White dwarf cosmochronology and M4

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Abstract. We discuss the accuracy of the present theoretical models of white dwarfs (WDs). We find that for ages higher than about 6-7 Gyr, the uncertainty in the predicted cooling times is of the order of 25\%. The bulk of this uncertainty is due to the chemical profile in the Carbon/Oxygen core and the treatment of the conductive opacity. As a first application of our WD models, we chose the globular cluster M4, for which very deep HST images are available. Notwithstanding the very high quality, the quoted data do not reach the peak of the WD luminosity function, thence allowing one to set a lower limit to the age of M4 of $\sim$9 Gyr. No reliable upper limit can however be set. In order to achieve this goal one needs observations several magnitudes deeper than the limit obtained so far.

Key words. Stars: evolution – white dwarf; Globular clusters: age

1. Introduction

As recognized several decades ago, white dwarfs (WDs) can be adopted as cosmic clocks. However, the extension of this method to the globular clusters (GCs) has remained thus far just a dream due to both the extreme faintness of the WDs in GCs and the complexity of high density matter behavior (Fontaine, Brassard & Bergeron 2001). The advent of the HST has significantly improved the situation, as witnessed for instance by the very deep images of the GC M4 obtained with the WFPC2 camera in 2001. A first analysis of this data set (Richer et al. 2002) produced the result that the age of M4 is 12.7$\pm$0.7 Gyr (Hansen et al. 2002). However, as discussed by Prada Moroni & Straniero 2002, the current theoretical uncertainty in the predicted cooling times is of the order of 25\% for ages suitable for GCs. Furthermore, the quoted data do not reach the peak of the WD luminosity function, thence allowing one to only set a lower limit to the age of the cluster. In order to set a reliable upper limit, one needs observations several magnitudes deeper than the limit obtained so far (see e.g. De Marchi et al. 2003 for a more detailed analysis).

2. Main theoretical uncertainties

In a recent work (Prada Moroni & Straniero 2002) we analyzed the uncertainties due to the assumptions about the basic ingredients (equa-
Fig. 1. Theoretical cooling sequences for a C/O DA WD of 0.6 $M_\odot$. Left panel: Cooling sequences obtained for the two extreme central Carbon abundances, namely $X_{12C}=$0.15 (dashed line) and $X_{12C}=$0.51 (solid line). Right panel: Theoretical models under different prescriptions for the conductive opacity: I93 (dot-dashed line); HL69 (dashed line); I93 for the fully degenerate regime ($\theta \leq 0.1$) and HL69 for the partially degenerate regime (solid line); Potekhin et al. 1999 (dotted line).

Fig. 2. Comparison between the empirical and the theoretical WD LFs for $(m - M)_{814}=12.25$. Left panel: theoretical LFs with fixed slope of the IMF (dN/dm $\propto m^{-\alpha}$ with $\alpha=2.3$) and different ages in the range 8-14 Gyr. Right panel: theoretical LFs with fixed age (12 Gyr) and different slopes of the IMF.

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C/O abundances in the core are quite uncertain due to the poor knowledge of both the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate (Buchmann et al. 1996) and the convective mixing efficiency in the He-burning phase (Straniero et al. 2003). Note that any mechanism that increases the size of the well mixed region during the final part of the He-burning (mechanical overshoot, semiconvection, breathing pulses or rotational induced mixing) leads to a reduction of the resulting amount of Carbon in the central region of the WD (Imbriani et al. 2001). The combined effect of the uncertainty of the convective theory with the error on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate produces theoretical predictions for the central Carbon mass fraction in white dwarfs in a quite large range. As an example, theoretical models predict values between $X_{^{12}\text{C}} = 0.15$ and 0.51 for a $0.6\, M_\odot$ DA WD. Figure 1 (left panel) shows the cooling evolution corresponding to these two extreme central Carbon abundances. At the faint end of the cooling sequence ($\log L / L_\odot \sim -5.5$) the global uncertainty on the predicted age is about 9%. As is well known, in the highly degenerate regime typical of WD interior the energy transport is dominated by electron conduction. Figure 1 (right panel) shows the extreme sensitivity of the cooling rate on the adopted conductive opacities tables. The Hubbard & Lampe (1969, HL69) should not be used for the C/O core due to a dated treatment of the crystallization. On the other hand, the more recent opacities by Itoh and coworkers (Itoh et al. 1983, Mitake et al. 1984, Itoh et al. 1993, hereinafter I93) are accurate only in the high-degenerate regime typical of WD interior the energy transport is dominated by electron conduction. Figure 1 also shows a model with a combination of the two previous opacities. The test shows that the discrepancy mainly concerns the conductive opacity in the weakly-degenerate regime.

### 3. The WD cooling sequence of M4

The data used in this contribution have been obtained with the WFPC2 as part of programme 8679 and are described in Richer et al. (2002). The target is a region located at $\sim 5'$ E of the nominal center of M4 and has been imaged through the F606W and the F814W filters. We adopted the data reduced by De Marchi et al. 2003 which show a WD sequence extending down to the $5\sigma$ detection limit at $m_{814}=26.8$, where the completeness reaches the 50% level.

The cooling sequence is abruptly terminated at exactly this limit as expected by detection statistic. The best way to date a cluster with WDs is to use their luminosity function (LF). Several sources of uncertainty come into play here. Firstly, the theoretical WD LF depends on the relationship between the mass of the progenitor and the mass of the WD, the time spent by the progenitor on the MS phase, the initial mass function (IMF) of the progenitors and the cooling time of the WD. The shape of the WD LF is particularly sensitive to the slope of the IMF, although the same is not true for the location of its peak. A sharp maximum is indeed expected in the WD LF of old stellar systems, a characteristic pile up that marks their age (Fontaine et al. 2001). Note that only when photometry is deep enough to reveal the peak of the WD LF one can safely constrain the cluster’s age. Otherwise, one can only provide a lower limit to it. Secondly, the WD LF is only meaningful if it is cleaned of as many contaminating field stars as possible. The CMD of M4 leaves no doubt that the contamination is strong. To clean the WD sequence one can use proper motion information derived from previous epochs of observation. Unfortunately, the only relevant observations in this case, conducted with the WFPC2 in 1995, are considerably less deep than those taken in 2001 and, therefore, the magnitude range over which one
can study the WD LF with some reliability is further reduced. By relaxing the detection limit to $3\sigma$ for the first epoch alone, one can lower the limit to $m_{814} \leq 26.5$. Below such a threshold, most stars detected in the second epoch do not have a corresponding match in the first one. Figure 2 shows the comparison between the observed WD LF of M4 and the theoretical predictions for fixed IMF, namely the Salpeter, and ages in the range 8-14 Gyr (left panel) and for fixed age, namely 12 Gyr, and different IMF slopes (right panel). The agreement is quite good, in all cases except 8 Gyr. These figures show that the available data do not allow to distinguish between different IMF slopes and ages.

The good match between theory and observation provides an important confirmation of the validity of the present description of the complex physical phenomena operating in the WDs. Unfortunately, little can be concluded from this data set about the M4’s age. In fact, as shown in figure 2, one can safely conclude only that M4 is older than 9-10 Gyr. No evidence of a maximum of the WD LF is obtained and, in turn, no upper limit for the age of M4 can be set on the basis of these data (see e.g. De Marchi et al. 2003 for a more detailed discussion).

References