosity is offered by the diffusion of Helium and metals. As shown by Cassisi et al. (1999), if element diffusion is properly taken into account, then the luminosity of HB models at the RR Lyrae gap decreases by 0.03-0.04 mag.

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